

- 2 The graph of $y = f(x)$ is transformed to the graph of $y = 1 + f(\frac{1}{2}x)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

- ① Stretch, scale factor 2, parallel to x -axis.
- ② Translation by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- 1 The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

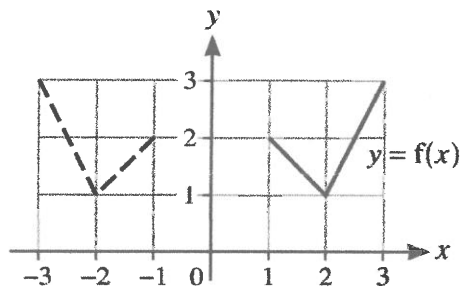
Describe fully, in the correct order, the two transformations that have been combined.

[4]

- ① Reflection in the x -axis ($f(x) \rightarrow -f(x)$)
- ② Translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

- 3 In each of parts (a), (b) and (c), the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$.

(a)

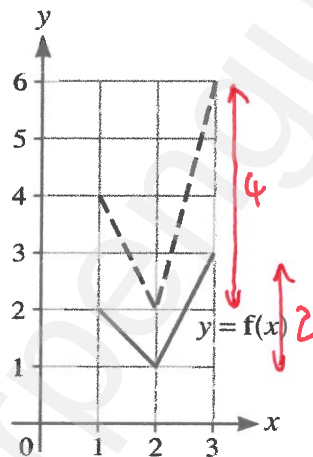


State, in terms of f , the equation of the graph shown with broken lines.

[1]

$$y = f(-x)$$

(b)

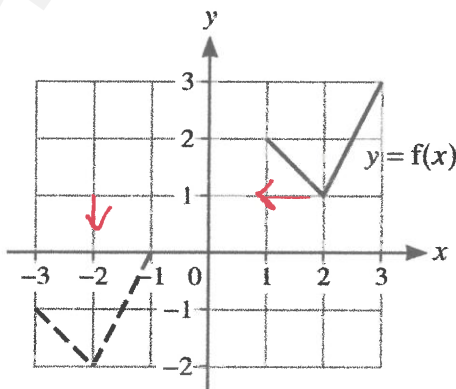


State, in terms of f , the equation of the graph shown with broken lines.

[1]

$$y = 2f(x)$$

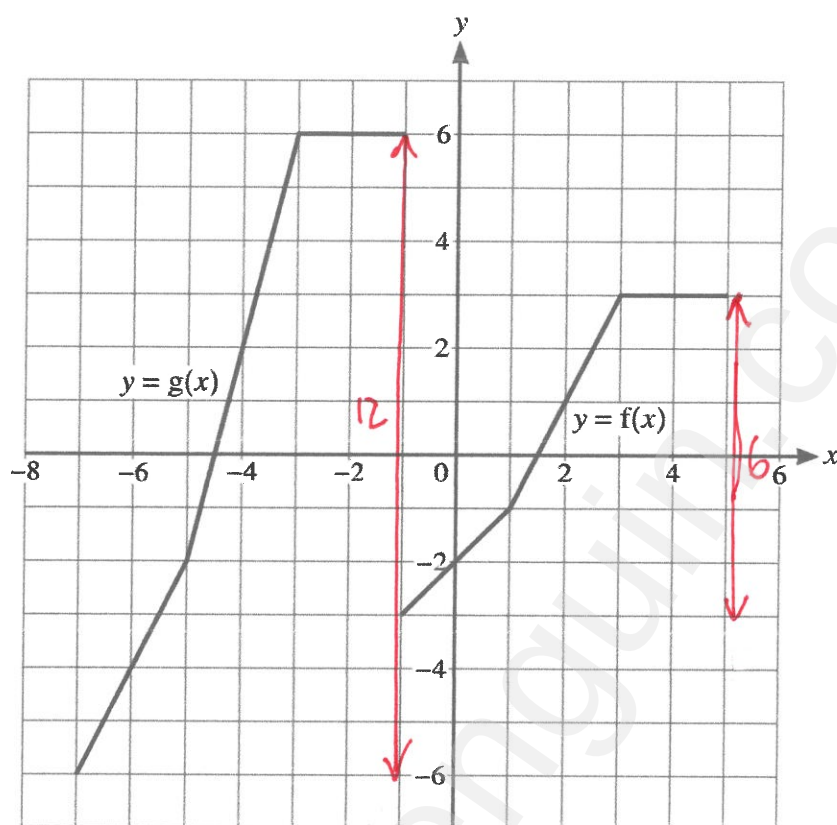
(c)



State, in terms of f , the equation of the graph shown with broken lines.

[2]

$$f(x+4) - 3$$



The diagram shows graphs with equations $y = f(x)$ and $y = g(x)$.

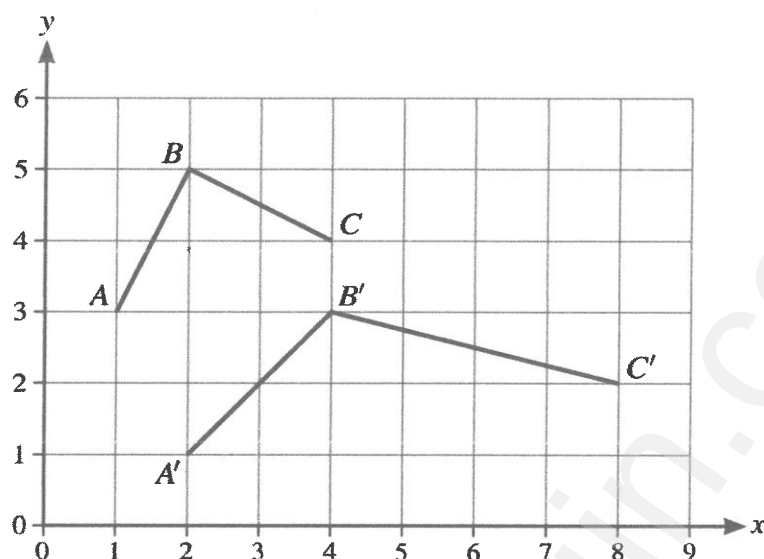
Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to $y = g(x)$. [4]

$f(x) \rightarrow g(x)$:

① Stretch, parallel to y-axis, scale factor 2

② Translation by $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$

1



The diagram shows the graph of $y = f(x)$, which consists of the two straight lines AB and BC . The lines $A'B'$ and $B'C'$ form the graph of $y = g(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = f(x)$.

State fully the two transformations.

[4]

① Stretch, parallel to x -axis, scale factor 2

② Translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

- 2 The curve $y = x^2$ is transformed to the curve $y = 4(x-3)^2 - 8$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations have been applied. [5]

$$x^2 \xrightarrow{\textcircled{1}} (x-3)^2 \xrightarrow{\textcircled{2}} 4(x-3)^2 \xrightarrow{\textcircled{3}} 4(x-3)^2 - 8$$

$$f(x) \longrightarrow f(x-3) \longrightarrow 4f(x-3) \longrightarrow 4f(x-3) - 8$$

① Translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

② Stretch, parallel to y-axis, scale factor 4

③ Translation by $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$

- 1 (a) Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

$$(x + 3)^2 - 9 + 5$$

$$\underline{(x + 3)^2 - 4}$$

- (b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved.

[2]

$$y = x^2 \rightarrow y = (x + 3)^2 - 4$$

$$f(x) \rightarrow f(x + 3) - 4$$

so: Translation by $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

- 2 (a) The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

① Translation by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

② Stretch parallel to y-axis, scale factor 2.

- (b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis^① and then stretched by scale factor $\frac{1}{3}$ ^② in the x-direction.

Write down the equation of the transformed curve.

[2]

① $f(x) \rightarrow f(-x): \sin(-2x) - 5(-x)$
 $= \sin(-2x) + 5x$

② $f(x) \rightarrow f(3x): \sin(-2 \times 3x) + 5(3x)$
 $= \underline{\underline{\sin(-6x) + 15x}}$

- 2 A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

- ① Stretch parallel to the x -axis with scale factor $\frac{1}{2}$
- ② Reflection in the y -axis
- ③ Stretch parallel to the y -axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a , b and c are constants.

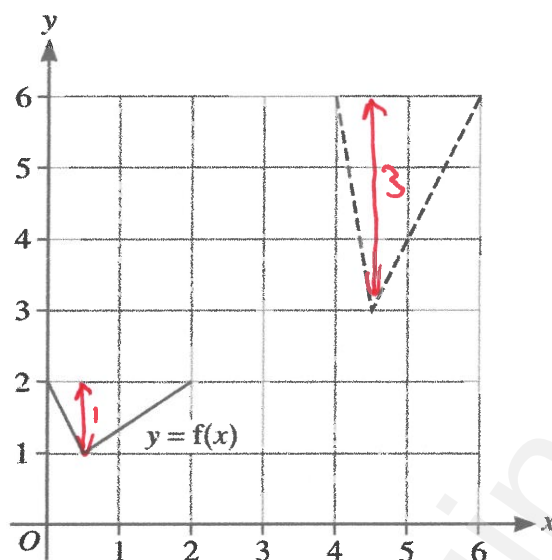
[4]

$$\begin{aligned} \text{① } f(x) &\rightarrow f(2x): (2x)^2 - 2(2x) + 5 \\ &= 4x^2 - 4x + 5 \end{aligned}$$

$$\begin{aligned} \text{② } f(x) &\rightarrow f(-x): 4(-x)^2 - 4(-x) + 5 \\ &= 4x^2 + 4x + 5 \end{aligned}$$

$$\begin{aligned} \text{③ } f(x) &\rightarrow 3f(x): 3[4x^2 + 4x + 5] \\ &= \underline{12x^2 + 12x + 15} \end{aligned}$$

5



In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

- (a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]

① Translation by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

② Stretch, scale factor 3, parallel to y-axis

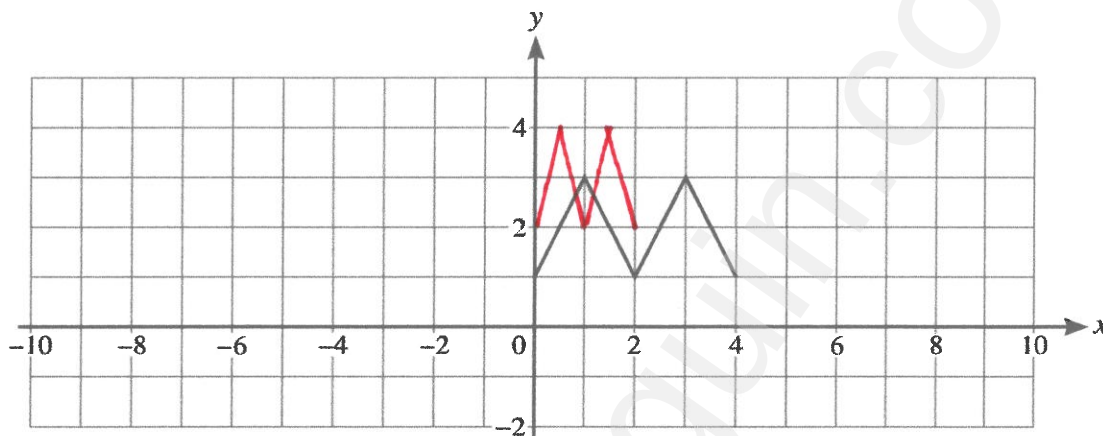
- (b) State in terms of y , f and x , the equation of the graph shown with broken lines. [2]

$$\underline{y = 3f(x - 4)}$$

- 5 The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x -direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) The diagram below shows the graph of $y = f(x)$.

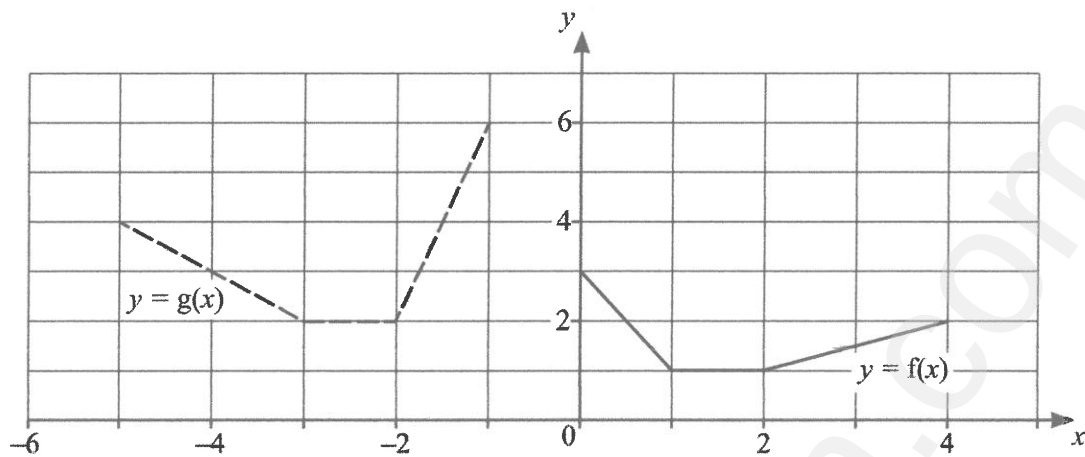
On the diagram sketch the graph of $y = g(x)$. [3]



(b) Find an expression for $g(x)$ in terms of $f(x)$. [2]

①: $f(x) \rightarrow f(2x)$

②: $f(2x) \rightarrow \underline{\underline{f(2x) + 1}}$



In the diagram, the graph with equation $y = f(x)$ is shown with solid lines and the graph with equation $y = g(x)$ is shown with broken lines.

- (a) Describe fully a sequence of three transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [6]

① Reflection in y -axis

② Translation by $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

③ Stretch in y -direction, factor 2.

- (b) Find an expression for $g(x)$ in the form $af(bx+c)$, where a , b and c are integers. [2]

!! Careful: ① & ② one both affecting x -coords:

①: $f(x) \rightarrow f(-x)$

②: $f(-x) \rightarrow f(-(x+1)) = f(-x-1)$ ← difficult

③: $f(-x-1) \rightarrow \underline{\underline{2f(-x-1)}}$



- 6 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$

$$g(x) = x^2 + 4x + 13.$$

- (a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x+p) + q$, where p and q are constants. [4]

$$f(x) = (x-1)^2 - 1 + 5$$

$$f(x) = (x-1)^2 + 4$$

$$g(x) = (x+2)^2 - 4 + 13$$

$$g(x) = (x+2)^2 + 9$$

$$\rightarrow \underline{g(x) = f(x+3) + 5}$$

- (b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

Translation by $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$

- 6 The equation of a curve is $y = x^2 - 8x + 5$.

(a) Find the coordinates of the minimum point of the curve.

[2]

Complete the square:

$$y = (x - 4)^2 - 16 + 5$$

$$y = (x - 4)^2 - 11$$

Minimum Point : (4, -11)

The curve is stretched by a factor of 2 parallel to the y-axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(b) Find the coordinates of the minimum point of the transformed curve.

[2]

① Multiply y-coord by 2:

$$(4, -22)$$

② Add 4 to x-coordinate and 1 to y-coordinate:

$$\underline{(8, -21)}$$

- (c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a , b and c are integers to be found. [4]

$$f(x) \xrightarrow{(1)} 2f(x) \xrightarrow{(2)} 2f(x-4) + 1$$

$$f(x) = x^2 - 8x + 5$$

$$2f(x) = 2x^2 - 16x + 10$$

$$2f(x-4) + 1 = 2(x-4)^2 - 16(x-4) + 10 + 1$$

$$= 2(x^2 - 8x + 16) - 16x + 64 + 11$$

$$= 2x^2 - 16x + 32 - 16x + 75$$

$$= \underline{\underline{2x^2 - 32x + 107}}$$

- 2 The graph of $y = f(x)$ is transformed to the graph of $y = f(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

- ① Stretch parallel to x -axis, scale factor $\frac{1}{2}$
- ② Translation by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

The point $P(5, 6)$ lies on the transformed curve $y = f(2x) - 3$.

(b) State the coordinates of the corresponding point on the original curve $y = f(x)$. [2]

$(5, 6)$ is on $f(2x) - 3$

undo transformation: $x \div 2$ $\downarrow$ $y + 3$ $\downarrow$

$(10, 9)$ is on $f(x)$

- 5 (a) Express $2x^2 - 8x + 14$ in the form $2[(x - a)^2 + b]$.

[2]

$$2[x^2 - 4x + 7]$$

$$2[(x-2)^2 - 4 + 7]$$

$$\underline{\underline{2[(x-2)^2 + 3]}}$$

The functions f and g are defined by

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R},$$

$$g(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

- (b) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied.

[4]

Either: A $2[(x-2)^2 + 3]$

① Translation by $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

② Stretch parallel to y -axis, scale factor 2.

Or: B $2(x-2)^2 + 6$

① Stretch parallel to y -axis, scale factor 2.

② Translation by $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$

- 9 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$

$$g(x) = 2x^2 + 4x + 12.$$

- (a) Express $f(x)$ in the form $(x - a)^2 + b$.

[1]

$$x^2 - 4x + 9 = (x - 2)^2 - 4 + 9$$

$$= (x - 2)^2 + 5$$

- (b) Express $g(x)$ in the form $2[(x + c)^2 + d]$.

[2]

$$2x^2 + 4x + 12 = 2[x^2 + 2x + 6]$$

$$= 2[(x + 1)^2 - 1 + 6]$$

$$= 2[(x + 1)^2 + 5]$$

- (c) Express $g(x)$ in the form $kf(x+h)$, where k and h are integers.

[1]

$$(x-2)^2 + 5 \rightarrow 2[(x+1)^2 + 5]$$

$$f(x) \rightarrow \underline{2f(x+3)}$$

- (d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$.

[4]

① Translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

② Stretch parallel to y-axis, scale factor 2.

- 4 (a) The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

Translation of $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$: $f(x+1) + 3$

so: $f(x) = x^2 + 2x - 5$

$$\begin{aligned} f(x+1) &= (x+1)^2 + 2(x+1) - 5 \\ &= x^2 + 2x + 1 + 2x + 2 - 5 \\ &= x^2 + 4x - 2 \end{aligned}$$

$$\begin{aligned} f(x+1) + 3 &= x^2 + 4x - 2 + 3 \\ &= \underline{x^2 + 4x + 1} \end{aligned}$$

- (b) The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied.

[2]

$$f(x) = x^2 + 2x - 5$$

$$\begin{aligned} f(2x) &= (2x)^2 + 2(2x) - 5 \\ &= 4x^2 + 4x - 5 \end{aligned}$$

→ Stretch parallel to x -axis, scale factor $\frac{1}{2}$.

- 4 The transformation R denotes a reflection in the x-axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

- (a) Find the equation, $y = g(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

$$f(x) \xrightarrow{R} -f(x) \xrightarrow{T} -f(x-3) - 1$$

$$\rightarrow \underline{g(x) = -(x-3)^2 - 1}$$

- (b) Find the equation, $y = h(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

$$f(x) \xrightarrow{T} f(x-3) - 1 \xrightarrow{R} -f(x-3) + 1$$

$$\rightarrow h(x) = -(x-3)^2 + 1$$

- (c) State fully the transformation that maps the curve $y = g(x)$ onto the curve $y = h(x)$. [2]

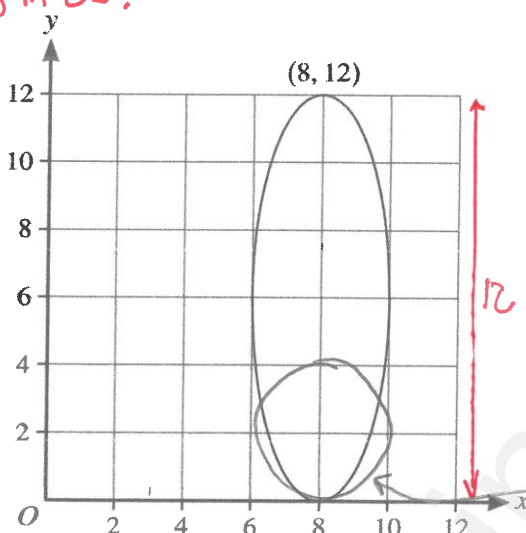
$$\begin{aligned} g(x) &= -(x-3)^2 - 1 \\ h(x) &= -(x-3)^2 + 1 \end{aligned} \quad \downarrow +2$$

$$\rightarrow \underline{\underline{\text{Translation by } \begin{pmatrix} 0 \\ 2 \end{pmatrix}}}$$

5

6

Watch out for
numbers going up in 2s!



The diagram shows a curve which has a maximum point at $(8, 12)$ and a minimum point at $(8, 0)$. The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y -direction.

- (a) State the scale factor of the stretch.

[1]

Looking at the width, the diameter of the circle must have been 4, so the stretch was scale factor 3

- (b) State the radius of the original circle.

[1]

Radius = 2 (again - careful with axes)

- (c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied.

[2]

(8, 2)

- (d) State the coordinates of the centre of the original circle.

[2]

(1, 5)

(8-7=1, 2+3=5)