

- 8 (a) Express $3 \cos 2x - \sqrt{3} \sin 2x$ in the form $R \cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and α . [3]

$$3 \cos 2x - \sqrt{3} \sin 2x = R \cos 2x \cos \alpha - R \sin 2x \sin \alpha$$

$$R \cos \alpha = 3 \quad (1) \quad R \sin \alpha = \sqrt{3} \quad (2)$$

$$(2) \div (1): \tan \alpha = \frac{\sqrt{3}}{3}$$

$$\alpha = \frac{\pi}{6}$$

$$\begin{aligned} (1)^2 + (2)^2: R^2 &= (\sqrt{3})^2 + 3^2 \\ &= 3 + 9 \\ &= 12 \\ R &= \sqrt{12} \\ &= 2\sqrt{3} \end{aligned}$$

$$\rightarrow \underline{2\sqrt{3} \cos\left(2x + \frac{\pi}{6}\right)}$$

- 2 (a) Express $5 \sin x - 3 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

$$5 \sin x - 3 \cos x = R \sin x \cos \alpha - R \cos x \sin \alpha$$

$$R \cos \alpha = 5 \quad (1) \quad R \sin \alpha = 3 \quad (2)$$

$$(2) \div (1): \tan \alpha = \frac{3}{5}$$

$$\alpha = 0.54 \text{ STO}$$

$$(1)^2 + (2)^2: R^2 = 5^2 + 3^2$$

$$= 25 + 9$$

$$R = \sqrt{34}$$

$$\underline{\underline{\sqrt{34} \sin(x - 0.54)}}$$

- (b) Hence state the greatest and least possible values of $(5 \sin x - 3 \cos x)^2$. [2]

$$\text{from (a): } 5 \sin x - 3 \cos x = \sqrt{34} \sin(x - 0.54)$$

$$\rightarrow (\sqrt{34} \sin(x - 0.54))^2$$

$$\text{max. of } \sqrt{34} \sin(x - 0.54) \text{ is } \sqrt{34}, \text{ minimum is } -\sqrt{34}$$

$$\text{greatest possible value} = (\sqrt{34})^2 = \underline{\underline{34}}$$

$$\text{least possible value} = (0)^2 = \underline{\underline{0}}$$

- 4 (a) Express $4 \cos x - \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

$$4 \cos x - \sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$$

$$R \cos \alpha = 4 \quad (1) \quad R \sin \alpha = 1 \quad (2)$$

$$(2) \div (1): \tan \alpha = \frac{1}{4}$$

$$\alpha = 14.04^\circ \text{ STO}$$

$$(1)^2 + (2)^2: R^2 = 4^2 + 1^2$$

$$= 16 + 1$$

$$R = \sqrt{17}$$

$$\underline{\sqrt{17} \cos(x + 14.04^\circ)}$$

- (b) Hence solve the equation $4 \cos 2x - \sin 2x = 3$ for $0^\circ < x < 180^\circ$. [5]

from (a): $4 \cos x - \sin x = \sqrt{17} \cos(x + 14.04^\circ)$

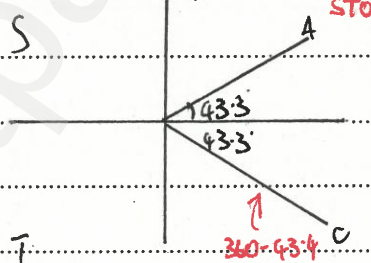
so: $4 \cos 2x - \sin 2x = \sqrt{17} \cos(2x + 14.04^\circ)$

$$\sqrt{17} \cos(2x + 14.04^\circ) = 3$$

$$\cos(2x + 14.04^\circ) = \frac{3}{\sqrt{17}}$$

$$2x + 14.04^\circ = 43.31^\circ$$

$0 < x < 180$
$0 < 2x < 360$
$14 < 2x + 14.04 < 374$



$$2x + 14.04^\circ = 43.31^\circ, 316.68^\circ$$

$$2x = 29.27^\circ, 302.64^\circ$$

$$x = 14.6^\circ, 151.3^\circ$$

- 5 (a) Express $\sqrt{7} \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

$$\sqrt{7} \sin x + 2 \cos x = R \sin x \cos \alpha + R \cos x \sin \alpha$$

$$R \cos \alpha = \sqrt{7} \quad (1) \quad R \sin \alpha = 2 \quad (2)$$

$$(2) \div (1): \tan \alpha = \frac{2}{\sqrt{7}}$$

$$\alpha = \underline{37.09^\circ}$$

$$(1)^2 + (2)^2: R^2 = (\sqrt{7})^2 + 2^2$$

$$= 7 + 4$$

$$R = \sqrt{11}$$

$$\underline{\sqrt{11} \sin(x + 37.09^\circ)}$$

(b) Hence solve the equation $\sqrt{7} \sin 2\theta + 2 \cos 2\theta = 1$, for $0^\circ < \theta < 180^\circ$.

[5]

from (a): $\sqrt{7} \sin x + 2 \cos x = \sqrt{11} \sin(x + 37.09^\circ)$

so: $\sqrt{7} \sin 2\theta + 2 \cos 2\theta = \sqrt{11} \sin(2\theta + 37.09^\circ)$

$$\sqrt{11} \sin(2\theta + 37.09^\circ) = 1$$

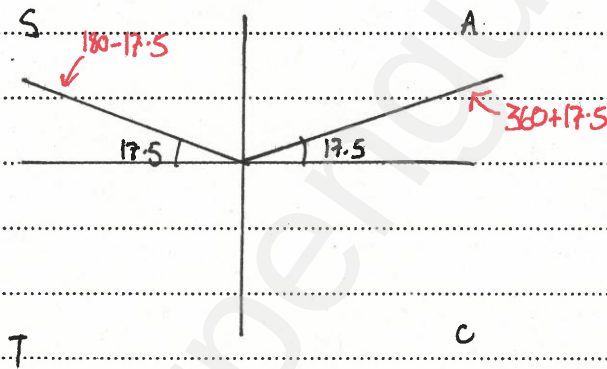
$$\sin(2\theta + 37.09^\circ) = \frac{1}{\sqrt{11}}$$

$$0 < \theta < 180$$

$$0 < 2\theta < 360$$

$$37.09 < 2\theta + 37.09 < 397.09$$

$$2\theta + 37.09 = 17.54$$



$$2\theta + 37.09 = 17.54^\circ, 162.45^\circ, 377.54^\circ$$

$$2\theta = -19.53^\circ, 125.36^\circ, 340.46^\circ$$

$$\theta = -9.8^\circ, 62.7^\circ, 170.2^\circ$$

$$\theta = \underline{62.7^\circ, 170.2^\circ}$$

- 5 (a) Express $\sqrt{2} \cos x - \sqrt{5} \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

$$\sqrt{2} \cos x - \sqrt{5} \sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$$

$$R \cos \alpha = \sqrt{2} \quad (1) \quad R \sin \alpha = \sqrt{5} \quad (2)$$

$$(2) \div (1): \tan \alpha = \frac{\sqrt{5}}{\sqrt{2}}$$

$$\alpha = 57.688^\circ$$

$$\begin{aligned} (1)^2 + (2)^2: \quad R^2 &= (\sqrt{2})^2 + (\sqrt{5})^2 \\ &= 2 + 5 \\ R &= \sqrt{7} \end{aligned}$$

$$\underline{\underline{\sqrt{7} \cos(x + 57.688^\circ)}}$$

(b) Hence solve the equation $\sqrt{2} \cos 2\theta - \sqrt{5} \sin 2\theta = 1$, for $0^\circ < \theta < 180^\circ$.

[4]

from part (a): $\sqrt{2} \cos x - \sqrt{5} \sin x = \sqrt{7} \cos(x + 57.688)$

so: $\sqrt{2} \cos 2\theta - \sqrt{5} \sin 2\theta = \sqrt{7} \cos(2\theta + 57.688)$

$$\sqrt{7} \cos(2\theta + 57.688) = 1$$

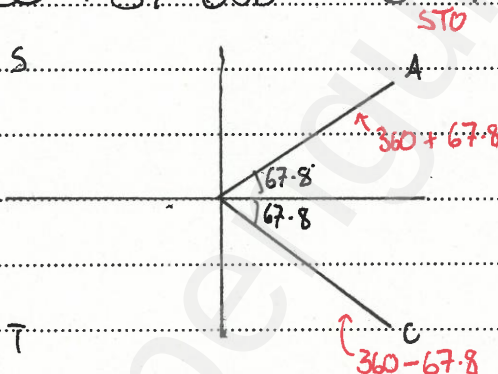
$$\cos(2\theta + 57.688) = \frac{1}{\sqrt{7}}$$

$$0 < \theta < 180$$

$$0 < 2\theta < 360$$

$$57.7 < 2\theta + 57.688 < 417.7$$

$$2\theta + 57.688 = 67.79...$$



$$2\theta + 57.688 = 67.79..., 292.20..., \text{ and } 427.79...$$

$$2\theta = 10.10..., 234.51...$$

$$\theta = 5.05..., 117.25...$$

$$\theta = \underline{5.1^\circ}, \underline{117.3^\circ}$$

- 6 (a) Express $5 \sin \theta + 12 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

$$5 \sin \theta + 12 \cos \theta = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

$$12 \cos \theta + 5 \sin \theta = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

$$R \cos \alpha = 12 \quad (1) \quad R \sin \alpha = 5 \quad (2)$$

$$(2) \div (1): \frac{R \sin \alpha}{R \cos \alpha} = \frac{5}{12}$$

$$\tan \alpha = \frac{5}{12}$$

$$\alpha = 0.395 \quad \text{STO}$$

$$(1)^2 + (2)^2: R^2 = 12^2 + 5^2$$

$$R^2 = 144 + 25$$

$$R^2 = 169$$

$$R = 13$$

$$\rightarrow \underline{13 \cos(\theta - 0.395)}$$

(b) Hence solve the equation $5 \sin 2x + 12 \cos 2x = 6$ for $0 \leq x \leq \pi$.

[4]

from (a): $5 \sin \theta + 12 \cos \theta = 13 \cos(\theta - 0.395)$

replace θ with $2x$:

$$5 \sin 2x + 12 \cos 2x = 13 \cos(2x - 0.395)$$

$$\rightarrow 13 \cos(2x - 0.395) = 6$$

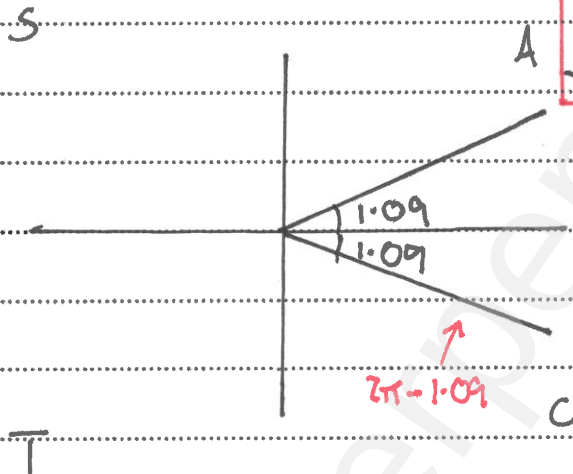
$$\cos(2x - 0.395) = \frac{6}{13}$$

$$2x - 0.395 = 1.09 \text{ STO}$$

$$0 \leq x \leq \pi$$

$$0 \leq 2x \leq 2\pi$$

$$-0.395 \leq 2x - 0.395 \leq 2\pi - 0.395$$



$$2x - 0.395 = 1.09, 5.19$$

$$2x = 1.486, 5.587$$

$$x = 0.743, 2.79$$

- 6 (a) By first expanding $\cos(x - 60^\circ)$, show that the expression

$$2 \cos(x - 60^\circ) + \cos x$$

can be written in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

$$\begin{aligned} \cos(x - 60^\circ) &= \cos x \cos 60^\circ + \sin x \sin 60^\circ \\ &= \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \end{aligned}$$

$$\begin{aligned} 2 \cos(x - 60^\circ) + \cos x &= 2 \left[\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right] + \cos x \\ &= \cos x + \sqrt{3} \sin x + \cos x \\ &= 2 \cos x + \sqrt{3} \sin x \end{aligned}$$

$$2 \cos x + \sqrt{3} \sin x = R \cos x \cos \alpha + R \sin x \sin \alpha$$

$$R \cos \alpha = 2 \quad (1)$$

$$R \sin \alpha = \sqrt{3} \quad (2)$$

$$\begin{aligned} (2) \div (1): \quad \tan \alpha &= \frac{\sqrt{3}}{2} \\ \alpha &= 40.89^\circ \end{aligned}$$

$$\begin{aligned} (1)^2 + (2)^2: \quad R^2 &= 2^2 + (\sqrt{3})^2 \\ &= 4 + 3 \end{aligned}$$

$$R = \sqrt{7}$$

$$\Rightarrow \underline{\underline{\sqrt{7} \cos(x - 40.89^\circ)}}$$

- (b) Hence find the value of x in the interval $0^\circ < x < 360^\circ$ for which $2 \cos(x - 60^\circ) + \cos x$ takes its least possible value. [2]

$$\text{least value of } \sqrt{7} \cos(x - 40.89^\circ) \text{ is } -\sqrt{7}:$$

$$\sqrt{7} \cos(x - 40.89^\circ) = -\sqrt{7}$$

$$\cos(x - 40.89^\circ) = -1$$

$$x - 40.89^\circ = 180^\circ$$

$$\underline{\underline{x = 220.9^\circ}}$$

- 6 (a) Express $3 \cos x + 2 \cos(x - 60^\circ)$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [4]

$$\begin{aligned} 2 \cos(x - 60^\circ) &= 2[\cos x \cos 60^\circ + \sin x \sin 60^\circ] \\ &= 2\left[\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x\right] \\ &= \cos x + \sqrt{3} \sin x \end{aligned}$$

$$\begin{aligned} \rightarrow 3 \cos x + 2 \cos(x - 60^\circ) &= 3 \cos x + \cos x + \sqrt{3} \sin x \\ &= 4 \cos x + \sqrt{3} \sin x \end{aligned}$$

$$\rightarrow 4 \cos x + \sqrt{3} \sin x = R \cos x \cos \alpha + R \sin x \sin \alpha$$

$$R \cos \alpha = 4 \quad (1) \qquad R \sin \alpha = \sqrt{3} \quad (2)$$

$$(2) \div (1): \frac{R \sin \alpha}{R \cos \alpha} = \frac{\sqrt{3}}{4}$$

$$\tan \alpha = \frac{\sqrt{3}}{4}$$

$$\alpha = 23.41^\circ$$

$$\begin{aligned} (1)^2 + (2)^2: R^2 &= 4^2 + (\sqrt{3})^2 \\ &= 16 + 3 \\ &= 19 \\ R &= \sqrt{19} \end{aligned}$$

$$\rightarrow \underline{\underline{\sqrt{19} \cos(x - 23.41^\circ)}}$$

(b) Hence solve the equation

$$3 \cos 2\theta + 2 \cos(2\theta - 60^\circ) = 2.5$$

for $0^\circ < \theta < 180^\circ$.

[4]

replace x with 2θ in part (a):

$$\sqrt{19} \cos(2\theta - 23.41) = 2.5$$

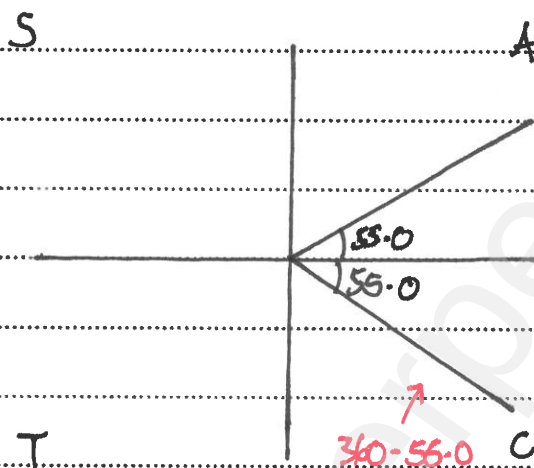
$$\cos(2\theta - 23.41) = \frac{2.5}{\sqrt{19}}$$

$$2\theta - 23.41 = 55.0$$

$$0 < \theta < 180$$

$$0 < 2\theta < 360$$

$$-23.41 < 2\theta - 23.41 < 336.59$$



$$2\theta - 23.41 = 55.0, 305.0$$

$$2\theta = 78.4, 328.4$$

$$\theta = 39.2^\circ, 164.2^\circ$$

- 6 (a) Express $\sqrt{6} \cos \theta + 3 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

$$\sqrt{6} \cos \theta + 3 \sin \theta = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

$$R \cos \alpha = \sqrt{6} \quad (1) \quad R \sin \alpha = 3 \quad (2)$$

$$(2) \div (1): \tan \alpha = \frac{3}{\sqrt{6}}$$

$$\alpha = 50.77^\circ$$

STO

$$(1)^2 + (2)^2: R^2 = (\sqrt{6})^2 + 3^2$$

$$= 6 + 9$$

$$R = \sqrt{15}$$

$$\underline{\underline{\sqrt{15} \cos(\theta - 50.77^\circ)}}$$

(b) Hence solve the equation $\sqrt{6} \cos \frac{1}{3}x + 3 \sin \frac{1}{3}x = 2.5$, for $0^\circ < x < 360^\circ$.

[4]

from (a) : $\sqrt{6} \cos \theta + 3 \sin \theta = \sqrt{15} \cos(\theta - 50.77^\circ)$

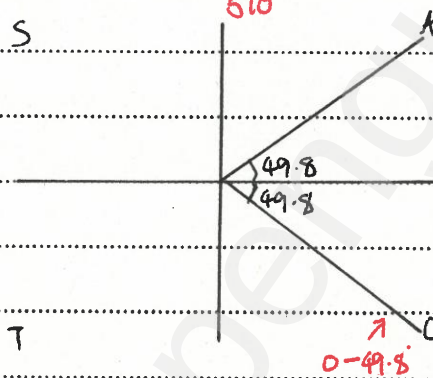
so : $\sqrt{6} \cos\left(\frac{x}{3}\right) + 3 \sin\left(\frac{x}{3}\right) = \sqrt{15} \cos\left(\frac{x}{3} - 50.77^\circ\right)$

$$\sqrt{15} \cos\left(\frac{x}{3} - 50.77^\circ\right) = 2.5$$

$$\cos\left(\frac{x}{3} - 50.77^\circ\right) = \frac{2.5}{\sqrt{15}}$$

$0 < x < 360$
$0 < \frac{x}{3} < 120$
$-50.8 < \frac{x}{3} - 50.77 < 69.2$

$$\frac{x}{3} - 50.77 = 49.79 \dots$$



$$\frac{x}{3} - 50.77 = 49.79 \dots, -49.79 \dots$$

$$\frac{x}{3} = 100.56 \dots, 0.97 \dots$$

$$x = 301.69 \dots, 2.91 \dots$$

$$\underline{x = 2.9^\circ, 301.7^\circ}$$

- 7 (a) Show that the equation $\sqrt{5} \sec x + \tan x = 4$ can be expressed as $R \cos(x + \alpha) = \sqrt{5}$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 2 decimal places. [4]

$$\sqrt{5} \sec x + \tan x = 4$$

$$\frac{\sqrt{5}}{\cos x} + \frac{\sin x}{\cos x} = 4$$

$$\sqrt{5} + \sin x = 4 \cos x$$

$$4 \cos x - \sin x = \sqrt{5}$$

$$4 \cos x - \sin x = R \cos x \cos \alpha + R \sin x \sin \alpha$$

$$R \cos \alpha = 4 \quad (1)$$

$$R \sin \alpha = 1 \quad (2)$$

$$(2) \div (1): \tan \alpha = \frac{1}{4}$$

$$\alpha = 14.04^\circ$$

$$(1)^2 + (2)^2: R^2 = 4^2 + 1^2$$

$$= 16 + 1$$

$$R = \sqrt{17}$$

$$\underline{\underline{\sqrt{17} \cos(x + 14.04^\circ) = \sqrt{5}}}$$

(b) Hence solve the equation $\sqrt{5} \sec 2x + \tan 2x = 4$, for $0^\circ < x < 180^\circ$.

[4]

from (a): $\sqrt{5} \sec x + \tan x = 4 \Leftrightarrow \sqrt{17} \cos(x + 14.04) = \sqrt{5}$

so: $\sqrt{5} \sec 2x + \tan 2x = 4 \Leftrightarrow \sqrt{17} \cos(2x + 14.04) = \sqrt{5}$

$$\sqrt{17} \cos(2x + 14.04) = \sqrt{5}$$

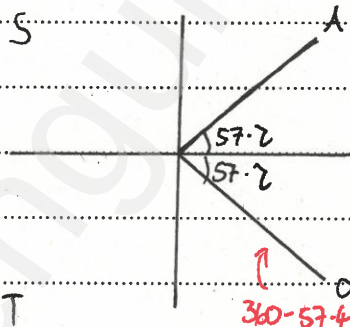
$$\cos(2x + 14.04) = \frac{\sqrt{5}}{\sqrt{17}}$$

$$0 < x < 180$$

$$2x + 14.04 = 57.15^\circ$$

$$0 < 2x < 360$$

$$14 < 2x + 14.04 < 374$$



$$2x + 14.04 = 57.15^\circ, 302.84^\circ$$

$$2x = 43.12^\circ, 288.80^\circ$$

$$x = \underline{21.6^\circ}, \underline{144.4^\circ}$$

- 8 (a) Express $3\sin x + 2\sqrt{2}\cos\left(x + \frac{1}{4}\pi\right)$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. State the exact value of R and give α correct to 3 decimal places. [4]

$$\begin{aligned} 2\sqrt{2}\cos\left(x + \frac{\pi}{4}\right) &= 2\sqrt{2}\left[\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4}\right] \\ &= 2\sqrt{2}\left[\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right] \\ &= \underline{2\cos x - 2\sin x} \end{aligned}$$

$$\begin{aligned} 3\sin x + \underline{2\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)} &= 3\sin x + 2\cos x - 2\sin x \\ &= \sin x + 2\cos x \end{aligned}$$

$$\rightarrow \sin x + 2\cos x = R\sin x \cos \alpha + R\cos x \sin \alpha$$

$$R\cos \alpha = 1 \quad (1) \qquad R\sin \alpha = 2 \quad (2)$$

$$(2) \div (1): \tan \alpha = \frac{2}{1}$$

$$\alpha = \underline{1.107} \text{ sf3}$$

$$\begin{aligned} (1)^2 + (2)^2: R^2 &= 1^2 + 2^2 \\ &= 1 + 4 \\ R &= \sqrt{5} \end{aligned}$$

$$\rightarrow \underline{\sqrt{5} \sin(x + 1.107)}$$

(b) Hence solve the equation

$$6 \sin \frac{1}{2}\theta + 4\sqrt{2} \cos\left(\frac{1}{2}\theta + \frac{1}{4}\pi\right) = 3$$

for $-4\pi < \theta < 4\pi$.

[5]

from (a): $3 \sin x + 2\sqrt{2} \cos\left(x + \frac{\pi}{4}\right) = \sqrt{5} \sin(x + 1.107)$) $\times 2$
 $6 \sin x + 4\sqrt{2} \cos\left(x + \frac{\pi}{4}\right) = 2\sqrt{5} \sin(x + 1.107)$) $x \rightarrow \frac{1}{2}\theta$
 $6 \sin \frac{1}{2}\theta + 4\sqrt{2} \cos\left(\frac{1}{2}\theta + \frac{\pi}{4}\right) = 2\sqrt{5} \sin\left(\frac{1}{2}\theta + 1.107\right)$

$$\rightarrow 2\sqrt{5} \sin\left(\frac{1}{2}\theta + 1.107\right) = 3$$

Range of θ is big for this question: expect lots of solutions:

$$-4\pi < \theta < 4\pi \quad \div 2$$

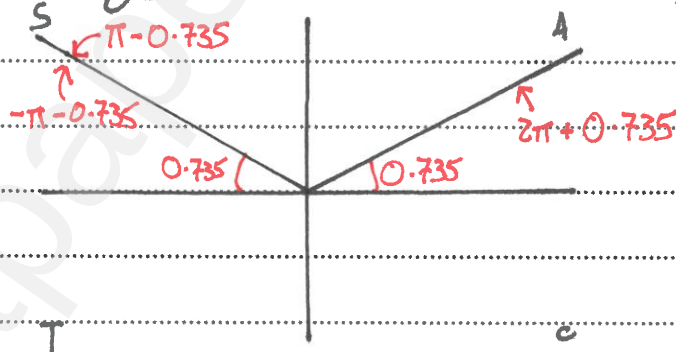
$$-2\pi < \frac{1}{2}\theta < 2\pi \quad + 1.107$$

$$\underline{-5.18 < \frac{1}{2}\theta + 1.107 < 7.39}$$

$$2\sqrt{5} \sin\left(\frac{1}{2}\theta + 1.107\right) = 3$$

$$\sin\left(\frac{1}{2}\theta + 1.107\right) = \frac{3}{2\sqrt{5}}$$

$$\frac{1}{2}\theta + 1.107 = 0.735 \quad \text{STO}$$



$$\frac{1}{2}\theta + 1.107 = -3.8769, 0.7353, 2.4063, 7.0185$$

$$\frac{1}{2}\theta = -4.9841, -0.3718, 1.2991, 5.9114$$

$$\underline{\underline{\theta = -9.97, -0.744, 2.60, 11.8}}$$