

- 3 (a) Given that $\cos(x - 30^\circ) = 2 \sin(x + 30^\circ)$, show that $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$.

[4]

$$\begin{aligned} \cos(x - 30^\circ) &= 2 \sin(x + 30^\circ) \\ \cos x \cos 30^\circ + \sin x \sin 30^\circ &= 2 [\sin x \cos 30^\circ + \cos x \sin 30^\circ] \\ \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x &= 2 \left(\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \right) \\ \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x &= \sqrt{3} \sin x + \cos x \\ \sqrt{3} \cos x + \sin x &= 2\sqrt{3} \sin x + 2 \cos x \\ \sin x - 2\sqrt{3} \sin x &= 2 \cos x - \sqrt{3} \cos x \\ \sin x (1 - 2\sqrt{3}) &= \cos x (2 - \sqrt{3}) \\ \frac{\sin x}{\cos x} &= \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}} \\ \tan x &= \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}} \quad \text{QED} \end{aligned}$$

- (b) Hence solve the equation

$$\cos(x - 30^\circ) = 2 \sin(x + 30^\circ),$$

for $0^\circ < x < 360^\circ$.

[2]

from (a): $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$

$$\begin{aligned} x &= \tan^{-1} \left(\frac{2 - \sqrt{3}}{1 - 2\sqrt{3}} \right) \\ &= -6.2^\circ \end{aligned}$$

STO

$x = 173.8^\circ, 353.8^\circ$

- 2 Solve the equation $3 \cos 2\theta = 3 \cos \theta + 2$, for $0^\circ \leq \theta \leq 360^\circ$.

[5]

$$3 \cos 2\theta = 3 \cos \theta + 2$$

$$3(2 \cos^2 \theta - 1) = 3 \cos \theta + 2$$

$$6 \cos^2 \theta - 3 = 3 \cos \theta + 2$$

$$6 \cos^2 \theta - 3 \cos \theta - 5 = 0$$

let $y = \cos \theta$:

$$6y^2 - 3y - 5 = 0$$

$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(6)(-5)}}{2(6)}$$

$$y = \frac{3 + \sqrt{129}}{12}$$

$$\text{or } y = \frac{3 - \sqrt{129}}{12}$$

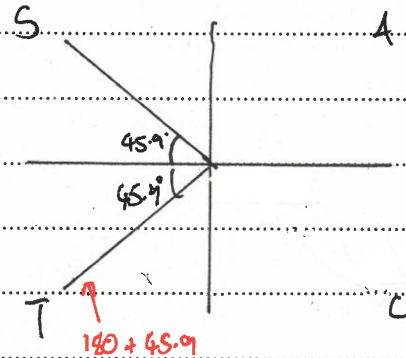
$$\cos \theta = \frac{3 + \sqrt{129}}{12}$$

$$\text{or } \cos \theta = \frac{3 - \sqrt{129}}{12}$$

X no sol's

$$\theta = 134.1^\circ$$

STO



$$\theta = \underline{134.1^\circ}, \underline{225.9^\circ}$$

- 4 By first expressing the equation $\tan(x-60^\circ) = 2 \cot x$ as a quadratic equation in $\tan x$, solve the equation for $0^\circ \leq x \leq 180^\circ$. [6]

$$\tan(x-60) = 2 \times \frac{1}{\tan x}$$

$$\frac{\tan x - \tan 60}{1 + \tan x \tan 60} = \frac{2}{\tan x}$$

$$\frac{\tan x - \sqrt{3}}{1 + \sqrt{3} \tan x} = \frac{2}{\tan x}$$

$$\tan x (\tan x - \sqrt{3}) = 2(1 + \sqrt{3} \tan x)$$

$$\tan^2 x - \sqrt{3} \tan x = 2 + 2\sqrt{3} \tan x$$

$$\tan^2 x - 3\sqrt{3} \tan x - 2 = 0$$

Let $Y = \tan x$:

$$Y^2 - 3\sqrt{3} Y - 2 = 0$$

$$Y = \frac{3\sqrt{3} \pm \sqrt{(-3\sqrt{3})^2 - 4(1)(-2)}}{2(1)}$$

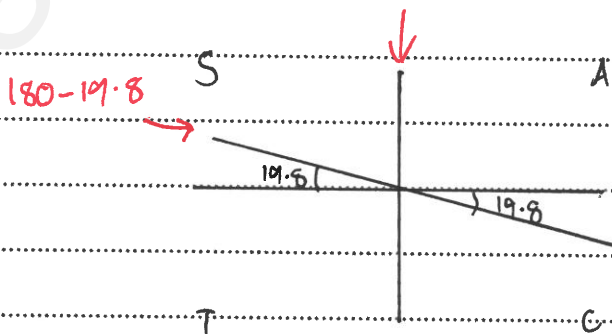
$$Y = 5.556 \dots \text{ or } Y = -0.3599 \dots$$

$$\tan x = 5.556 \dots$$

$$\tan x = -0.3599 \dots$$

$$x = 79.8^\circ$$

$$x = -19.8^\circ$$



$$x = 160.2^\circ$$

$$\rightarrow x = 79.8^\circ, 160.2^\circ$$



- 5 By first expressing the equation

$$\tan \theta \tan(\theta + 45^\circ) = 2 \cot 2\theta$$

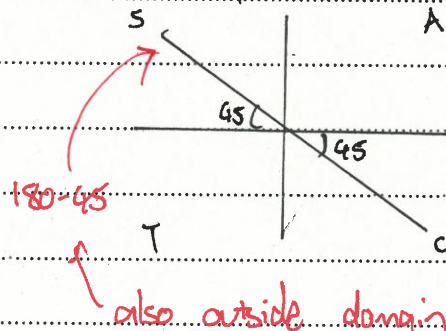
as a quadratic equation in $\tan \theta$, solve the equation for $0^\circ < \theta < 90^\circ$.

[6]

$$\begin{aligned} \tan \theta \tan(\theta + 45^\circ) &= 2 \cot 2\theta \\ \tan \theta \times \frac{\tan \theta + \tan 45^\circ}{1 - \tan \theta \tan 45^\circ} &= 2 \times \frac{1}{\tan 2\theta} \quad \leftarrow \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ \tan \theta \times \frac{\tan \theta + 1}{1 - \tan \theta} &= 2 \times \frac{1 - \tan^2 \theta}{2 \tan \theta} \\ \frac{\tan^2 \theta + \tan \theta}{1 - \tan \theta} &= \frac{2 - 2 \tan^2 \theta}{2 \tan \theta} \\ 2 \tan \theta (\tan^2 \theta + \tan \theta) &= (1 - \tan \theta)(2 - 2 \tan^2 \theta) \\ 2 \tan^3 \theta + 2 \tan^2 \theta &= 2 - 2 \tan^2 \theta - 2 \tan \theta + 2 \tan^3 \theta \\ 4 \tan^2 \theta + 2 \tan \theta - 2 &= 0 \\ 2 \tan^2 \theta + \tan \theta - 1 &= 0 \\ \text{let } y = \tan \theta: \quad 2y^2 + y - 1 &= 0 \\ (2y - 1)(y + 1) &= 0 \\ y = \frac{1}{2} \quad \text{or} \quad y = -1 \\ \tan \theta = \frac{1}{2} \quad \text{or} \quad \tan \theta = -1 \end{aligned}$$

$$\theta = 26.6^\circ$$

$$\theta = -45^\circ$$



- 4 (a) Show that the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$ can be written in the form

$$\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0.$$

[3]

$$\begin{aligned} \tan(\theta + 60^\circ) &= 2 \cot \theta \\ \frac{\tan \theta + \tan 60^\circ}{1 - \tan \theta \tan 60^\circ} &= \frac{2}{\tan \theta} \\ \frac{\tan \theta + \sqrt{3}}{1 - \sqrt{3} \tan \theta} &\neq \frac{2}{\tan \theta} \end{aligned}$$

$$\begin{aligned} \tan \theta (\tan \theta + \sqrt{3}) &= 2(1 - \sqrt{3} \tan \theta) \\ \tan^2 \theta + \sqrt{3} \tan \theta &= 2 - 2\sqrt{3} \tan \theta \\ \tan^2 \theta + 3\sqrt{3} \tan \theta - 2 &= 0 \quad \text{QED} \end{aligned}$$

(b) Hence solve the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$, for $0^\circ < \theta < 180^\circ$.

[3]

from part (a): $\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0$

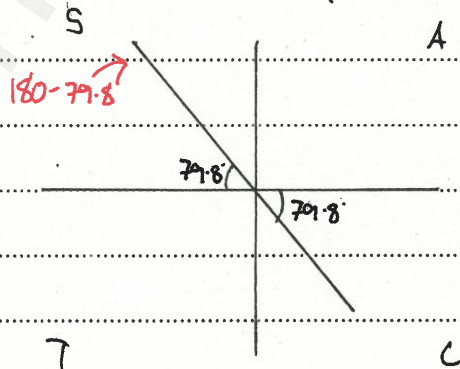
let $y = \tan \theta$: $y^2 + 3\sqrt{3}y - 2 = 0$

$$y = \frac{-(3\sqrt{3}) \pm \sqrt{(3\sqrt{3})^2 - 4(1)(-2)}}{2(1)}$$

$$y = \frac{\sqrt{35} - 3\sqrt{3}}{2} \quad \text{or} \quad y = \frac{-\sqrt{35} - 3\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{35} - 3\sqrt{3}}{2} \quad \text{or} \quad \tan \theta = \frac{-\sqrt{35} - 3\sqrt{3}}{2}$$

$$\theta = 19.8^\circ \quad \text{or} \quad \theta = -79.8^\circ$$



$$\theta = \underline{19.8^\circ}, \underline{100.2^\circ}$$

- 5 (a) Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in $\tan \theta$.

[3]

$$\frac{1}{\tan 2\theta} + \frac{1}{\tan \theta} = 2$$

$$\frac{1 - \tan^2 \theta}{2 \tan \theta} + \frac{1}{\tan \theta} = 2$$

$\times 2 \tan \theta$ $\times 2 \tan \theta$ $\times 2 \tan \theta$

$$1 - \tan^2 \theta + 2 = 4 \tan \theta$$

$$3 - \tan^2 \theta = 4 \tan \theta$$

$$-\tan^2 \theta - 4 \tan \theta + 3 = 0$$

$$\tan^2 \theta + 4 \tan \theta - 3 = 0$$

QED

- (b) Hence solve the equation $\cot 2\theta + \cot \theta = 2$, for $0 < \theta < \pi$, giving your answers correct to 3 decimal places.

[3]

$$\tan^2 \theta + 4 \tan \theta - 3 = 0$$

let $y = \tan \theta$: $y^2 + 4y - 3 = 0$

$$y = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$$

$$y = -2 + \sqrt{7} \quad \text{or} \quad y = -2 - \sqrt{7}$$

$$\tan \theta = -2 + \sqrt{7} \quad \tan \theta = -2 - \sqrt{7}$$

$$\theta = 0.573 \quad \theta = -1.36$$

outside domain

$$\theta = 0.573 \quad \theta = 1.783$$

- 6 (a) Show that the equation $\cot^2 \theta + 2 \cos 2\theta = 4$ can be written in the form

$$4 \sin^4 \theta + 3 \sin^2 \theta - 1 = 0.$$

[3]

$$\frac{\cos^2 \theta}{\sin^2 \theta} + 2 \cos 2\theta = 4$$

$$\cos^2 \theta + 2 \cos 2\theta \times \sin^2 \theta = 4 \sin^2 \theta$$

$$\cos^2 \theta + 2(1 - 2\sin^2 \theta) \sin^2 \theta = 4 \sin^2 \theta$$

$$1 - \sin^2 \theta + 2\sin^2 \theta - 4\sin^4 \theta = 4\sin^2 \theta$$

$$1 + \sin^2 \theta - 4\sin^4 \theta = 4\sin^2 \theta$$

$$1 - 3\sin^2 \theta - 4\sin^4 \theta = 0$$

 $\times -1$ $\times -1$ $\times -1$ $\times -1$

$$-1 + 3\sin^2 \theta + 4\sin^4 \theta = 0$$

$$\underline{4\sin^4 \theta + 3\sin^2 \theta - 1 = 0} \quad \text{QED}$$

(b) Hence solve the equation $\cot^2 \theta + 2 \cos 2\theta = 4$, for $0^\circ < \theta < 360^\circ$.

[3]

from (a): $4 \sin^4 \theta + 3 \sin^2 \theta - 1 = 0$

Let $Y = \sin^2 \theta$:

$$4Y^2 + 3Y - 1 = 0$$

$$(4Y - 1)(Y + 1) = 0$$

$$4Y - 1 = 0 \quad \text{or} \quad Y + 1 = 0$$

$$Y = \frac{1}{4}$$

$$Y = -1$$

$$\sin^2 \theta = \frac{1}{4}$$

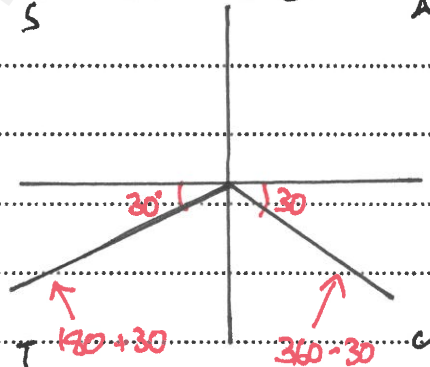
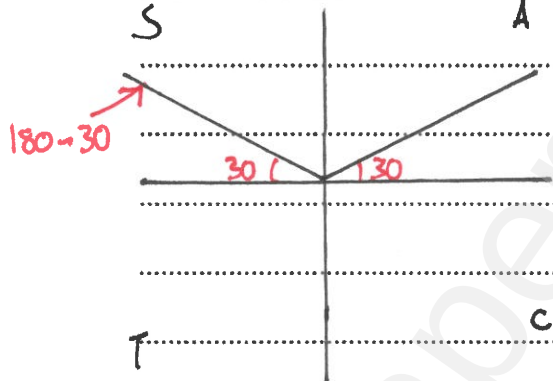
$$\sin^2 \theta = -1$$

X no solns

$$\sin \theta = \frac{1}{2} \quad \text{or} \quad \sin \theta = -\frac{1}{2}$$

$$\theta = 30^\circ$$

$$\theta = -30^\circ$$



$$\theta = 30^\circ, 150^\circ$$

$$\theta = 210^\circ, 330^\circ$$

$$\rightarrow \underline{\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ}$$

- 6 (a) Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4\theta$.

[4]

LHS: $\cos 4\theta + 4\cos 2\theta + 3$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 4\theta = 2\cos^2 2\theta - 1$

$2\cos^2 2\theta - 1 + 4(2\cos^2\theta - 1) + 3$

$\cos 2\theta = 2\cos^2\theta - 1$

$2(2\cos^2\theta - 1)^2 - 1 + 4(2\cos^2\theta - 1) + 3$

$2(4\cos^4\theta - 4\cos^2\theta + 1) - 1 + 8\cos^2\theta - 4 + 3$

$8\cos^4\theta - 8\cos^2\theta + 2 + 8\cos^2\theta - 4 + 3$

$= \underline{8\cos^4\theta} = \text{RHS}$

QED

(b) Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 4$ for $0^\circ \leq \theta \leq 180^\circ$.

[3]

from part (a): $\cos 4\theta + 4\cos 2\theta = 8\cos^4\theta - 3$

Sub into new eqⁿ:

$$8\cos^4\theta - 3 = 4$$

$$8\cos^4\theta = 7$$

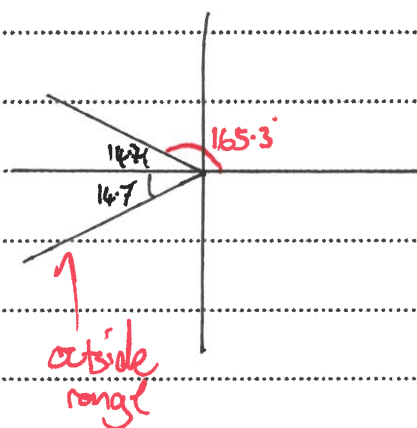
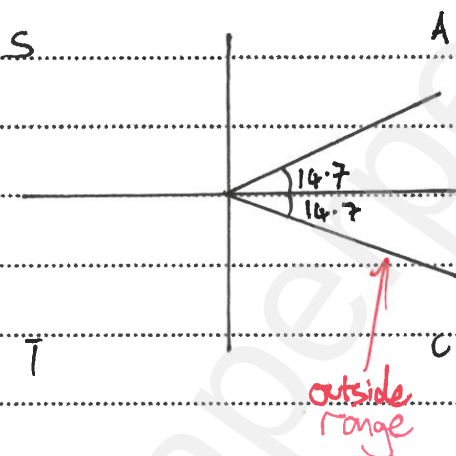
$$\cos^4\theta = \frac{7}{8}$$

$$\cos\theta = \sqrt[4]{\frac{7}{8}}$$

$$\text{or } \cos\theta = -\sqrt[4]{\frac{7}{8}}$$

$$\theta = 14.7^\circ$$

$$\theta = 165.3^\circ$$



$$\theta = \underline{14.7^\circ}, \underline{165.3^\circ}$$

3 Solve the equation $2 \cot 2x + 3 \cot x = 5$, for $0^\circ < x < 180^\circ$.

[6]

$$\frac{2 \cot 2x}{1 - \tan^2 x} \rightarrow \frac{2}{\tan 2x} + \frac{3}{\tan x} = 5$$

$$\frac{2}{\frac{2 \tan x}{1 - \tan^2 x}} + \frac{3}{\tan x} = 5$$

$$\frac{2(1 - \tan^2 x)}{2 \tan x} + \frac{3}{\tan x} = 5$$

$$\frac{1 - \tan^2 x}{\tan x} + \frac{3}{\tan x} = 5$$

$$\frac{4 - \tan^2 x}{\tan x} = 5$$

$$4 - \tan^2 x = 5 \tan x$$

$$\tan^2 x + 5 \tan x - 4 = 0$$

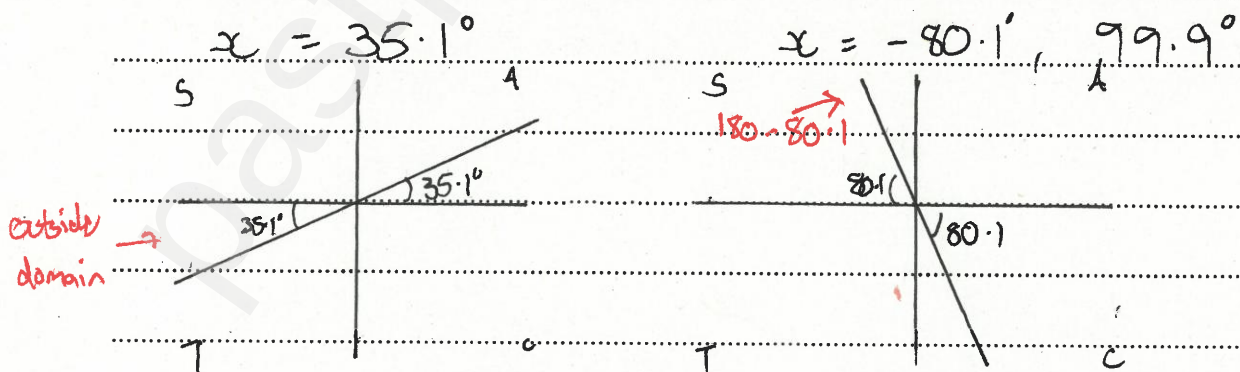
let $Y = \tan x$:

$$Y^2 + 5Y - 4 = 0$$

$$Y = \frac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2(1)}$$

$$Y = \frac{-5 + \sqrt{41}}{2} \quad \text{or} \quad Y = \frac{-5 - \sqrt{41}}{2}$$

$$\tan x = \frac{-5 + \sqrt{41}}{2} \quad \text{or} \quad \tan x = \frac{-5 - \sqrt{41}}{2}$$





- 7 (a) Show that the equation $\tan^3 x + 2 \tan 2x - \tan x = 0$ may be expressed as

$$\tan^4 x - 2 \tan^2 x - 3 = 0$$

for $\tan x \neq 0$.

[3]

$$\tan^3 x + 2 \tan 2x - \tan x = 0$$

$$\uparrow \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\tan^3 x + 2 \left(\frac{2 \tan x}{1 - \tan^2 x} \right) - \tan x = 0$$

$$\tan^3 x + \frac{4 \tan x}{1 - \tan^2 x} - \tan x = 0 \quad \times (1 - \tan^2 x)$$

$$(1 - \tan^2 x)(\tan^3 x) + 4 \tan x - (1 - \tan^2 x)(\tan x) = 0$$

$$\tan^3 x - \tan^5 x + 4 \tan x - \tan x + \tan^3 x = 0$$

$$- \tan^5 x + 2 \tan^3 x + 3 \tan x = 0 \quad \times -1$$

$$\tan^5 x - 2 \tan^3 x - 3 \tan x = 0$$

$$\tan x (\tan^4 x - 2 \tan^2 x - 3) = 0$$

$$\tan x = 0 \quad \text{or} \quad \tan^4 x - 2 \tan^2 x - 3 = 0$$

$$\times \tan x \neq 0$$

$$\rightarrow \underline{\tan^4 x - 2 \tan^2 x - 3 = 0} \quad \text{QED}$$



$$0 < 2\theta < 2\pi$$

- (b) Hence solve the equation $\tan^3 2\theta + 2\tan 4\theta - \tan 2\theta = 0$ for $0 < \theta < \pi$. Give your answers in exact form. [3]

from (a): $\tan^4 2\theta - 2\tan^2 2\theta - 3 = 0$

Let $Y = \tan^2 2\theta$:

$$Y^2 - 2Y - 3 = 0$$

$$(Y - 3)(Y + 1) = 0$$

$$Y = 3 \quad \text{or} \quad Y = -1$$

$$\tan^2 2\theta = 3$$

$$\tan^2 2\theta = -1$$

$$\tan 2\theta = \pm\sqrt{3}$$

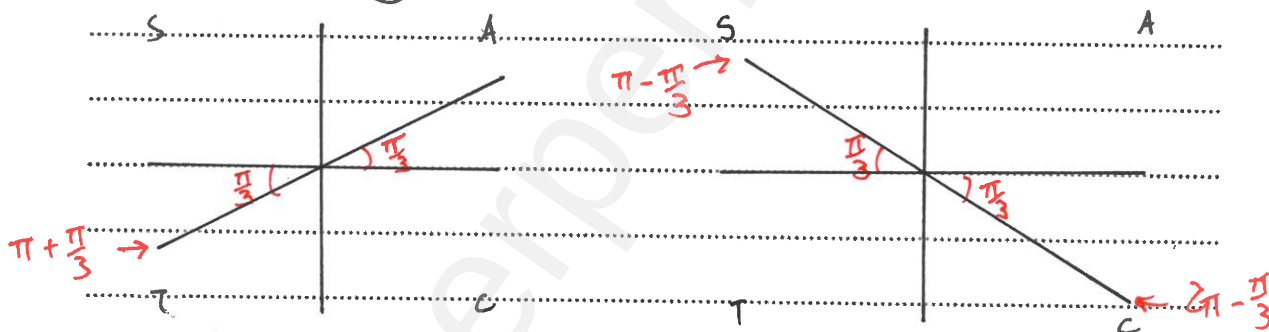
x No solns

$$\tan 2\theta = \sqrt{3}$$

$$2\theta = \frac{\pi}{3}$$

$$\tan 2\theta = -\sqrt{3}$$

$$2\theta = -\frac{\pi}{3}$$



$$2\theta = \frac{\pi}{3}, \frac{4\pi}{3}$$

$$2\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{6}, \frac{2\pi}{3}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$$

- 7 (a) By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$.

[3]

$$\begin{aligned}
 \cos 3\theta &= \cos(2\theta + \theta) \\
 &= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta \\
 &\quad \begin{array}{l} \text{red arrows: } 2\cos^2\theta - 1 \text{ and } 2\sin\theta\cos\theta \end{array} \\
 &= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta \times \sin\theta \\
 &= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta \\
 &\quad \text{red arrow: } 1 - \cos^2\theta \\
 &= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta \\
 &= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \\
 &= \underline{4\cos^3\theta - 3\cos\theta} \quad \text{QED}
 \end{aligned}$$

(b) Hence solve the equation

$$\cos 3\theta + \cos \theta \cos 2\theta = \cos^2 \theta$$

for $0^\circ \leq \theta \leq 180^\circ$.

[5]

from (a): $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$

$$\rightarrow 4\cos^3 \theta - 3\cos \theta + \cos \theta \cos 2\theta = \cos^2 \theta$$

$$\uparrow 2\cos^2 \theta - 1$$

$$4\cos^3 \theta - 3\cos \theta + \cos \theta (2\cos^2 \theta - 1) = \cos^2 \theta$$

$$4\cos^3 \theta - 3\cos \theta + 2\cos^3 \theta - \cos \theta = \cos^2 \theta$$

$$6\cos^3 \theta - 4\cos \theta = \cos^2 \theta$$

$$6\cos^3 \theta - \cos^2 \theta - 4\cos \theta = 0$$

$$\cos \theta (6\cos^2 \theta - \cos \theta - 4) = 0$$

$$\uparrow$$

$$\cos \theta = 0$$

$$\theta = 90^\circ$$

$$\text{let } Y = \cos \theta:$$

$$6Y^2 - Y - 4 = 0$$

$$Y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-4)}}{2(6)}$$

$$Y = \frac{1 + \sqrt{97}}{12}$$

$$\text{or } Y = \frac{1 - \sqrt{97}}{12}$$

$$\cos \theta = \frac{1 + \sqrt{97}}{12}$$

$$\cos \theta = \frac{1 - \sqrt{97}}{12}$$

$$\theta = 25.3^\circ$$

$$\theta = 137.5^\circ$$

$$\underline{\theta = 25.3^\circ, 90^\circ, 137.5^\circ}$$



5 (a) Show that $\cos^4 \theta - \sin^4 \theta - 4 \sin^2 \theta \cos^2 \theta \equiv \cos^2 2\theta + \cos 2\theta - 1$. [3]

LHS: $\cos^4 \theta - \sin^4 \theta - 4 \sin^2 \theta \cos^2 \theta$
 $= (\cos^2 \theta)^2 - (\sin^2 \theta)^2 - (2 \sin \theta \cos \theta)^2$
 $\quad \quad \quad \uparrow \cos 2\theta = 2 \cos^2 \theta - 1$
 $\quad \quad \quad \cos 2\theta + 1 = 2 \cos^2 \theta$
 $\quad \quad \quad \cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$
 $= \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta\right)^2 - (\sin^2 \theta)^2 - (2 \sin \theta \cos \theta)^2$
 $\quad \quad \quad \uparrow \sin 2\theta = 2 \sin \theta \cos \theta$
 $= \frac{1}{4} + \cos 2\theta + \frac{1}{4} \cos^2 2\theta - (\sin^2 \theta)^2 - (\sin 2\theta)^2$
 $\quad \quad \quad \uparrow \cos 2\theta = 1 - 2 \sin^2 \theta$
 $\quad \quad \quad 2 \sin^2 \theta = 1 - \cos 2\theta$
 $\quad \quad \quad \sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$
 $= \frac{1}{4} + \frac{1}{2} \cos 2\theta + \frac{1}{4} \cos^2 2\theta - \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta\right)^2 - (\sin 2\theta)^2$
 $= \frac{1}{4} + \frac{1}{2} \cos 2\theta + \frac{1}{4} \cos^2 2\theta - \left(\frac{1}{4} - \frac{1}{2} \cos 2\theta + \frac{1}{4} \cos^2 2\theta\right) - \sin^2 2\theta$
 $= \cancel{\frac{1}{4}} + \frac{1}{2} \cos 2\theta + \cancel{\frac{1}{4} \cos^2 2\theta} - \cancel{\frac{1}{4}} + \frac{1}{2} \cos 2\theta - \cancel{\frac{1}{4} \cos^2 2\theta} - \sin^2 2\theta$
 $= \cos 2\theta - \sin^2 2\theta$
 $\quad \quad \quad \uparrow \sin^2 A + \cos^2 A = 1$
 $\quad \quad \quad \sin^2 2\theta + \cos^2 2\theta = 1$
 $\quad \quad \quad \sin^2 2\theta = 1 - \cos^2 2\theta$
 $= \cos 2\theta - (1 - \cos^2 2\theta)$
 $= \cos 2\theta - 1 + \cos^2 2\theta$
 $= \underline{\cos^2 2\theta + \cos 2\theta - 1} \quad \text{QED}$





$$0^\circ \leq 2\alpha \leq 360^\circ$$



(b) Solve the equation $\cos^4 \alpha - \sin^4 \alpha = 4 \sin^2 \alpha \cos^2 \alpha$ for $0^\circ \leq \alpha \leq 180^\circ$.

[3]

Rearrange: $\cos^4 \alpha - \sin^4 \alpha - 4 \sin^2 \alpha \cos^2 \alpha = 0$

from (a): $\cos^2 2\alpha + \cos 2\alpha - 1 = 0$

Let $Y = \cos 2\alpha$:

$$Y^2 + Y - 1 = 0$$

$$Y = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$Y = \frac{-1 + \sqrt{5}}{2}$$

or

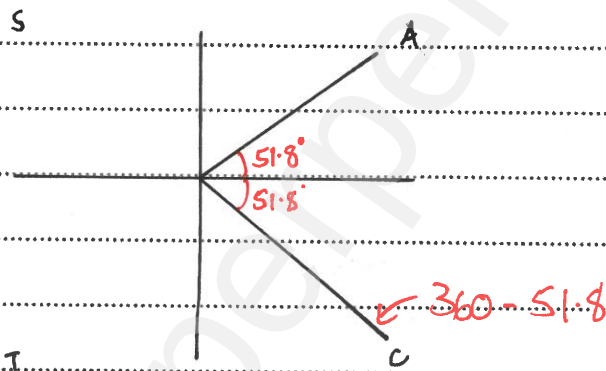
$$Y = \frac{-1 - \sqrt{5}}{2}$$

$$\cos 2\alpha = \frac{-1 + \sqrt{5}}{2}$$

$$\cos 2\alpha = \frac{-1 - \sqrt{5}}{2}$$

No sol's

$$2\alpha = 51.827^\circ$$



$$2\alpha = 51.827, 308.172$$

$$\alpha = \underline{\underline{25.9^\circ, 154.1^\circ}}$$

- 8 (a) By first expanding $(\cos^2 \theta + \sin^2 \theta)^2$, show that

$$\cos^4 \theta + \sin^4 \theta \equiv 1 - \frac{1}{2} \sin^2 2\theta.$$

[3]

$$(\cos^2 \theta + \sin^2 \theta)^2 = 1$$

$$(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) = 1$$

$$\cos^4 \theta + 2\sin^2 \theta \cos^2 \theta + \sin^4 \theta = 1$$

$$\cos^4 \theta + \sin^4 \theta = 1 - 2\sin^2 \theta \cos^2 \theta$$

$$\cos^4 \theta + \sin^4 \theta = 1 - 2(\sin \theta \cos \theta)^2$$

$$= 1 - 2\left(\frac{1}{2} \sin 2\theta\right)^2$$

$$= 1 - 2 \times \frac{1}{4} \sin^2 2\theta$$

$$= 1 - \frac{1}{2} \sin^2 2\theta$$

$$\cos^4 \theta + \sin^4 \theta = 1 - \frac{1}{2} \sin^2 2\theta \quad \underline{\text{QED}}$$

(b) Hence solve the equation

$$\cos^4 \theta + \sin^4 \theta = \frac{5}{9},$$

for $0^\circ < \theta < 180^\circ$.

[4]

from part (a): $1 - \frac{1}{2} \sin^2 2\theta = \frac{5}{9}$

$$-\frac{1}{2} \sin^2 2\theta = -\frac{4}{9}$$

$$\sin^2 2\theta = \frac{8}{9}$$

$$0 < \theta < 180$$

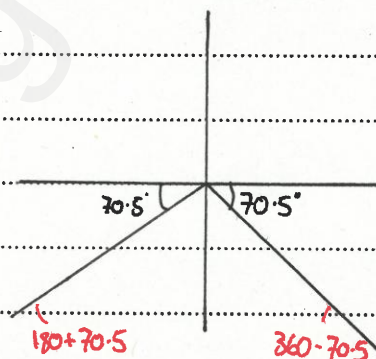
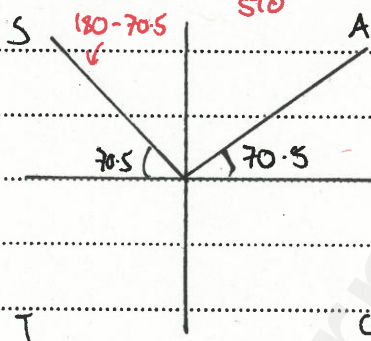
$$0 < 2\theta < 360$$

$$\sin 2\theta = \sqrt{\frac{8}{9}}$$

$$\text{or } \sin 2\theta = -\sqrt{\frac{8}{9}}$$

$$2\theta = 70.5^\circ$$

$$2\theta = -70.5^\circ$$



$$2\theta = 70.5^\circ, 109.5^\circ$$

$$\theta = 35.3^\circ, 54.7^\circ$$

$$2\theta = -70.5^\circ, 289.5^\circ, 299.5^\circ$$

$$\theta = -35.3^\circ, 125.3^\circ, 144.7^\circ$$

$$\theta = \underline{35.3^\circ, 54.7^\circ, 125.3^\circ, 144.7^\circ}$$