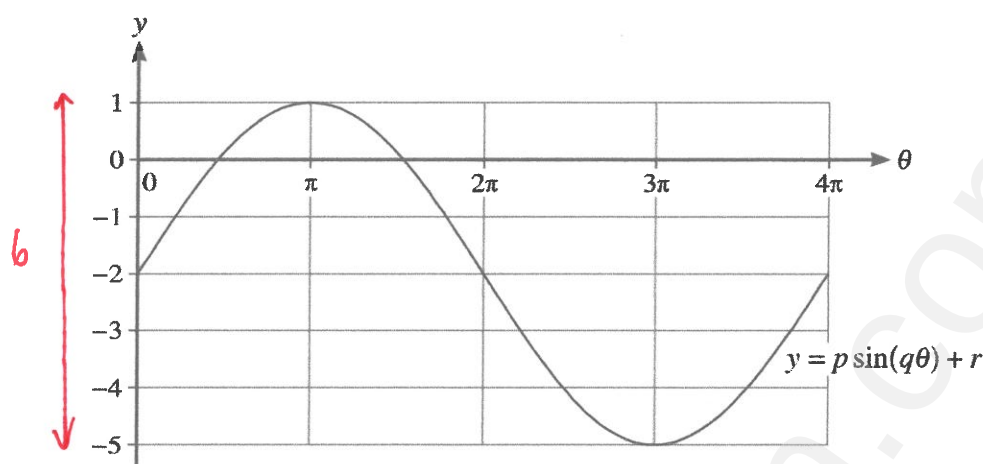




The diagram shows part of the curve with equation $y = p \sin(q\theta) + r$, where p , q and r are constants.

- (a) State the value of p . [1]

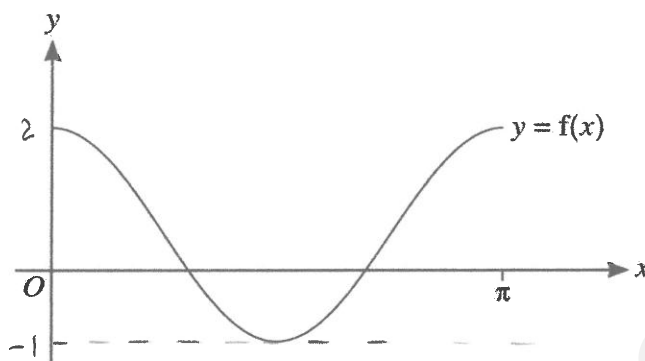
Amplitude is 6, where it is usually 2, so
 $p = 3$

- (b) State the value of q . [1]

Usually, one period is 2π , but here it is
 4π , so $q = \frac{1}{2}$

- (c) State the value of r . [1]

Graph starts at $(0, -2)$ instead of $(0, 0)$,
 so $r = -2$



The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2} \cos 2x + \frac{1}{2}$ for $0 \leq x \leq \pi$.

- (a) State the range of f .

[2]

Amplitude has been multiplied by $\frac{3}{2}$, so $-\frac{3}{2} \leq y \leq \frac{3}{2}$,
but then we've added $\frac{1}{2}$, so $-1 \leq y \leq 2$

A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x -axis is a tangent to the curve $y = g(x)$.

- (b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$.

[2]

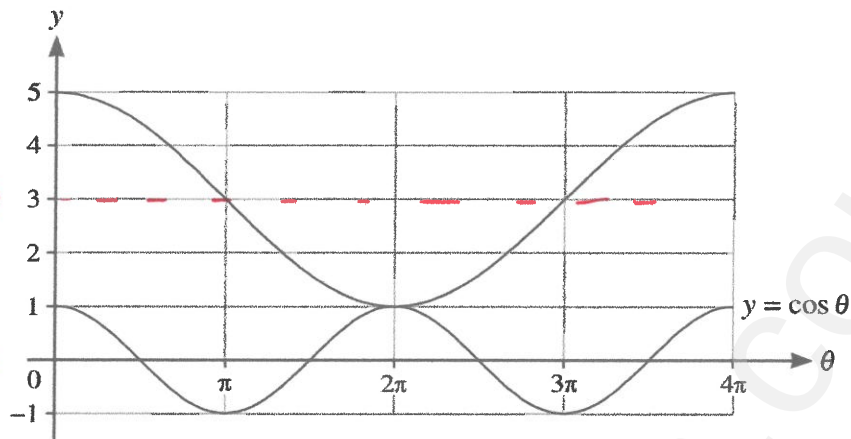
If the x -axis is a tangent, the curve must
have been shifted up by 1, so $k=1$ and
translation by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- (c) State the equation of the curve which is the reflection of $y = f(x)$ in the x -axis. Give your answer in the form $y = a \cos 2x + b$, where a and b are constants.

[1]

Reflection in x -axis: $f(x) \rightarrow -f(x)$:
 $y = -\left[\frac{3}{2} \cos 2x + \frac{1}{2}\right]$

$$y = -\frac{3}{2} \cos 2x - \frac{1}{2}$$



In the diagram, the lower curve has equation $y = \cos \theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos \theta$.

Find, in terms of a cosine function, the equation of the upper curve.

[3]

① period has doubled, so $f(x) \rightarrow f(\frac{1}{2}x)$

② amplitude has also doubled so $f(\frac{1}{2}x) \rightarrow 2f(\frac{1}{2}x)$

③ the "middle" of the graph (vertically) has shifted from 0 to 3, so $2f(\frac{1}{2}x) + 3$

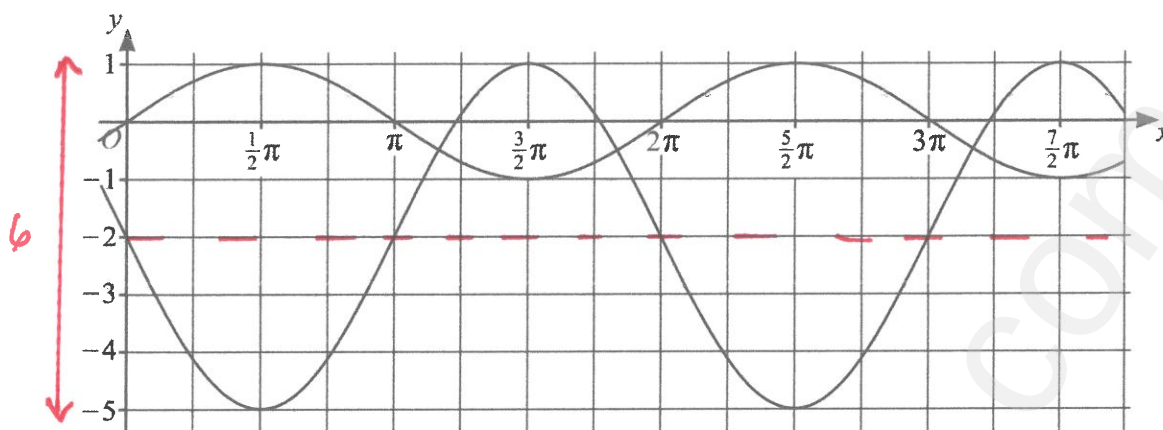
$$\rightarrow f(\theta) = \cos \theta$$

$$f(\frac{1}{2}\theta) = \cos(\frac{1}{2}\theta)$$

$$2f(\frac{1}{2}\theta) = 2\cos(\frac{1}{2}\theta)$$

$$2f(\frac{1}{2}\theta) + 3 = \underline{\underline{2\cos(\frac{1}{2}\theta) + 3}}$$

2



The diagram shows two curves. One curve has equation $y = \sin x$ and the other curve has equation $y = f(x)$.

- (a) In order to transform the curve $y = \sin x$ to the curve $y = f(x)$, the curve $y = \sin x$ is first reflected in the x -axis.

Describe fully a sequence of two further transformations which are required.

[4]

Amplitude of $f(x)$ is 6, where it is 2 for $y = \sin x$.

"middle" of graph vertically is at -2 , where it is usually at 0 .

so: ① Stretch parallel to y -axis, scale factor 3.

② Translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

- (b) Find $f(x)$ in terms of $\sin x$.

[2]

$\sin x \xrightarrow{\text{reflect in } x\text{-axis}} -\sin x \xrightarrow{\text{stretch in } y} -3\sin x \xrightarrow{\text{translate}} -3\sin x - 2$

$$\rightarrow \underline{\underline{f(x) = -3\sin x - 2}}$$

- 7 A curve has equation $y = 2 + 3 \sin \frac{1}{2}x$ for $0 \leq x \leq 4\pi$.

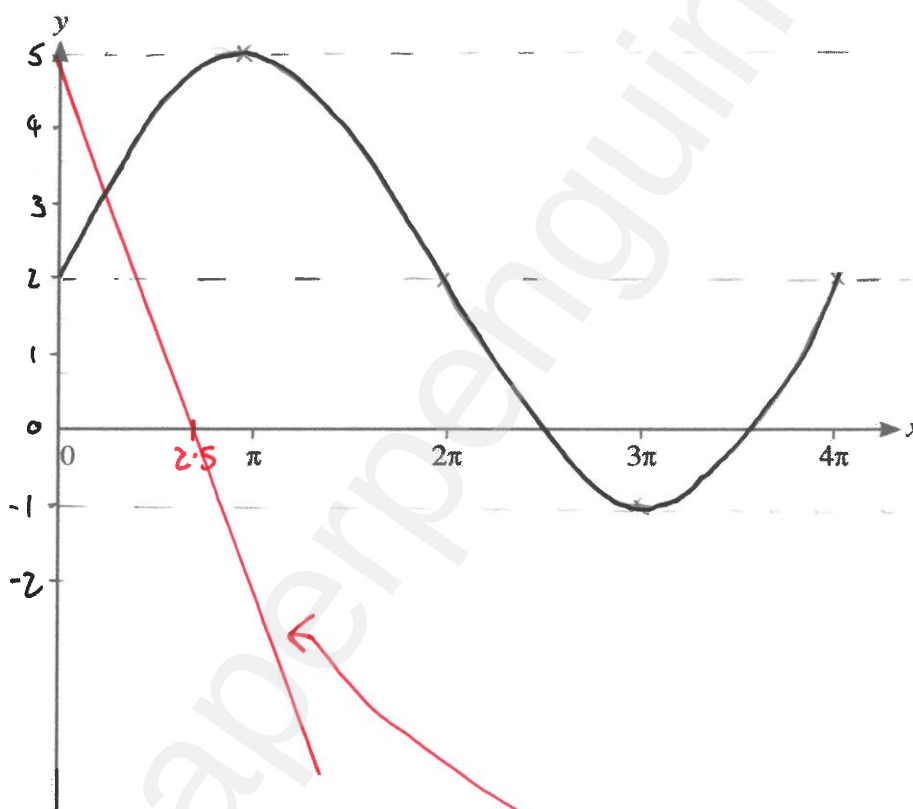
(a) State greatest and least values of y .

[2]

Amplitude has been multiplied by 3, so $-3 \leq y \leq 3$,
but then we've added 2, so $-1 \leq y \leq 5$

(b) Sketch the curve.

[2]



(c) State the number of solutions of the equation

$$2 + 3 \sin \frac{1}{2}x = 5 - 2x$$

for $0 \leq x \leq 4\pi$.

[1]

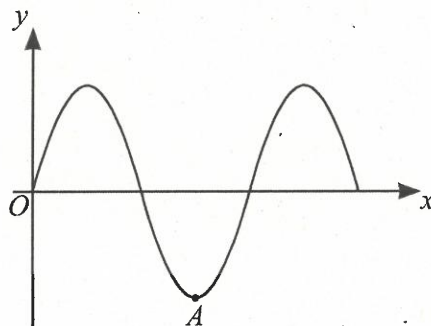
$y = 5 - 2x$: y intercept: (0, 5)

x intercept: $0 = 5 - 2x$

$$2x = 5$$

$$x = \frac{5}{2}$$

$\therefore$ Only one solution.



The diagram shows part of the curve with equation $y = k \sin \frac{1}{2}x$, where k is a positive constant and x is measured in radians. The curve has a minimum point A .

- (a) State the coordinates of A .

[1]

On a normal sine graph, the coordinates of A would be $(\frac{3\pi}{2}, -1)$. Given the transformation $k f(\frac{1}{2}x)$, the y -coords have been multiplied by k and the x -coords have been multiplied by 2 , so: $(3\pi, -k)$

- (b) A sequence of transformations is applied to the curve in the following order.

Translation of 2 units in the negative y -direction

Reflection in the x -axis

Find the equation of the new curve and determine the coordinates of the point on the new curve corresponding to A .

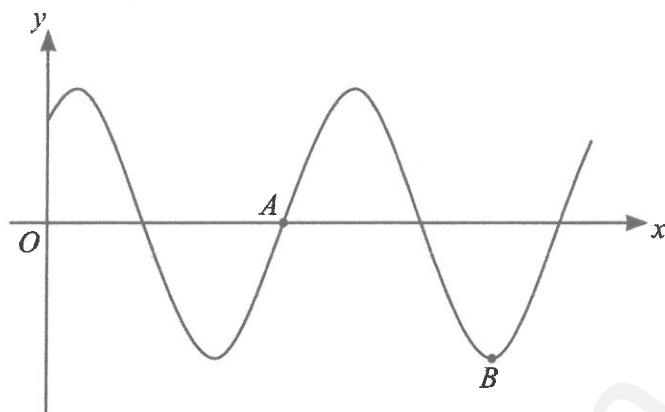
[3]

$$f(x) \xrightarrow{\begin{pmatrix} 0 \\ -2 \end{pmatrix}} f(x) - 2 \xrightarrow{x \rightarrow -x} -f(x) + 2$$

Equation: $k \sin(\frac{1}{2}x) \rightarrow k \sin(\frac{1}{2}x) - 2 \rightarrow \underline{\underline{-k \sin(\frac{1}{2}x) + 2}}$

point A : $(3\pi, -k) \rightarrow (3\pi, -k-2) \rightarrow \underline{\underline{(3\pi, k+2)}}$

2 (a)



The diagram shows the curve $y = k \cos(x - \frac{1}{6}\pi)$ where k is a positive constant and x is measured in radians. The curve crosses the x -axis at point A and B is a minimum point.

Find the coordinates of A and B .

[3]

Cosine graph has been translated by $(\frac{\pi}{6}, 0)$. On a normal cosine graph, A would be at $\frac{3\pi}{2}$. $\frac{3\pi}{2} + \frac{\pi}{6} = \frac{5\pi}{3}$

$$\rightarrow \underline{A = (\frac{5\pi}{3}, 0)}$$

x -coordinate of B on a normal graph is 3π : $3\pi + \frac{\pi}{6} = \frac{19\pi}{6}$
 y -coordinates have been multiplied by k : $k \times -1 = -k$
 $\rightarrow \underline{B = (\frac{19\pi}{6}, -k)}$

(b) Find the exact value of t that satisfies the equation

$$3 \sin^{-1}(3t) + 2 \cos^{-1}(\frac{1}{2}\sqrt{2}) = \pi. \quad [2]$$

$$2 \cos^{-1}(\frac{1}{2}\sqrt{2}) = \frac{\pi}{2}:$$

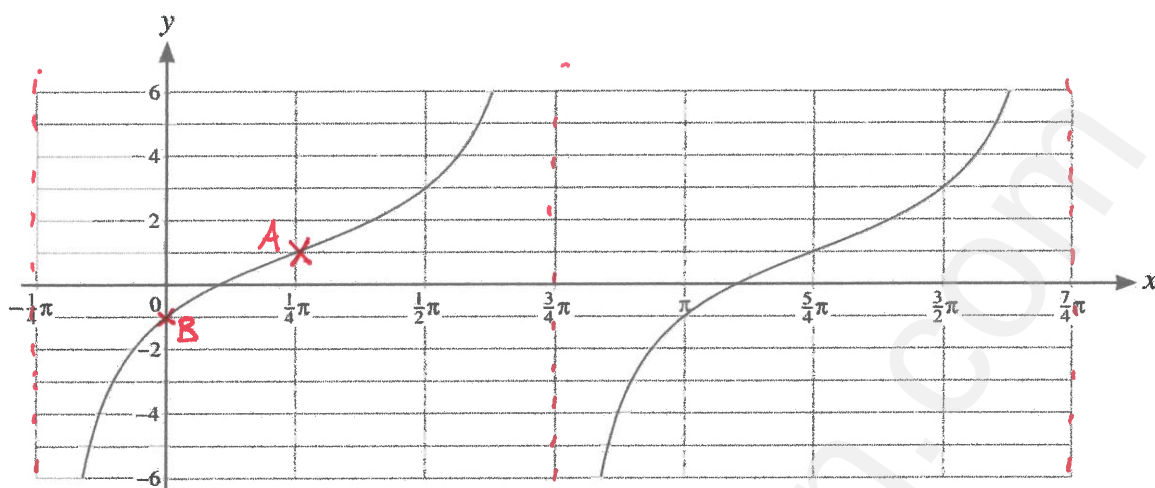
$$\rightarrow 3 \sin^{-1}(3t) + \frac{\pi}{2} = \pi$$

$$3 \sin^{-1}(3t) = \frac{\pi}{2}$$

$$\sin^{-1}(3t) = \frac{\pi}{6}$$

$$3t = \frac{1}{2}$$

$$\underline{t = \frac{1}{6}}$$



The diagram shows part of the graph of $y = a \tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a , b and c .

[3]

Using the position of the asymptotes, we can see the graph has been shifted right by $\frac{\pi}{4}$, so $b = \frac{\pi}{4}$

Then, using point A on the graph, we can see that this is the middle of one period and should be on the x -axis, so it has been shifted up by 1. This means $c = 1$

Now we can use a coordinate to find the final value, e.g. B: $(0, -1)$:

$$\begin{aligned} y &= a \tan\left(x - \frac{\pi}{4}\right) + 1 \\ -1 &= a \tan\left(0 - \frac{\pi}{4}\right) + 1 \\ -1 &= a \times (-1) + 1 \\ -2 &= -a \\ \underline{a} &= \underline{2} \end{aligned}$$

- 8 (a) The curve $y = \sin x$ is transformed to the curve $y = 4 \sin(\frac{1}{2}x - 30^\circ)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

!!! Two transformations to x -coords, so the order is not what you might expect:

① Translation by $\begin{pmatrix} 30^\circ \\ 0 \end{pmatrix}$

② Stretch parallel to x -axis, scale factor 2

③ Stretch parallel to y -axis, scale factor 4

- 9 Functions f and g are such that

$$f(x) = 2 - 3 \sin 2x \quad \text{for } 0 \leq x \leq \pi,$$

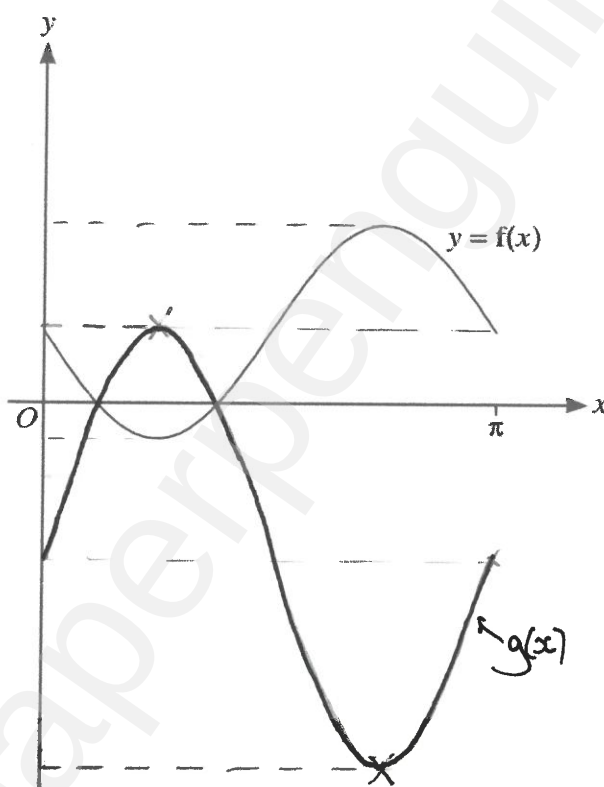
$$g(x) = -2f(x) \quad \text{for } 0 \leq x \leq \pi.$$

- (a) State the ranges of f and g .

[3]

$f(x)$: Amplitude has been multiplied by -3 , so $-3 \leq y \leq 3$, but then we've added 2 , so: $-1 \leq f(x) \leq 5$
 $g(x)$: $f(x)$ has been multiplied by -2 , so: $-10 \leq g(x) \leq 2$

The diagram below shows the graph of $y = f(x)$.



- (b) Sketch, on this diagram, the graph of $y = g(x)$.

[2]

The function h is such that

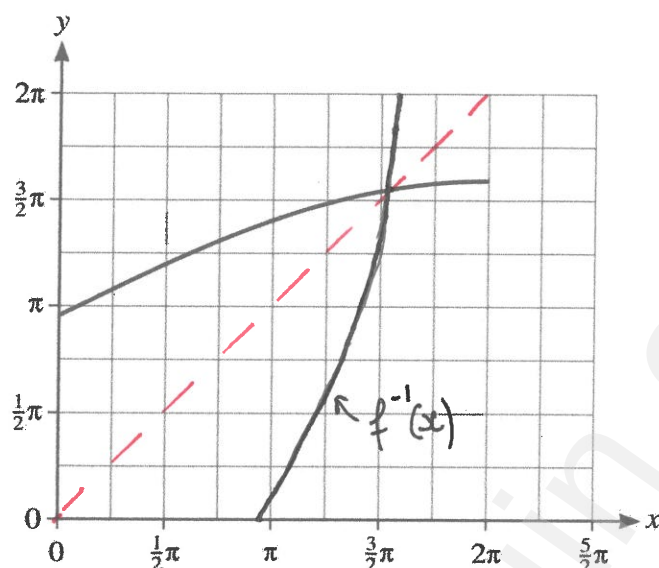
$$h(x) = g(x + \pi) \quad \text{for } -\pi \leq x \leq 0.$$

- (c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$.

[3]

$$f(x) \rightarrow g(x) \rightarrow h(x) : f(x) \rightarrow -2f(x) \rightarrow -2f(x + \pi)$$

- ① Translation by $\begin{pmatrix} -\pi \\ 0 \end{pmatrix}$ ② Stretch parallel to y -axis, scale factor 2 .
 ③ Reflection in x -axis.



The diagram shows the graph of $y = f(x)$ where the function f is defined by

$$f(x) = 3 + 2 \sin \frac{1}{4}x \text{ for } 0 \leq x \leq 2\pi.$$

- (a) On the diagram above, sketch the graph of $y = f^{-1}(x)$.

[2]

reflect in $y=x$

- (b) Find an expression for $f^{-1}(x)$.

[2]

$$y = 3 + 2 \sin\left(\frac{x}{4}\right)$$

$$y - 3 = 2 \sin\left(\frac{x}{4}\right)$$

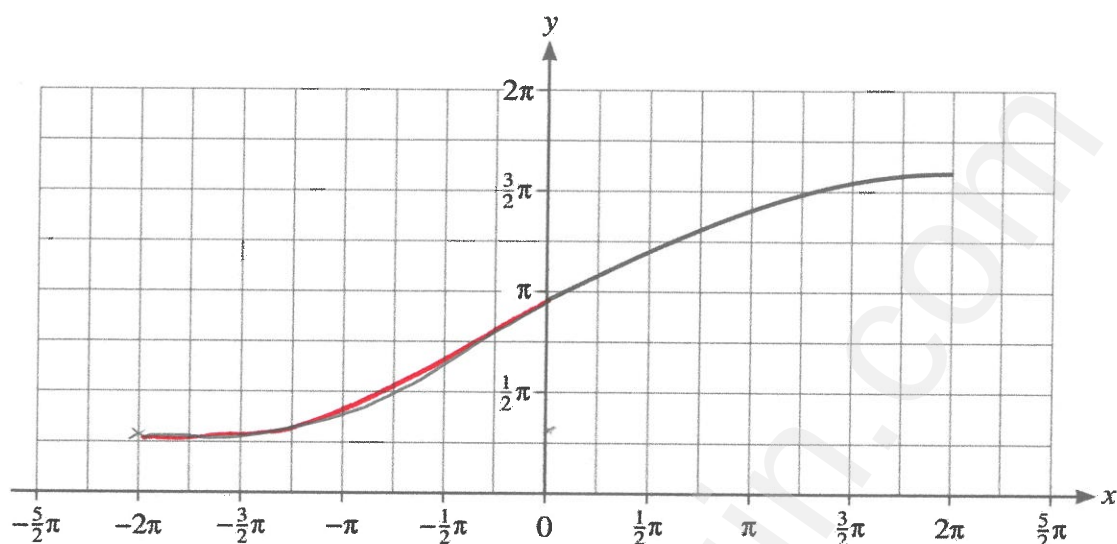
$$\frac{y-3}{2} = \sin\left(\frac{x}{4}\right)$$

$$\sin^{-1}\left(\frac{y-3}{2}\right) = \frac{x}{4}$$

$$x = 4 \sin^{-1}\left(\frac{y-3}{2}\right)$$

$$\underline{f^{-1}(x) = 4 \sin^{-1}\left(\frac{x-3}{2}\right)}$$

(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2 \sin \frac{1}{4}x$ for $-2\pi \leq x \leq 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

Yes, it has an inverse because it is one-to-one.

- (d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leq x \leq \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

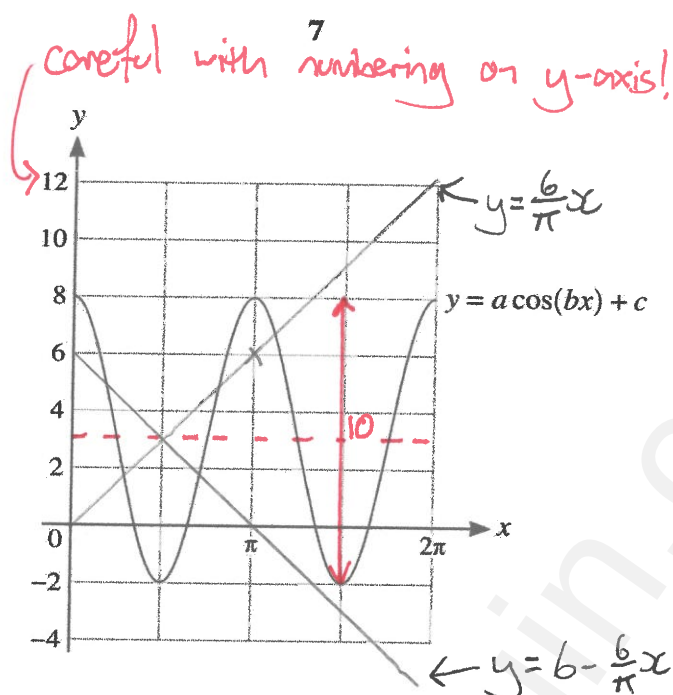
$$\sin x \rightarrow 3 + 2 \sin\left(\frac{1}{4}x\right) :$$

$$f(x) \rightarrow f\left(\frac{1}{4}x\right) \rightarrow 2f\left(\frac{1}{4}x\right) \rightarrow 2f\left(\frac{1}{4}x\right) + 3$$

① Stretch, parallel to x -axis, scale factor 4.

② Stretch parallel to y -axis, scale factor 2.

③ Translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$



The diagram shows part of the graph of $y = a \cos(bx) + c$.

- (a) Find the values of the positive integers a , b and c .

[3]

Amplitude is 10, where it is usually 2, so $a = 5$

Period is π , where it is usually 2π , so $b = 2$

"middle" of graph vertically is 3, where it is usually 0, so $c = 3$

- (b) For these values of a , b and c , use the given diagram to determine the number of solutions in the interval $0 \leq x \leq 2\pi$ for each of the following equations.

(i) $a \cos(bx) + c = \frac{6}{\pi}x$ → $y = \frac{6}{\pi}x$

[1]

$y = \frac{6}{\pi}x$ passes through the points $(0,0)$ and $(\pi,6)$
→ see diagram. Crosses in 3 places, so 3 solutions.

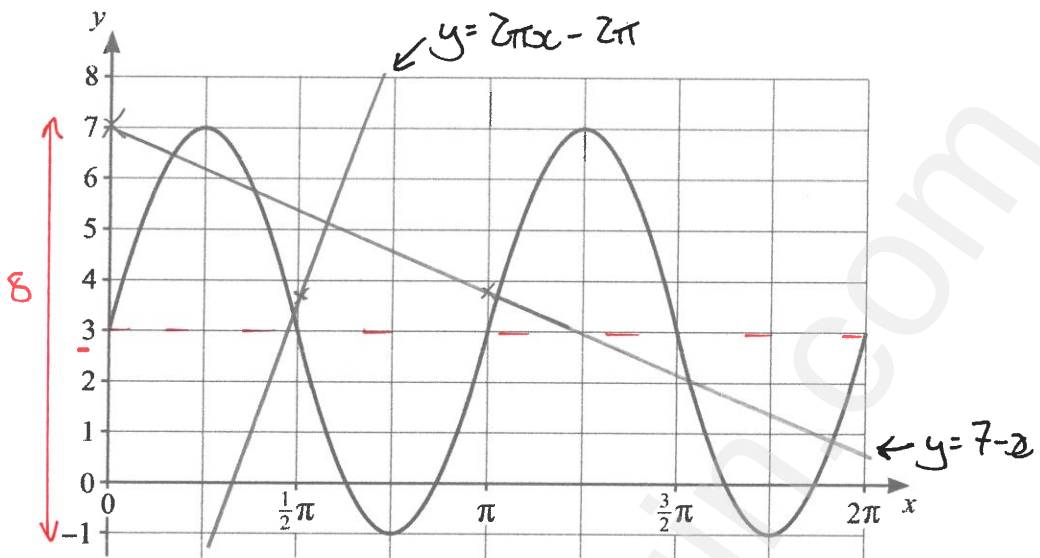
(ii) $a \cos(bx) + c = 6 - \frac{6}{\pi}x$ → $y = 6 - \frac{6}{\pi}x$

[1]

$y = 6 - \frac{6}{\pi}x$ passes through $(0,6)$ and $(\pi,0)$ → see diagram
Crosses in 2 places, so 2 solutions.



1



The diagram shows the curve with equation $y = a \sin(bx) + c$ for $0 \leq x \leq 2\pi$, where a , b and c are positive constants.

- (a) State the values of a , b and c .

[3]

Amplitude has been multiplied by 4, so $a = 4$

Period is π , where it is usually 2π , so $b = 2$

"middle" of graph vertically is 3, where it is usually 0, so $c = 3$

- (b) For these values of a , b and c , determine the number of solutions in the interval $0 \leq x \leq 2\pi$ for each of the following equations:

(i) $a \sin(bx) + c = 7 - x$

[1]

Plot graph of $y = 7 - x$ (passes through $(0, 7)$ and $(\pi, 7 - \pi)$)
Crosses in 5 places, so 5 solutions.

(ii) $a \sin(bx) + c = 2\pi(x - 1)$

[1]

Plot $y = 2\pi x - 2\pi$ (passes through $(0, -2\pi)$ and $(\frac{\pi}{2}, \pi^2 - 2\pi)$)
Crosses in 1 place, so 1 solution.

(Not many points fit on the axes)

- 11 A curve has equation $y = 3 \cos 2x + 2$ for $0 \leq x \leq \pi$.

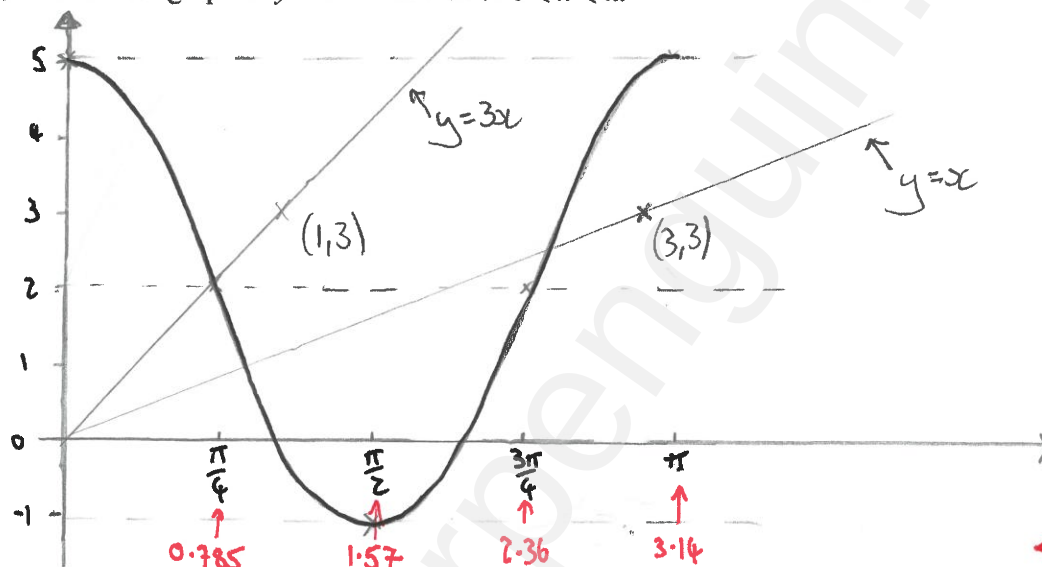
(a) State the greatest and least values of y .

[2]

Amplitude has been multiplied by 3, so $-3 \leq y \leq 3$,
but then we've added 2, so: $-1 \leq y \leq 5$

(b) Sketch the graph of $y = 3 \cos 2x + 2$ for $0 \leq x \leq \pi$.

[2]



(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3 \cos 2x + 2 = kx$ for $0 \leq x \leq \pi$ in each of the following cases.

(i) $k = -3$

[1]

No solutions (goes through (0,0) with negative gradient and misses graph.)

(ii) $k = 1$

[1]

2 solutions: See $y=x$ on diagram

(iii) $k = 3$

[1]

1 solution: See $y=3x$ on diagram

Functions f , g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3 \cos 2x + 2,$$

$$g(x) = f(2x) + 4,$$

$$h(x) = 2f\left(x + \frac{1}{2}\pi\right).$$

- (d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

$$g(x) = f(2x) + 4$$

$$\rightarrow f(x) \rightarrow f(2x) \rightarrow f(2x) + 4$$

① Stretch parallel to x -axis, scale factor $\frac{1}{2}$

② Translation by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

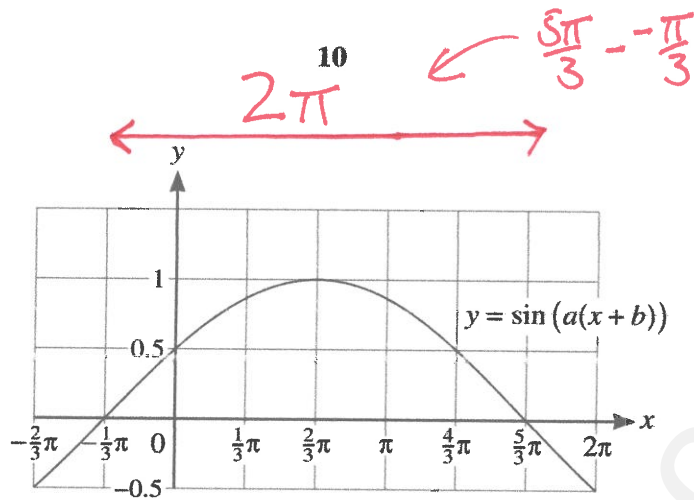
- (e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

$$h(x) = 2f\left(x + \frac{\pi}{2}\right)$$

$$\rightarrow f(x) \rightarrow f\left(x + \frac{\pi}{2}\right) \rightarrow 2f\left(x + \frac{\pi}{2}\right)$$

① Translation by $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$

② Stretch parallel to y -axis, scale factor 2



The diagram shows part of the graph of $y = \sin(a(x+b))$, where a and b are positive constants.

- (a) State the value of a and one possible value of b .

[2]

Distance between $-\frac{\pi}{3}$ and $\frac{5\pi}{3}$ is 2π , so period of graph = 4π

→ Stretch factor of 2, so $a = \frac{1}{2}$

$$y = \sin\left(\frac{1}{2}(x+b)\right)$$

sub. $(-\frac{\pi}{3}, 0)$:

$$0 = \sin\left(\frac{1}{2}\left(-\frac{\pi}{3}+b\right)\right)$$

$$0 = \frac{1}{2}\left(-\frac{\pi}{3}+b\right)$$

$$0 = -\frac{\pi}{6} + \frac{b}{2}$$

$$\frac{b}{2} = \frac{\pi}{6}$$

$$b = \frac{\pi}{3}$$

Another curve, with equation $y = f(x)$, has a single stationary point at the point (p, q) , where p and q are constants. This curve is transformed to a curve with equation

$$y = -3f\left(\frac{1}{4}(x+8)\right) \rightarrow y = -3f\left(\frac{1}{4}x+2\right)$$

- (b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q .

[3]

Careful: two transformations to x -coords, so the order isn't what you expect: Translate first.

x -coord:

$$p \xrightarrow{+2} p-2 \xrightarrow{\times 4} 4p-8$$

y -coord:

$$q \xrightarrow{\times -3} -3q$$

$$(4p-8, -3q)$$