2 Let $f(x) = 2x^3 - 5x^2 + 4$.

(a) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\frac{4}{5 - 2x_n}}$$

converges, then it converges to a root of the equation $f(x) = 0$. [2]

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(b) The equation has a root close to 1.2 .

Use the iterative formula from part **(a)** and an initial value of 1.2 to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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5 **(a)** It is given that the equation $e^{2x} = 5 + \cos 3x$ has only one root.

Show by calculation that this root lies in the interval $0.7 < x < 0.8$. [2]

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(b) Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(5 + \cos 3x_n\right)$$

converges then it converges to the root of the equation in part **(a)**. [1]

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(c) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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6 The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α.

(a) Show by calculation that α lies between 0.5 and 1. [2]

(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\left(\frac{1}{3x_n}\right)\right)$$

converges, then it converges to α. [2]

(c) Use this iterative formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(a)** By sketching a suitable pair of graphs, show that the equation $x^5 = 2 + x$ has exactly one real root. [2]

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(b) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{4x_n^5 + 2}{5x_n^4 - 1}$$

converges, then it converges to the root of the equation in part **(a)**. [2]

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(c) Use the iterative formula with initial value $x_1 = 1.5$ to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

2 **(a)** By sketching a suitable pair of graphs, show that the equation $\cot 2x = \sec x$ has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(b) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\tan^{-1}(\cos x_n)$$

converges, then it converges to the root in part **(a)**. [1]

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5 **(a)** By sketching a suitable pair of graphs, show that the equation $\ln x = 3x - x^2$ has one real root. [2]

(b) Verify by calculation that the root lies between 2 and 2.8. [2]

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(c) Use the iterative formula $x_{n+1} = \sqrt{3x_n - \ln x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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3 **(a)** By sketching a suitable pair of graphs, show that the equation $\sec x = 2 - \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \frac{1}{2}\pi$. [2]

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(b) Verify by calculation that this root lies between 0.8 and 1. [2]

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(c) Use the iterative formula $x_{n+1} = \cos^{-1}\left(\dfrac{2}{4 - x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 2 - \cos x$$

has one root in the interval $0 < x \leqslant \frac{1}{2}\pi$. [2]

(b) Show by calculation that this root lies between 0.6 and 0.8. [2]

(c) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\dfrac{1}{2 - \cos x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\sqrt{x} = e^x - 3$$

has only one root. [2]

(b) Show by calculation that this root lies between 1 and 2. [2]

(c) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \ln\left(3 + \sqrt{x_n}\right)$$

converges, then it converges to the root of the equation in **(a)**. [1]

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(d) Use the iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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5 **(a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

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(b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1}\left(\frac{1}{e^{-\frac{1}{2}x_n} + 1}\right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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5 **(a)** By sketching a suitable pair of graphs, show that the equation $2 + e^{-0.2x} = \ln(1 + x)$ has only one root. [2]

(b) Show by calculation that this root lies between 7 and 9. [2]

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(c) Use the iterative formula

$$x_{n+1} = \exp\left(2 + e^{-0.2x_n}\right) - 1$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[exp(x) is an alternative notation for e^x.] [3]

7 **(a)** By sketching a suitable pair of graphs, show that the equation $4 - x^2 = \sec \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 2. [2]

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(c) Use the iterative formula $x_{n+1} = \sqrt{4 - \sec \frac{1}{2}x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 **(a)** By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leqslant \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 1.5. [2]

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(c) Use the iterative formula $x_{n+1} = 2\tan^{-1}\left(\dfrac{1}{1+\mathrm{e}^{-x_n}}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec}\frac{1}{2}x = e^x - 3$ has exactly one root, denoted by α, in the interval $0 < x < \pi$. [2]

(b) Verify by calculation that α lies between 1 and 2. [2]

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(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula

$$x_{n+1} = \ln(\operatorname{cosec} \tfrac{1}{2}x_n + 3)$$

converges, then it converges to α. [1]

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(d) Use this iterative formula with an initial value of 1.4 to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(e) State the minimum number of calculated iterations needed with this initial value to determine α correct to 2 decimal places. [1]

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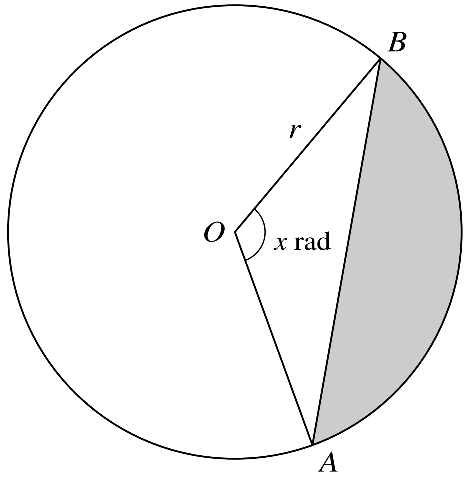

The diagram shows a circle with centre O and radius r. The angle of the **minor** sector AOB of the circle is x radians. The area of the **major** sector of the circle is 3 times the area of the shaded region.

(a) Show that $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$. [4]

(b) Show by calculation that the root of the equation in **(a)** lies between 2 and 2.5. [2]

(c) Use an iterative formula based on the equation in **(a)** to calculate this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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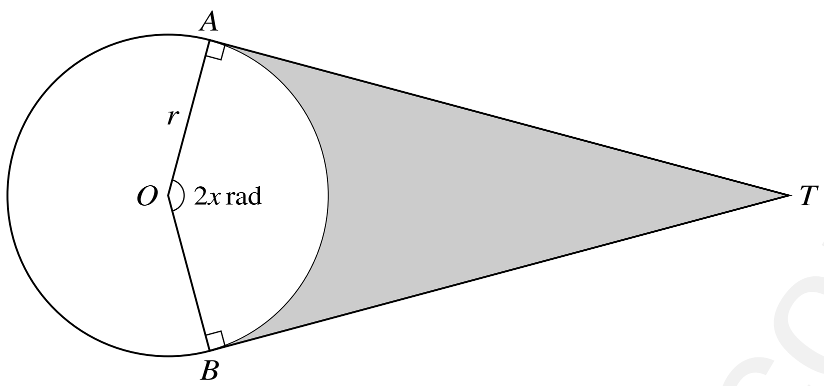

The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The area of the shaded region is equal to the area of the circle.

(a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

(b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

(c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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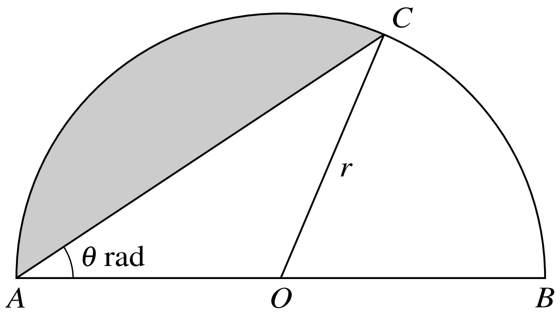

The diagram shows a semicircle with diameter AB, centre O and radius r. The shaded region is the minor segment on the chord AC and its area is one third of the area of the semicircle. The angle CAB is θ radians.

(a) Show that $\theta = \frac{1}{3}(\pi - 1.5\sin 2\theta)$. [4]

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(b) Verify by calculation that $0.5 < \theta < 0.7$. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine θ correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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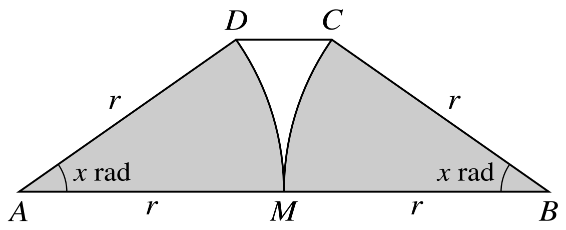

The diagram shows a trapezium $ABCD$ in which $AD = BC = r$ and $AB = 2r$. The acute angles BAD and ABC are both equal to x radians. Circular arcs of radius r with centres A and B meet at M, the midpoint of AB.

(a) Given that the sum of the areas of the shaded sectors is 90% of the area of the trapezium, show that x satisfies the equation $x = 0.9(2 - \cos x)\sin x$. [3]

(b) Verify by calculation that x lies between 0.5 and 0.7. [2]

(c) Show that if a sequence of values in the interval $0 < x < \frac{1}{2}\pi$ given by the iterative formula

$$x_{n+1} = \cos^{-1}\left(2 - \frac{x_n}{0.9\sin x_n}\right)$$

converges, then it converges to the root of the equation in part **(a)**. [2]

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(d) Use this iterative formula to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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