3 The variables x and y satisfy the relation $2^y = 3^{1-2x}$.

(a) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

$$\ln 2^y = \ln 3^{1-2x}$$

$$y \ln 2 = (1-2x)\ln 3$$

$$y \ln 2 = \ln 3 - 2x \ln 3$$

$$y \ln 2 = -2x \ln 3 + \ln 3$$

$$y = \frac{-2\ln 3}{\ln 2}x + \frac{\ln 3}{\ln 2}$$

of form $y = mx + c$, with $m = \frac{-2\ln 3}{\ln 2}$

$\left(\text{and } c = \frac{\ln 3}{\ln 2}\right)$

(b) Find the exact x-coordinate of the point of intersection of this line with the line $y = 3x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [2]

$$3x = \frac{-2\ln 3}{\ln 2}x + \frac{\ln 3}{\ln 2}$$

$$3x \ln 2 = -2\ln 3 x + \ln 3$$

$$3x\ln 2 + 2x \ln 3 = \ln 3$$

$$x(3\ln 2 + 2\ln 3) = \ln 3$$

$$x(\ln 2^3 + \ln 3^2) = \ln 3$$

$$x(\ln 8 + \ln 9) = \ln 3$$

$$x \ln 72 = \ln 3$$

$$x = \frac{\ln 3}{\ln 72}$$

3 The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

$x = A(3^{-y})$

$\ln x = \ln(A(3^{-y}))$

$\ln x = \ln A + \ln 3^{-y}$

$\ln x = \ln A - y\ln 3$

$y\ln 3 + \ln x = \ln A$

$y\ln 3 = -\ln x + \ln A$

$y = \frac{-1}{\ln 3}\ln x + \frac{\ln A}{\ln 3}$

this is of the form $y = mx + c$, so makes a straight line, where $m = \frac{-1}{\ln 3}$, $x = \ln x$ and $c = \frac{\ln A}{\ln 3}$

It is given that the line intersects the y-axis at the point where $y = 1.3$.

(b) Calculate the value of A, giving your answer correct to 2 decimal places. [2]

$(0, 1.3)$: $1.3 = \frac{-1}{\ln 3}(0) + \frac{\ln A}{\ln 3}$

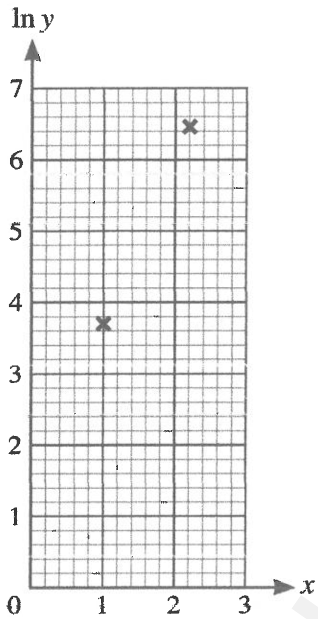
$1.3 = 0 + \frac{\ln A}{\ln 3}$

$\ln A = 1.3\ln 3$

$A = e^{1.3\ln 3}$

$A = 4.17$

3



The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are (1, 3.7) and (2.2, 6.46).

Use this information to find the values of a and b. [4]

$y = ab^x$

$\ln y = \ln(ab^x)$

$\ln y = \ln a + \ln b^x$

$\ln y = \ln a + x\ln b$

sub. (1, 3.7):

$3.7 = \ln a + \ln b$ (1)

sub. (2.2, 6.46):

$6.46 = \ln a + 2.2\ln b$ (2)

(2) − (1):

$2.76 = 1.2\ln b$

$\ln b = \frac{2.76}{1.2}$

$\ln b = 2.3$

$b = e^{2.3}$

$= 9.97$ STO

→ (1):

$3.7 = \ln a + \ln(9.97)$

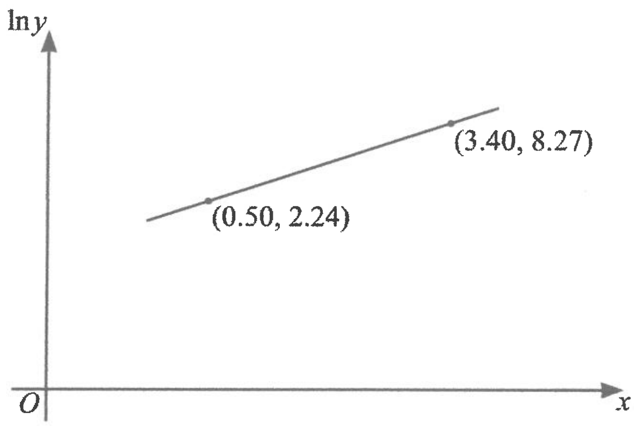
$3.7 = \ln a + 2.3$

$\ln a = 1.4$

$a = e^{1.4}$

$= 4.06$

6



The variables x and y satisfy the equation $ay = b^x$, where a and b are constants. The graph of $\ln y$ against x is a straight line passing through the points (0.50, 2.24) and (3.40, 8.27), as shown in the diagram.

Find the values of a and b. Give each value correct to 1 significant figure. [4]

$ay = b^x$

$\ln(ay) = \ln b^x$

$\ln a + \ln y = x \ln b$

$\ln y = x \ln b - \ln a$

Sub. (0.5, 2.24):

$2.24 = 0.5 \ln b - \ln a$ ①

Sub. (3.4, 8.27):

$8.27 = 3.4 \ln b - \ln a$ ②

② − ①:

$6.03 = 2.9 \ln b$

$\ln b = \frac{6.03}{2.9}$

$b = e^{\frac{6.03}{2.9}}$

$b = 7.998\ldots$ STO

$= 8$ (1sf)

→ ①:

$2.24 = 0.5\ln(7.998..) - \ln a$

$\ln a = 0.5\ln(7.998..) - 2.24$

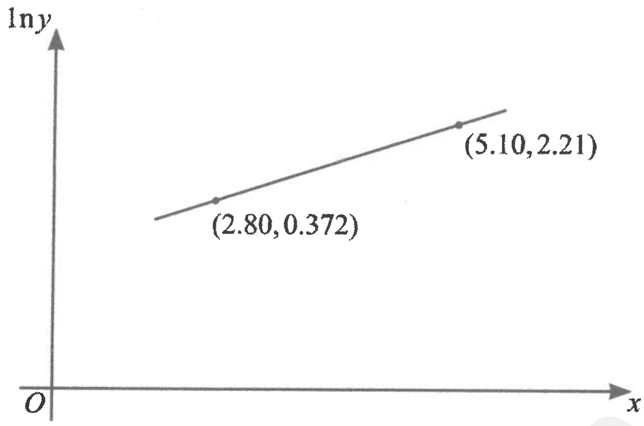
$\ln a = -1.200\ldots$

$a = e^{-1.200}$

$= 0.301$

$= 0.3$ (1sf)

4



The variables x and y satisfy the equation $ky = e^{cx}$, where k and c are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2.80, 0.372)$ and $(5.10, 2.21)$, as shown in the diagram.

Find the values of k and c. Give each value correct to 2 significant figures. [4]

$\ln(ky) = \ln(e^{cx})$

$\ln k + \ln y = cx$

$\ln y = cx - \ln k$

Sub. $(2.8, 0.372)$:

$0.372 = 2.8c - \ln k$ (1)

Sub. $(5.1, 2.21)$:

$2.21 = 5.1c - \ln k$ (2)

(2) − (1):

$1.838 = 2.3c$

$c = 0.7991\ldots$ STO

→ (1): $0.372 = 2.8(0.7991\ldots) - \ln k$

$\ln k = 2.2375\ldots - 0.372$

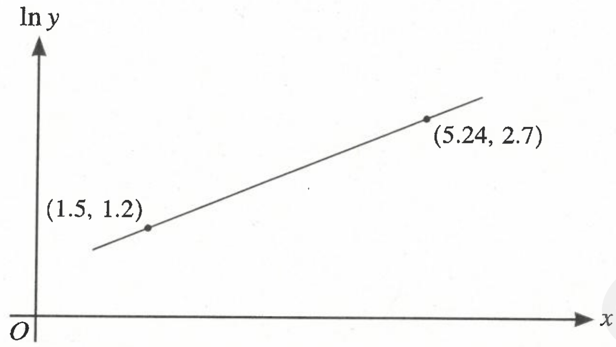
$\ln k = 1.865\ldots$

$k = e^{1.865\ldots}$

$k = 6.5$ (2sf)

$c = 0.80$ (2sf)

2



The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points (1.5, 1.2) and (5.24, 2.7) as shown in the diagram.

Find the values of A and k correct to 2 decimal places. [5]

$y^2 = Ae^{kx}$

$\ln y^2 = \ln(Ae^{kx})$

$2\ln y = \ln A + \ln e^{kx}$

$2\ln y = \ln A + kx$

$\ln y = \frac{1}{2}\ln A + \frac{1}{2}kx$

$\ln y = \frac{1}{2}kx + \frac{1}{2}\ln A$

$(1.5, 1.2)$:

$1.2 = \frac{1}{2}k(1.5) + \frac{1}{2}\ln A$

$2.4 = 1.5k + \ln A$ ①

$(5.24, 2.7)$:

$2.7 = \frac{1}{2}k(5.24) + \frac{1}{2}\ln A$

$5.4 = 5.24k + \ln A$ ②

② − ①: $3 = 3.74k$

$k = 0.80$

①:

$2.4 = 1.5(0.80\ldots) + \ln A$

$2.4 = 1.203\ldots + \ln A$

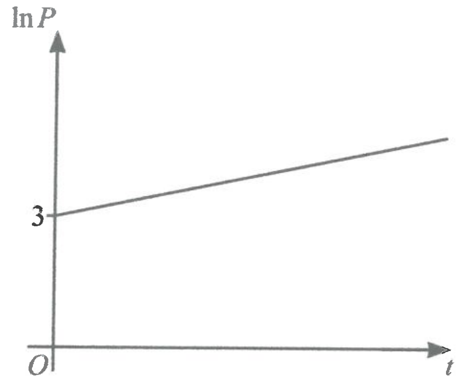
$\ln A = 1.196\ldots$

$A = e^{1.196\ldots}$

$A = 3.31$

DO NOT WRITE IN THIS MARGIN

3



The number of bacteria in a population, P, at time t hours is modelled by the equation $P = ae^{kt}$, where a and k are constants. The graph of $\ln P$ against t, shown in the diagram, has gradient $\frac{1}{20}$ and intersects the vertical axis at $(0, 3)$.

(a) State the value of k and find the value of a correct to 2 significant figures. [3]

$P = ae^{kt}$

$\ln P = \ln(ae^{kt})$

$\ln P = \ln a + \ln e^{kt}$

$\ln P = \ln a + kt$

$\ln P = \underset{\text{gradient}}{kt} + \underset{\text{intercept}}{\ln a}$

gradient $= \frac{1}{20}$: $\underline{k = \frac{1}{20}}$

sub. $(0,3)$: $\ln P = \frac{1}{20}t + \ln a$

$3 = 0 + \ln a$

$\ln a = 3$

$a = e^3$

$= \underline{20}$ (2sf)

$\rightarrow \underline{\ln P = \frac{1}{20}t + 3}$

(b) Find the time taken for P to double. Give your answer correct to the nearest hour. [2]

Easiest way is to sub. two values for P (one double the other) e.g. $P=1$ and $P=2$

$\underline{P=1:}$

$\ln 1 = \frac{1}{20}t + 3$

$0 = \frac{1}{20}t + 3$

$\frac{1}{20}t = -3$

$\underline{t = -60}$

(it doesn't matter that this is negative - the graph is a straight line so it would be the same in the $-t$ quadrants)

$\underline{P=2:}$

$\ln 2 = \frac{1}{20}t + 3$

$\frac{1}{20}t = \ln 2 - 3$

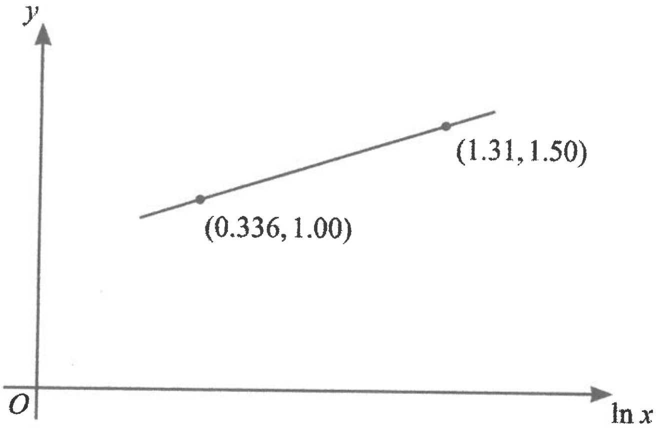
$\underline{t = 20(\ln 2 - 3)}$

difference: time $= 20(\ln 2 - 3) - -60$

$= 13.86$ hrs

$= \underline{14 \text{ hrs}}$ (nearest hr)

3



The variables x and y satisfy the equation $a^y = bx$, where a and b are constants. The graph of y against $\ln x$ is a straight line passing through the points (0.336, 1.00) and (1.31, 1.50), as shown in the diagram.

Find the values of a and b. Give each value correct to the nearest integer. [4]

$a^y = bx$

$\ln a^y = \ln(bx)$

$y\ln a = \ln b + \ln x$

Sub. (0.336, 1.00):

$1.00 \ln a = \ln b + 0.336$

$\ln a = \ln b + 0.336$ ①

Sub. (1.31, 1.50):

$1.50 \ln a = \ln b + 1.31$

$1.5 \ln a = \ln b + 1.31$ ②

② - ①:

$0.5 \ln a = 0.974$

$\ln a = 1.948$

$a = e^{1.948}$

$a = 7.0146..$ STO

① : $\ln(7.0146) = \ln b + 0.336$

$1.948 = \ln b + 0.336$

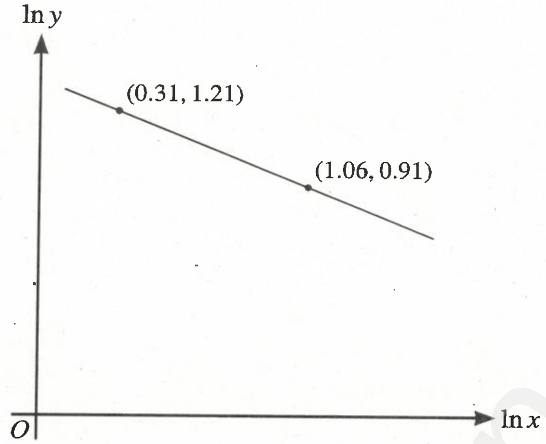
$\ln b = 1.612$

$b = e^{1.612}$

$b = 5.0128...$

$a = 7$, $b = 5$

3



The variables x and y satisfy the equation $x^n y^2 = C$, where n and C are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points (0.31, 1.21) and (1.06, 0.91), as shown in the diagram.

Find the value of n and find the value of C correct to 2 decimal places. [5]

$x^n y^2 = C$

$\ln(x^n y^2) = \ln C$

$\ln x^n + \ln y^2 = \ln C$

$n\ln x + 2\ln y = \ln C$

$2\ln y = -n\ln x + \ln C$

$\ln y = \frac{-n}{2}\ln x + \frac{1}{2}\ln C$

$(0.31, 1.21)$:

$1.21 = \frac{-n}{2}(0.31) + \frac{1}{2}\ln C$

$2.42 = -0.31n + \ln C$ ①

$(1.06, 0.91)$: $0.91 = \frac{-n}{2}(1.06) + \frac{1}{2}\ln C$

$1.82 = -1.06n + \ln C$ ②

① − ②: $0.6 = 0.75n$

$n = 0.80$

①: $2.42 = -0.31(0.8) + \ln C$

$2.42 = -0.248 + \ln C$

$2.668 = \ln C$

$C = e^{2.668}$

$C = 14.41$

3 The variables x and y satisfy the equation $a^{2y-1} = b^{x-y}$, where a and b are constants.

(a) Show that the graph of y against x is a straight line. [3]

$\ln a^{2y-1} = \ln b^{x-y}$

$(2y-1)\ln a = (x-y)\ln b$

$2y\ln a - \ln a = x\ln b - y\ln b$

$2y\ln a + y\ln b = x\ln b + \ln a$

$y(2\ln a + \ln b) = x\ln b + \ln a$

$y = \dfrac{\ln b}{2\ln a + \ln b}x + \dfrac{\ln a}{2\ln a + \ln b}$

So straight line with gradient and intercept

(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where p and q are rational numbers in their simplest form. [2]

Sub. $a = b^3$:

$y = \dfrac{\ln b}{2\ln b^3 + \ln b}x + \dfrac{\ln b^3}{2\ln b^3 + \ln b}$

$= \dfrac{\ln b}{\ln b^6 + \ln b}x + \dfrac{\ln b^3}{\ln b^6 + \ln b}$

$= \dfrac{\ln b}{\ln b^7}x + \dfrac{\ln b^3}{\ln b^7}$

$= \dfrac{\cancel{\ln b}}{7\cancel{\ln b}}x + \dfrac{3\cancel{\ln b}}{7\cancel{\ln b}}$

$y = \dfrac{1}{7}x + \dfrac{3}{7}$