6 **(a)** Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\tfrac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x.$$ [3]

$\sin 5x$: $\sin(3x + 2x) = \sin 3x \cos 2x + \cos 3x \sin 2x$
+ + +
$\sin x$: $\sin(3x - 2x) = \sin 3x \cos 2x - \cos 3x \sin 2x$

$\sin 5x + \sin x = 2\sin 3x \cos 2x$

$\div 2$ $\div 2$

$\frac{1}{2}(\sin 5x + \sin x) = \sin 3x \cos 2x$ QED

7 (a) Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$. [3]

Change LHS to either just $\cos\theta$ or $\sin\theta$:

LHS: $\cos^4\theta - (\sin^2\theta)^2$

$\sin^2\theta = 1-\cos^2\theta$

$= \cos^4\theta - (1-\cos^2\theta)^2$

$= \cos^4\theta - (1 - 2\cos^2\theta + \cos^4\theta)$

$= \cancel{\cos^4\theta} - 1 + 2\cos^2\theta - \cancel{\cos^4\theta}$

$= 2\cos^2\theta - 1$

$= \underline{\cos 2\theta}$ QED

2 Solve the equation $\cos(\theta - 60°) = 3\sin\theta$, for $0° \leqslant \theta \leqslant 360°$. [5]

$$\cos(\theta - 60) = 3\sin\theta$$

$$\cos\theta\cos 60 + \sin\theta\sin 60 = 3\sin\theta$$

$$\underset{\times 2}{\tfrac{1}{2}\cos\theta} + \underset{\times 2}{\tfrac{\sqrt{3}}{2}\sin\theta} = \underset{\times 2}{3\sin\theta}$$

$$\cos\theta + \sqrt{3}\sin\theta = 6\sin\theta$$

$$\cos\theta = 6\sin\theta - \sqrt{3}\sin\theta$$

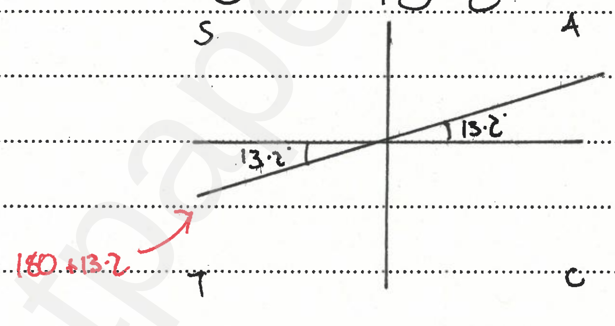
$$\cos\theta = (6 - \sqrt{3})\sin\theta$$

$$1 = (6 - \sqrt{3})\frac{\sin\theta}{\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{6 - \sqrt{3}}$$

$$\tan\theta = \frac{1}{6 - \sqrt{3}}$$

$$\theta = 13.2°$$



$$\theta = \underline{13.2°}, \underline{193.2°}$$

7 Let $f(x) = 8x^3 + 54x^2 - 17x - 21$.

(a) Show that $x + 7$ is a factor of $f(x)$. [1]

$f(-7) = 8(-7)^3 + 54(-7)^2 - 17(-7) - 21$

$= -2744 + 2646 + 119 - 21$

$= 0$

so $(x+7)$ is a factor

(b) Find the quotient when $f(x)$ is divided by $x + 7$. [2]

$$\begin{array}{r|l} & 8x^2 - 2x - 3 \\ x+7 & 8x^3 + 54x^2 - 17x - 21 \\ & \underline{8x^3 + 56x^2} \\ & \quad -2x^2 - 17x \\ & \quad \underline{-2x^2 - 14x} \\ & \qquad -3x - 21 \\ & \qquad \underline{-3x - 21} \end{array}$$

quotient : $8x^2 - 2x - 3$

(c) Hence solve the equation

$$8\cos^3\theta + 54\cos^2\theta - 17\cos\theta - 21 = 0,$$

for $0° \leqslant \theta \leqslant 360°$. [3]

Let $x = \cos\theta$:

$$8x^3 + 54x^2 - 17x - 21 = 0$$

$$(x+7)(8x^2 - 2x - 3) = 0$$

factorise: $ac = -24$; -6 and 4:

$8x^2 + 4x - 6x - 3$

$4x(2x+1) - 3(2x+1)$

$(4x-3)(2x+1)$

$\rightarrow (x+7)(4x-3)(2x+1) = 0$

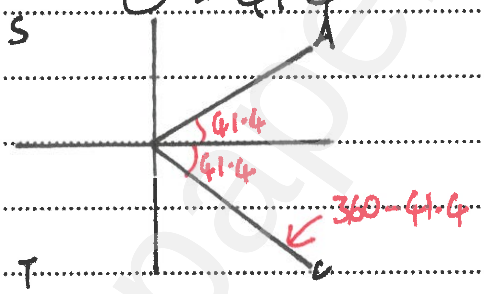
$x = -7 \quad x = \frac{3}{4} \quad x = \frac{-1}{2}$

$\cos\theta = -7 \quad \cos\theta = \frac{3}{4}$ ① $\quad \cos\theta = \frac{-1}{2}$ ②

✗ no sol^{n}s

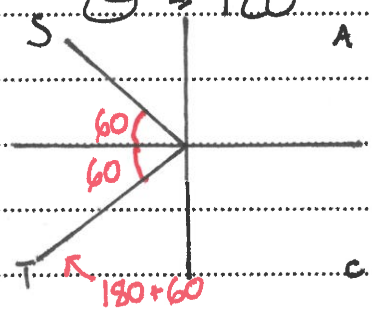
①: $\cos\theta = \frac{3}{4}$

$\theta = 41.4°$



②: $\cos\theta = \frac{-1}{2}$

$\theta = 120°$



$\theta = 41.4°, 318.6° \qquad \theta = 120°, 240°$

$\theta = 41.4°, 120°, 240°, 318.6°$

4 Solve the equation $2\cos x - \cos\frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

$\cos 2A = 2\cos^2 A - 1$

$\cos x = 2\cos^2(\frac{1}{2}x) - 1$

$2\cos x = 4\cos^2(\frac{1}{2}x) - 2$

$\rightarrow 4\cos^2(\frac{1}{2}x) - 2 - \cos(\frac{1}{2}x) = 1$

$4\cos^2(\frac{1}{2}x) - \cos(\frac{1}{2}x) - 3 = 0$

Let $Y = \cos(\frac{1}{2}x)$:

$4Y^2 - Y - 3 = 0$

$(4Y + 3)(Y - 1) = 0$

$4Y + 3 = 0$ or $Y - 1 = 0$

$4Y = -3$ $\quad$ $Y = 1$

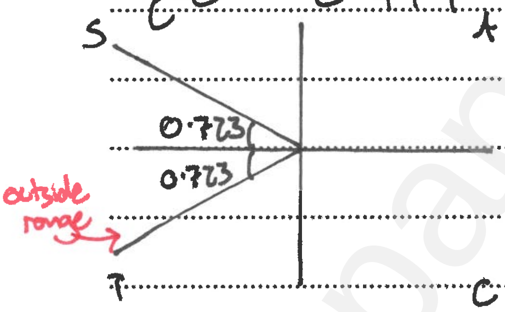
$Y = \frac{-3}{4}$

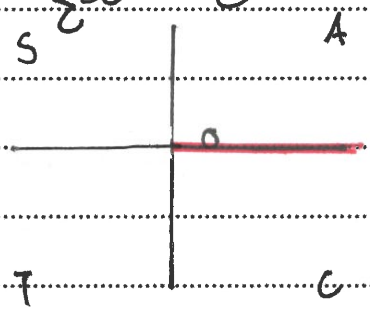
$\cos(\frac{1}{2}x) = \frac{-3}{4}$ or $\cos(\frac{1}{2}x) = 1$

$0 \leqslant x \leqslant 2\pi$

$0 \leqslant \frac{1}{2}x \leqslant \pi$

$\frac{1}{2}x = 2.419$ $\quad$ $\frac{1}{2}x = 0$





$\frac{1}{2}x = 0$ only

$\frac{1}{2}x = 0, 2.419$

$x = 0, 4.84$

$\tan(A \pm B)$

4 Solve the equation $\tan(x+45°) = 2\cot x$ for $0° < x < 180°$. [5]

$$\frac{\tan x + \tan 45}{1 - \tan x \tan 45} = \frac{2}{\tan x}$$

$$\frac{\tan x + 1}{1 - \tan x} = \frac{2}{\tan x}$$

$$\tan x(\tan x + 1) = 2(1 - \tan x)$$

$$\tan^2 x + \tan x = 2 - 2\tan x$$

$$\tan^2 x + 3\tan x - 2 = 0$$

let $y = \tan x$:

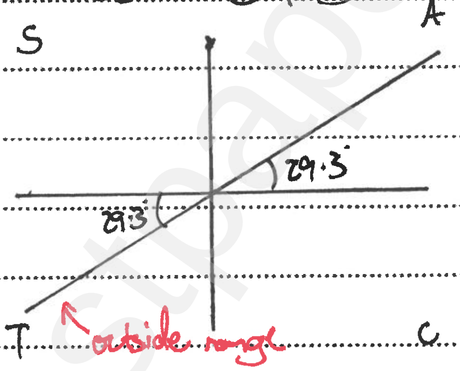
$$y^2 + 3y - 2 = 0$$

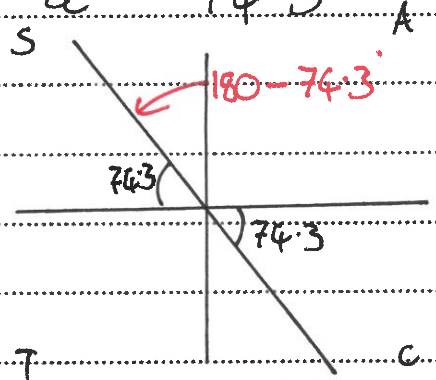
$$y = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)}$$

$$y = \frac{-3 + \sqrt{17}}{2} \quad \text{or} \quad y = \frac{-3 - \sqrt{17}}{2}$$

$$\tan x = \frac{-3 + \sqrt{17}}{2} \quad \text{or} \quad \tan x = \frac{-3 - \sqrt{17}}{2}$$

$$x = 29.3° \qquad x = -74.3°$$





$x = 105.7°$

$$x = 29.3°, \ 105.7°$$

5 (a) Show that $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 2\cot 2x$. [4]

LHS: $\dfrac{\cos 3x \cos x}{\sin x \cos x} + \dfrac{\sin 3x \sin x}{\sin x \cos x}$

$= \dfrac{\cos 3x \cos x + \sin 3x \sin x}{\sin x \cos x}$ using $\cos(A+B)$ and $\sin(A+B)$

$= \dfrac{(\cos 2x \cos x - \sin 2x \sin x)\cos x + (\sin 2x \cos x + \cos 2x \sin x)\sin x}{\sin x \cos x}$

$= \dfrac{\cos 2x \cos^2 x - \cancel{\sin 2x \sin x \cos x} + \cancel{\sin 2x \sin x \cos x} + \cos 2x \sin^2 x}{\sin x \cos x}$

$= \dfrac{\cos 2x \cos^2 x + \cos 2x \sin^2 x}{\sin x \cos x}$

$= \dfrac{\cos 2x (\cos^2 x + \sin^2 x)}{\sin x \cos x}$

$= \dfrac{\cos 2x}{\sin x \cos x}$ $\qquad \sin 2x = 2\sin x \cos x$

$\qquad \frac{1}{2}\sin 2x = \sin x \cos x$

$= \dfrac{\cos 2x \times 2}{\frac{1}{2}\sin 2x \times 2}$

$= \dfrac{2\cos 2x}{\sin 2x}$

$= \underline{2\cot 2x}$ = RHS

(b) Hence solve the equation $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 4$, for $0 < x < \pi$. [3]

from (a): $2\cot 2x = 4$

$\cot 2x = \frac{4}{2}$

$\cot 2x = 2$

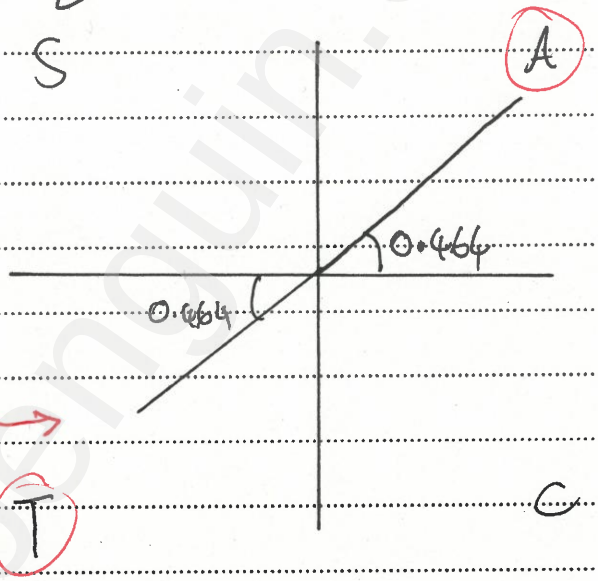
$\frac{1}{\tan 2x} = 2$

$\tan 2x = \frac{1}{2}$

$\boxed{\begin{array}{l} 0 < x < \pi \\ 0 < 2x < 2\pi \end{array}}$

$2x = \tan^{-1}\left(\frac{1}{2}\right)$

$2x = \underset{\boxed{STO}}{0.464...}$



$\pi + 0.464$

$\rightarrow 2x = 0.464..., \underset{\boxed{STO}}{3.605...}$

$x = \underline{0.232}, \underline{1.80}$

5 Solve the equation $\sin\theta = 3\cos 2\theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]

$\sin\theta = 3\cos 2\theta + 2$

$\sin\theta = 3(1 - 2\sin^2\theta) + 2$

$\sin\theta = 3 - 6\sin^2\theta + 2$

$\sin\theta = 5 - 6\sin^2\theta$

$6\sin^2\theta + \sin\theta - 5 = 0$

let $y = \sin\theta$:

$6y^2 + y - 5 = 0$

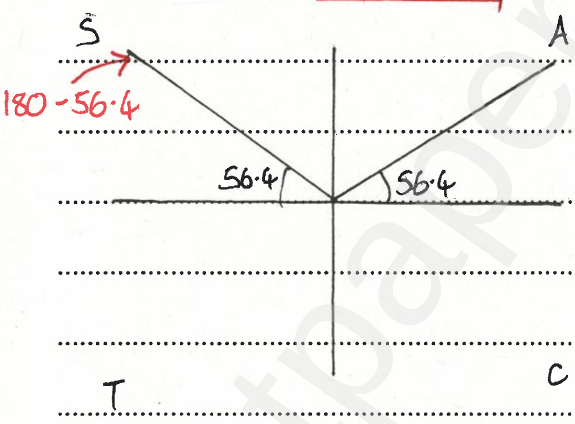
$(6y - 5)(y + 1) = 0$

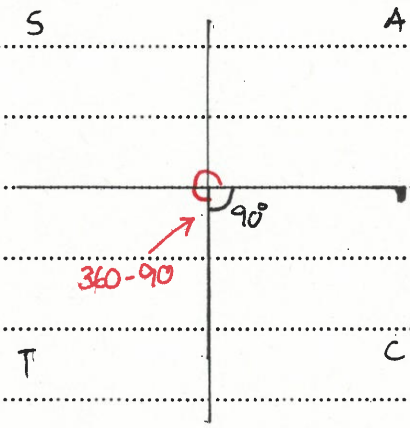
$6y - 5 = 0$ or $y + 1 = 0$

$y = \frac{5}{6}$ or $y = -1$

$\sin\theta = \frac{5}{6}$ or $\sin\theta = -1$

$\theta = 56.4°$ or $\theta = -90$





$\theta = 56.4°, 123.6°$ $\theta = 270°$

$\theta = 56.4°, 123.6°, 270°$

3 Express the equation $\tan(\theta + 60°) = 2 + \tan(60° - \theta)$ as a quadratic equation in $\tan\theta$, and hence solve the equation for $0° \leqslant \theta \leqslant 180°$. [6]

$$\tan(\theta + 60) = 2 + \tan(60 - \theta)$$

$$\tan(\theta + 60) - \tan(60 - \theta) = 2$$

$$\frac{\tan\theta + \tan 60}{1 - \tan\theta \tan 60} - \frac{\tan 60 - \tan\theta}{1 + \tan 60 \tan\theta} = 2$$

$$\frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} - \frac{\sqrt{3} - \tan\theta}{1 + \sqrt{3}\tan\theta} = 2$$

$$\frac{(\tan\theta + \sqrt{3})(1 + \sqrt{3}\tan\theta) - (\sqrt{3} - \tan\theta)(1 - \sqrt{3}\tan\theta)}{(1 - \sqrt{3}\tan\theta)(1 + \sqrt{3}\tan\theta)} = 2$$

$$\frac{\tan\theta + \cancel{\sqrt{3}\tan^2\theta} + \cancel{\sqrt{3}} + 3\tan\theta - \cancel{\sqrt{3}} + 3\tan\theta + \tan\theta - \cancel{\sqrt{3}\tan^2\theta}}{1 - 3\tan^2\theta} = 2$$

$$\frac{8\tan\theta}{1 - 3\tan^2\theta} = 2$$

$$8\tan\theta = 2(1 - 3\tan^2\theta)$$

$$8\tan\theta = 2 - 6\tan^2\theta$$

$$6\tan^2\theta + 8\tan\theta - 2 = 0$$

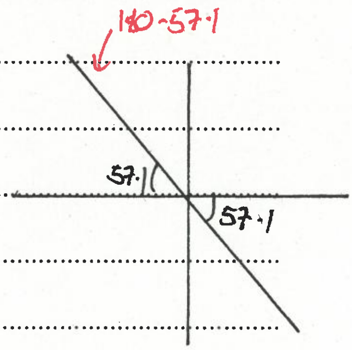
$$3\tan^2\theta + 4\tan\theta - 1 = 0$$

let $y = \tan\theta$: $3y^2 + 4y - 1 = 0$

$$y = \frac{-4 \pm \sqrt{4^2 - 4(3)(-1)}}{2(3)}$$

$$y = \frac{-2 + \sqrt{7}}{3} \quad \text{or} \quad y = \frac{-2 - \sqrt{7}}{3}$$

$$\tan\theta = \frac{-2 + \sqrt{7}}{3} \quad \text{or} \quad \tan\theta = \frac{-2 - \sqrt{7}}{3}$$



$\theta = 12.1°$ or $\theta = -57.1$

$\theta = 122.9°$

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3 By first expressing the equation $\tan(x + 45°) = 2\cot x + 1$ as a quadratic equation in $\tan x$, solve the equation for $0° < x < 180°$. [6]

$$\tan(x+45) = 2\cot x + 1$$

$$\frac{\tan x + \tan 45}{1 - \tan x \tan 45} = 2\cot x + 1$$

$$\frac{\tan x + 1}{1 - \tan x} = 2\cot x + 1$$

$$\tan x + 1 = (2\cot x + 1)(1 - \tan x)$$

$$\tan x + 1 = 2\cot x - 2\cot x \tan x + 1 - \tan x$$

$$\tan x + 1 = 2\frac{1}{\tan x} - 2\frac{1}{\tan x}\tan x + 1 - \tan x$$

$$\tan x + 1 = 2\frac{1}{\tan x} - 2 + 1 - \tan x$$

$$\tan x + 1 = \frac{2}{\tan x} - 1 - \tan x$$

$$2\tan x + 2 = \frac{2}{\tan x}$$

$$2\tan^2 x + 2\tan x = 2$$

$$2\tan^2 x + 2\tan x - 2 = 0$$

$$\tan^2 x + \tan x - 1 = 0$$

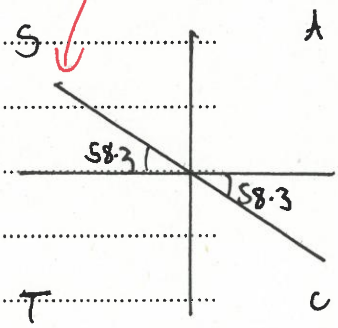
let $y = \tan x$: $y^2 + y - 1 = 0$

$$y = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$y = \frac{-1+\sqrt{5}}{2} \quad \text{or} \quad y = \frac{-1-\sqrt{5}}{2}$$

180−58.3

$$\tan x = \frac{-1+\sqrt{5}}{2} \quad \text{or} \quad \tan x = \frac{-1-\sqrt{5}}{2}$$



$x = \underline{31.7°}$ or $x = -58.3°$

$= \underline{121.7°}$

$x = \underline{31.7°, 121.7°}$

5 The angles α and β lie between 0° and 180° and are such that

$$\tan(\alpha + \beta) = 2 \quad ① \qquad \text{and} \qquad \tan\alpha = 3\tan\beta. \quad ②$$

Find the possible values of α and β. [6]

① $\tan(\alpha+\beta) = 2$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 2$$

②: $$\frac{3\tan\beta + \tan\beta}{1 - 3\tan^2\beta} = 2$$

$$4\tan\beta = 2(1 - 3\tan^2\beta)$$

$$4\tan\beta = 2 - 6\tan^2\beta$$

$$6\tan^2\beta + 4\tan\beta - 2 = 0$$

$$3\tan^2\beta + 2\tan\beta - 1 = 0$$

let $y = \tan\beta$:

$$3y^2 + 2y - 1 = 0$$

$$(3y - 1)(y + 1) = 0$$

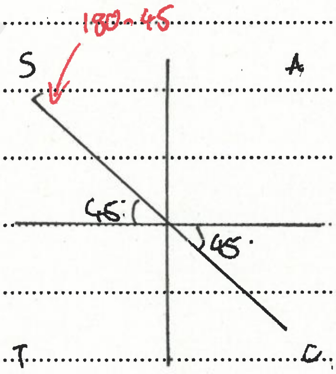
$3y - 1 = 0$ or $y + 1 = 0$

$y = \frac{1}{3}$ or $y = -1$

$\tan\beta = \frac{1}{3}$ or $\tan\beta = -1$

$\beta = 18.4°$ $\qquad$ $\beta = -45°$

no others in domain $\qquad$ $= 135°$



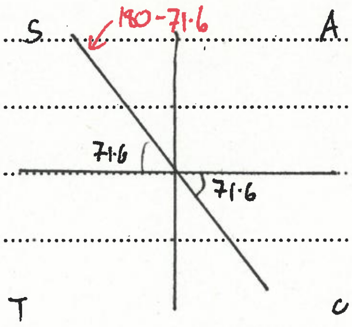
②: if $\tan\beta = \frac{1}{3}$, $\tan\alpha = 1$:

$\alpha = 45°$

②: if $\tan\beta = -1$, $\tan\alpha = -3$

$\alpha = -71.6°$

$\alpha = 108.4°$



so:
$\alpha = 45°$ and $\beta = 18.4°$

or:
$\alpha = 108.4°$ and $\beta = 135°$

5 **(a)** Given that

$$\sin(x + \tfrac{1}{6}\pi) - \sin(x - \tfrac{1}{6}\pi) = \cos(x + \tfrac{1}{3}\pi) - \cos(x - \tfrac{1}{3}\pi),$$

find the exact value of $\tan x$. [4]

$\sin\left(x + \frac{\pi}{6}\right) = \sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}$
$= \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$

$\sin\left(x - \frac{\pi}{6}\right) = \sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6}$
$= \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$

$\rightarrow \sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$
$= \underline{\cos x}$

$\cos\left(x + \frac{\pi}{3}\right) = \cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}$
$= \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

$\cos\left(x - \frac{\pi}{3}\right) = \cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}$
$= \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

$\rightarrow \cos\left(x + \frac{\pi}{3}\right) - \cos\left(x - \frac{\pi}{3}\right) = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$
$= \underline{-\sqrt{3}\sin x}$

Original equation becomes:

$\cos x = -\sqrt{3}\sin x$

$1 = \frac{-\sqrt{3}\sin x}{\cos x}$

$\tan x = \underline{\frac{-1}{\sqrt{3}}}$

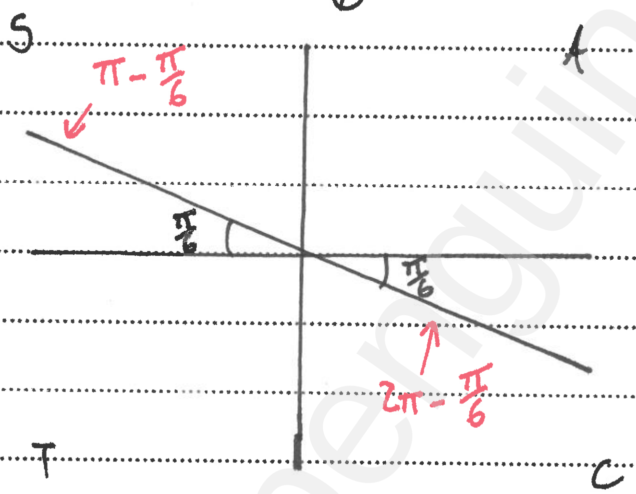
(b) Hence find the exact roots of the equation

$$\sin(x + \tfrac{1}{6}\pi) - \sin(x - \tfrac{1}{6}\pi) = \cos(x + \tfrac{1}{3}\pi) - \cos(x - \tfrac{1}{3}\pi)$$

for $0 \leqslant x \leqslant 2\pi$. [2]

from (a): $\tan x = \frac{-1}{\sqrt{3}}$

$x = \frac{-\pi}{6}$



$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

4 **(a)** Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0.$$ [2]

$\sin 2\theta + \cos 2\theta = 2\sin^2\theta$

$2\sin\theta\cos\theta$ ↗ ↖ $\cos^2\theta - \sin^2\theta$

$2\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta = 2\sin^2\theta$

$2\sin\theta\cos\theta + \cos^2\theta - 3\sin^2\theta = 0$

$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$ QED

(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ for $0° < \theta < 180°$. [4]

Factorise by splitting the middle: two numbers that multiply to -3 and sum to $+2$: -1 and $+3$

$\cos^2\theta + 3\sin\theta\cos\theta - 1\sin\theta\cos\theta - 3\sin^2\theta = 0$

$\cos\theta(\cos\theta + 3\sin\theta) - \sin\theta(\cos\theta + 3\sin\theta) = 0$

$(\cos\theta - \sin\theta)(\cos\theta + 3\sin\theta) = 0$

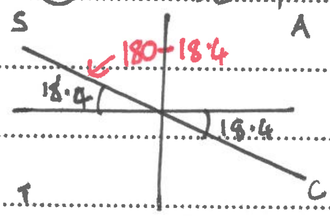
$\cos\theta - \sin\theta = 0$ or $\cos\theta + 3\sin\theta = 0$

$\sin\theta = \cos\theta$ | $3\sin\theta = -\cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = 1$ | $\dfrac{\sin\theta}{\cos\theta} = \dfrac{-1}{3}$

$\tan\theta = 1$ | $\tan\theta = -\dfrac{1}{3}$

$\theta = 45°$ | $\theta = -18.4$



$\theta = 161.6°$

$\theta = 45°, 161.6°$

4 (a) Show that $\sec^4\theta - \tan^4\theta \equiv 1 + 2\tan^2\theta$. [3]

LHS: $\sec^4\theta - \tan^4\theta$

$= (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$ (difference of two squares)

$1 + \tan^2\theta = \sec^2\theta$

$= (\sec^2\theta + \tan^2\theta)(1 + \tan^2\theta - \tan^2\theta)$

$= (\sec^2\theta + \tan^2\theta) \times 1$

$1 + \tan^2\theta = \sec^2\theta$

$= 1 + \tan^2\theta + \tan^2\theta$

$= \underline{1 + 2\tan^2\theta}$ QED

(b) Hence or otherwise solve the equation $\sec^4 2\alpha - \tan^4 2\alpha = 2\tan^2 2\alpha \sec^2 2\alpha$ for $0° < \alpha < 180°$. [5]

from (a): $\sec^4 2\alpha - \tan^4 2\alpha \equiv 1 + 2\tan^2 2\alpha$

$\rightarrow \quad 1 + 2\tan^2 2\alpha = 2\tan^2 2\alpha \sec^2 2\alpha$

$\sec^2 2\alpha = 1 + \tan^2 2\alpha$

$1 + 2\tan^2 2\alpha = 2\tan^2 2\alpha (1 + \tan^2 2\alpha)$

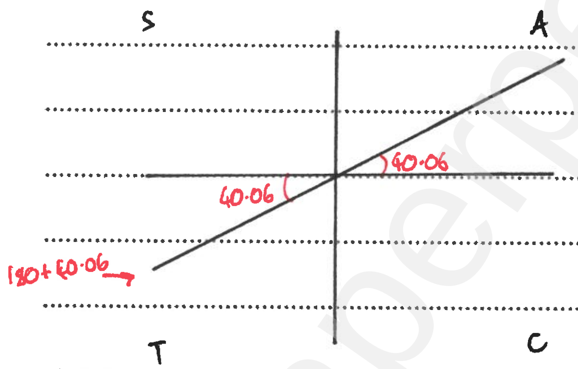
$1 + 2\tan^2 2\alpha = 2\tan^2 2\alpha + 2\tan^4 2\alpha$

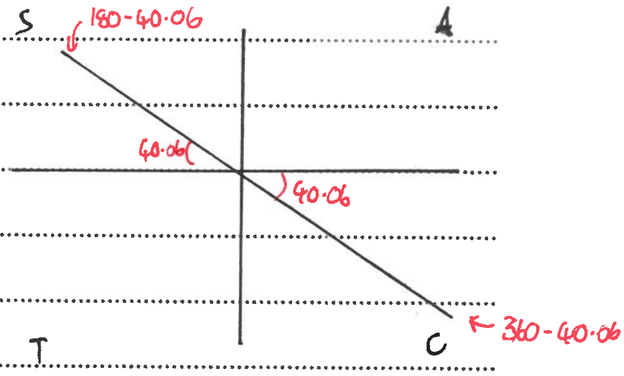
$1 = 2\tan^4 2\alpha$

$\tan^4 2\alpha = \frac{1}{2}$

$\tan 2\alpha = \sqrt[4]{\frac{1}{2}}$ or $\tan 2\alpha = -\sqrt[4]{\frac{1}{2}}$

$2\alpha = 40.06°$ $\qquad$ $2\alpha = -40.06°$





$2\alpha = 40.06°, 220.06°$ $\qquad$ $2\alpha = 139.94°, 319.94°$

$2\alpha = 40.06°, 139.94°, 220.06°, 319.94°$

$\alpha = 20.0°, 70.0°, 110.0°, 160.0°$

5 **(a)** By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2}\tan\theta$ may be expressed as $\tan^4\theta + 2\tan^2\theta - 7 = 0$. [4]

$$\tan(2\theta + 2\theta) = \frac{\tan 2\theta + \tan 2\theta}{1 - \tan 2\theta \tan 2\theta}$$

$$= \frac{2\tan 2\theta}{1 - \tan^2 2\theta} \quad \leftarrow \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}{1 - \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$$

$$= \frac{\frac{4\tan\theta}{1-\tan^2\theta}}{1 - \frac{4\tan^2\theta}{(1-\tan^2\theta)^2}} \quad \begin{matrix} \times(1-\tan^2\theta)^2 \\ \times(1-\tan^2\theta)^2 \end{matrix}$$

$\tan 4\theta = \frac{1}{2}\tan\theta$

$$\frac{1}{2}\tan\theta = \frac{4\tan\theta(1-\tan^2\theta)}{(1-\tan^2\theta)^2 - 4\tan^2\theta}$$

$$\frac{1}{2}\tan\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 2\tan^2\theta + \tan^4\theta - 4\tan^2\theta}$$

$$\frac{1}{2}\tan\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$$

$$\frac{1}{2}\tan\theta(1 - 6\tan^2\theta + \tan^4\theta) = 4\tan\theta - 4\tan^3\theta$$

$$\frac{1}{2}\tan\theta - 3\tan^3\theta + \frac{1}{2}\tan^5\theta = 4\tan\theta - 4\tan^3\theta$$

$$\tan\theta - 6\tan^3\theta + \tan^5\theta = 8\tan\theta - 8\tan^3\theta$$

$$\tan^5\theta + 2\tan^3\theta - 7\tan\theta = 0$$

$$\div\tan\theta \qquad \div\tan\theta \qquad \div\tan\theta \qquad \div\tan\theta$$

$$\underline{\tan^4\theta + 2\tan^2\theta - 7 = 0}$$

(b) Hence solve the equation $\tan 4\theta = \frac{1}{2}\tan\theta$, for $0° < \theta < 180°$. [3]

From part (a): $\tan^4\theta + 2\tan^2\theta - 7 = 0$

let $y = \tan^2\theta$: $y^2 + 2y - 7 = 0$

$$y = \frac{-2 \pm \sqrt{2^2 - 4(1)(-7)}}{2(1)}$$

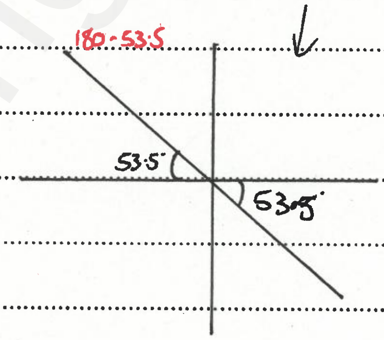
$y = -1 + 2\sqrt{2}$ or $y = -1 - 2\sqrt{2}$

$\tan^2\theta = -1 + 2\sqrt{2}$ or $\tan^2\theta = -1 - 2\sqrt{2}$

✗ no sol^s

$\tan\theta = 1.35\ldots$ or $\tan\theta = -1.35\ldots$

$\theta = 53.5°$ or $\theta = -53.5°, 126.5°$



$\theta = 53.5°, 126.5°$