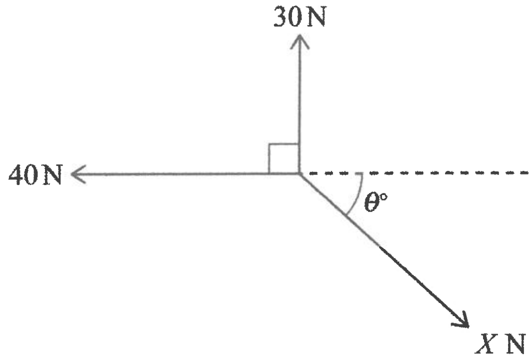
1



Three coplanar forces of magnitudes 40 N, 30 N and X N act at a point in the directions shown in the diagram.

Given that the forces are in equilibrium, find the values of θ and X. [4]

$R(\rightarrow)$: $X\cos\theta - 40 = 0$

$X\cos\theta = 40$ ①

$R(\uparrow)$: $30 - X\sin\theta = 0$

$X\sin\theta = 30$ ②

② ÷ ①: $\dfrac{X\sin\theta}{X\cos\theta} = \dfrac{30}{40}$

$\tan\theta = \dfrac{3}{4}$

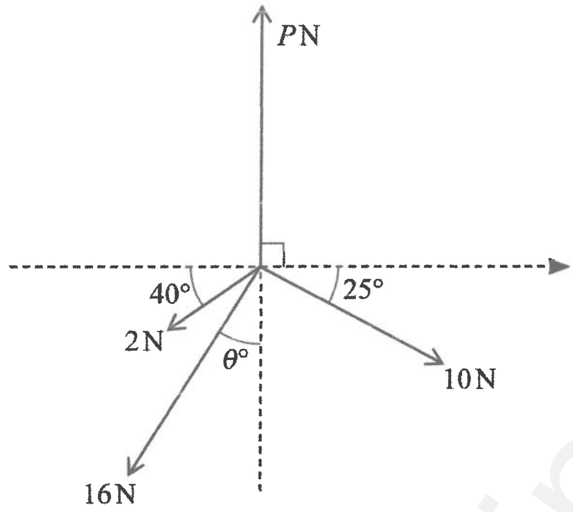
$\theta = 36.9°$ STO

→ ①: $X\cos(36.869...) = 40$

$X = \dfrac{40}{\cos(36.869..)}$

$X = 50$ N

3



Four coplanar forces of magnitude P N, 10 N, 16 N and 2 N act at a point in the directions shown in the diagram. It is given that the forces are in equilibrium.

Find the values of θ and P. [6]

$R(\rightarrow)$: $10\cos 25 - 16\sin\theta - 2\cos 40 = 0$

$16\sin\theta = 10\cos 25 - 2\cos 40$

$\sin\theta = \dfrac{10\cos 25 - 2\cos 40}{16}$

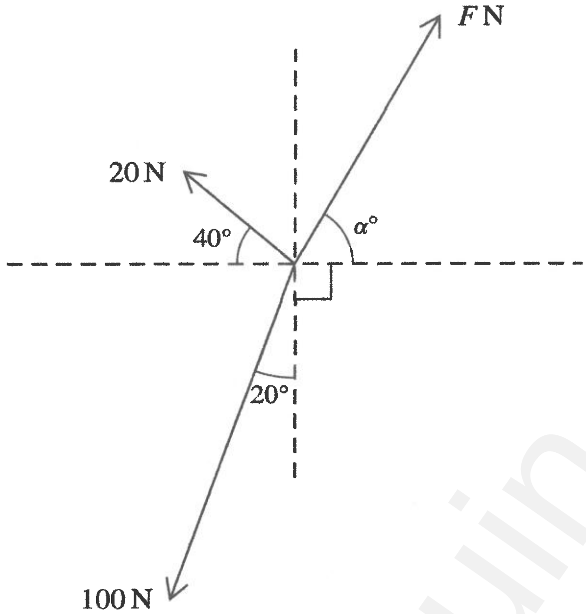
$\theta = \underline{28.1^\circ}$ (STORE full answer)

$R(\uparrow)$: $P - 10\sin 25 - 2\sin 40 - 16\cos(28.07\ldots) = 0$

$P = 10\sin 25 + 2\sin 40 + 16\cos(28.07\ldots)$

$= \underline{19.6\text{N}}$

4



Three coplanar forces of magnitudes 20 N, 100 N and F N act at a point. The directions of these forces are shown in the diagram.

Given that the three forces are in equilibrium, find F and α. [6]

$R(\uparrow)$: $F\sin\alpha + 20\sin 40 - 100\cos 20 = 0$

$F\sin\alpha = 100\cos 20 - 20\sin 40$

$F\sin\alpha = 81.114$ STO (1)

$R(\rightarrow)$: $F\cos\alpha - 20\cos 40 - 100\sin 20 = 0$

$F\cos\alpha = 100\sin 20 + 20\cos 40$

$F\cos\alpha = 49.523$ STO (2)

(1) ÷ (2): $\dfrac{F\sin\alpha}{F\cos\alpha} = \dfrac{81.114}{49.523}$

$\tan\alpha = 1.638$

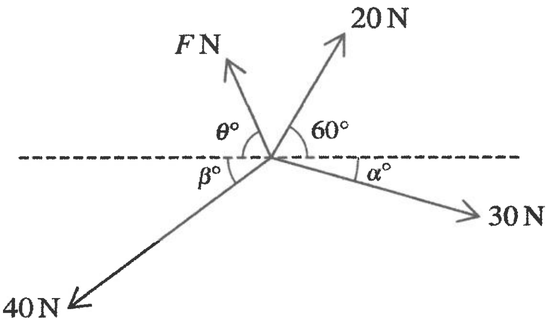
$\alpha = 58.6°$ STO

Sub into (1): $F\sin(58.6) = 81.114$

$F = \dfrac{81.114}{\sin(58.6)}$

$= 95.0\text{N}$

3



Four coplanar forces act at a point. The magnitudes of the forces are 20 N, 30 N, 40 N and F N. The directions of the forces are as shown in the diagram, where $\sin\alpha° = 0.28$ and $\sin\beta° = 0.6$.

$\alpha = 16.26$ STO $\beta = 36.87$ STO

Given that the forces are in equilibrium, find F and θ. [6]

$R(\uparrow)$: $F\sin\theta + 20\sin 60 - 40\sin\beta - 30\sin\alpha = 0$

$F\sin\theta + 20\sin 60 - 40\times 0.6 - 30\times 0.28 = 0$

$F\sin\theta + 20\sin 60 - 24 - 8.4 = 0$

$F\sin\theta = 32.4 - 20\sin 60$ (1)

$R(\rightarrow)$: $20\cos 60 + 30\cos\alpha - F\cos\theta - 40\cos\beta = 0$

$20\cos 60 + 30\cos(16.26) - F\cos\theta - 40\cos(36.87) = 0$

$F\cos\theta = 20\cos 60 + 30\cos(16.26) - 40\cos(36.87)$

$F\cos\theta = 6.8$ (2)

(1)÷(2): $\dfrac{F\sin\theta}{F\cos\theta} = \dfrac{32.4 - 20\sin 60}{6.8}$

$\tan\theta = 2.218$

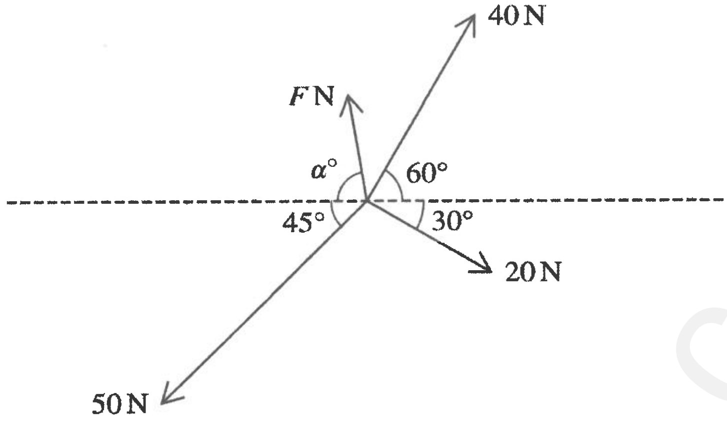
$\theta = \underline{65.7°}$ STO

Sub into (2): $F\cos(65.7) = 6.8$

$F = \dfrac{6.8}{\cos(65.7)}$

$F = \underline{16.5\text{ N}}$

3



Four coplanar forces of magnitudes 40 N, 20 N, 50 N and F N act at a point in the directions shown in the diagram. The four forces are in equilibrium.

Find F and α. [6]

$R(\uparrow): 40\sin 60 + F\sin\alpha - 20\sin 30 - 50\sin 45 = 0$

$F\sin\alpha = 20\sin 30 + 50\sin 45 - 40\sin 60$

$F\sin\alpha = 10.714$ STO ①

$R(\rightarrow): 40\cos 60 + 20\cos 30 - F\cos\alpha - 50\cos 45 = 0$

$F\cos\alpha = 40\cos 60 + 20\cos 30 - 50\cos 45$

$F\cos\alpha = 1.965$ STO ②

① ÷ ②: $\frac{F\sin\alpha}{F\cos\alpha} = \frac{10.714}{1.965}$

$\tan\alpha = 5.452\ldots$

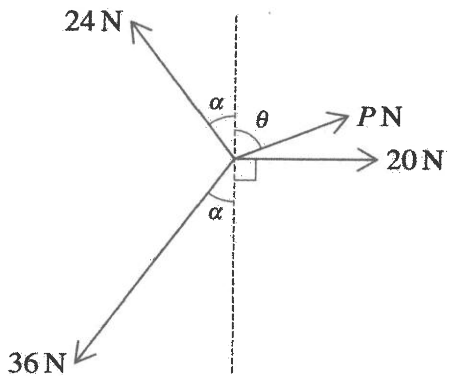
$\alpha = 79.6°$ STO

Sub into ①: $F\sin(79.6) = 10.714$

$F = \frac{10.714}{\sin(79.6)}$

$= 10.9\text{ N}$

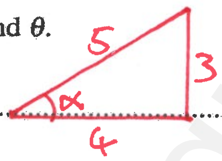
3



Coplanar forces of magnitudes 24 N, P N, 20 N and 36 N act at a point in the directions shown in the diagram. The system is in equilibrium.

Given that $\sin\alpha = \frac{3}{5}$, find the values of P and θ. [6]

$\sin\alpha = \frac{3}{5}$ →



$\cos\alpha = \frac{4}{5}$

$\tan\alpha = \frac{3}{4}$

$R(\uparrow)$: $24\cos\alpha + P\cos\theta - 36\cos\alpha = 0$

$24 \times \frac{4}{5} + P\cos\theta - 36 \times \frac{4}{5} = 0$

$19.2 + P\cos\theta - 28.8 = 0$

$P\cos\theta - 9.6 = 0$

$P\cos\theta = 9.6$ (1)

$R(\rightarrow)$: $20 + P\sin\theta - 24\sin\alpha - 36\sin\alpha = 0$

$20 + P\sin\theta - 24 \times \frac{3}{5} - 36 \times \frac{3}{5} = 0$

$20 + P\sin\theta - 14.4 - 21.6 = 0$

$P\sin\theta - 16 = 0$

$P\sin\theta = 16$ (2)

(2) ÷ (1): $\frac{P\sin\theta}{P\cos\theta} = \frac{16}{9.6}$

$\tan\theta = \frac{5}{3}$

$\theta = 59.0°$

STO

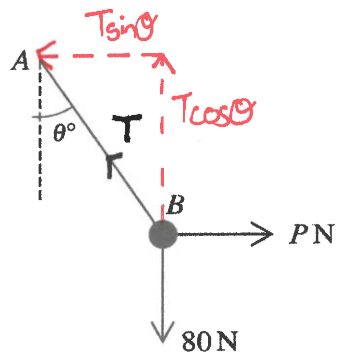
Sub into (1):

$P\cos(59.0) = 9.6$

$P = \frac{9.6}{\cos(59.0)}$

$= 18.7\text{N}$

5



A light string AB is fixed at A and has a particle of weight 80 N attached at B. A horizontal force of magnitude P N is applied at B such that the string makes an angle $\theta°$ to the vertical (see diagram).

(a) It is given that $P = 32$ and the system is in equilibrium.

Find the tension in the string and the value of θ. [4]

$R(\rightarrow)$: $32 - T\sin\theta = 0$

$T\sin\theta = 32$ ①

$R(\uparrow)$: $T\cos\theta - 80 = 0$

$T\cos\theta = 80$ ②

① ÷ ②: $\dfrac{T\sin\theta}{T\cos\theta} = \dfrac{32}{80}$

$\tan\theta = \dfrac{32}{80}$

$\theta = \underline{21.8°}$ STO

→①: $T\sin(21.8) = 32$

$T = \dfrac{32}{\sin(21.8)}$

$= \underline{86.2\text{N}}$

(b) It is given instead that the tension in the string is 120 N and that the particle attached at B still has weight 80 N.

Find the value of P and the value of θ. [4]

$R(\rightarrow)$: $P - 120\sin\theta = 0$

$P = 120\sin\theta$ ①

$R(\uparrow)$: $120\cos\theta - 80 = 0$

$120\cos\theta = 80$

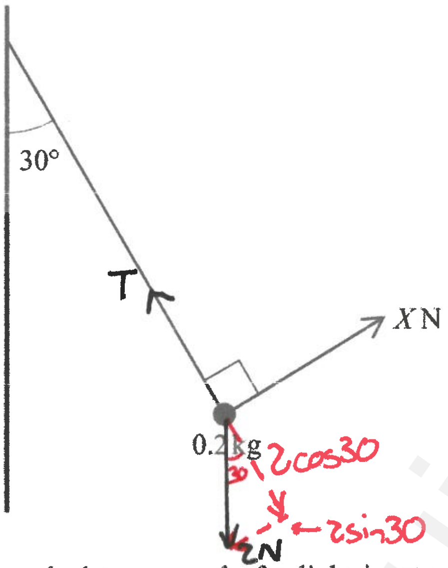
$\cos\theta = \frac{80}{120}$

$\underline{\theta = 48.2°}$ STO

→①: $P = 120\sin(48.2)$

$= \underline{89.4\text{ N}}$

2



A particle of mass 0.2 kg is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point on a vertical wall. The particle is held in equilibrium by a force of magnitude X N, perpendicular to the string, with the string taut and making an angle of 30° with the wall (see diagram).

Find the tension in the string and the value of X. [3]

Note: It's easier to resolve in the directions of T and X, rather than vertically and horizontally, but both methods work.

$R(\nwarrow)$: $T - 2\cos 30 = 0$

$T = 2\cos 30$

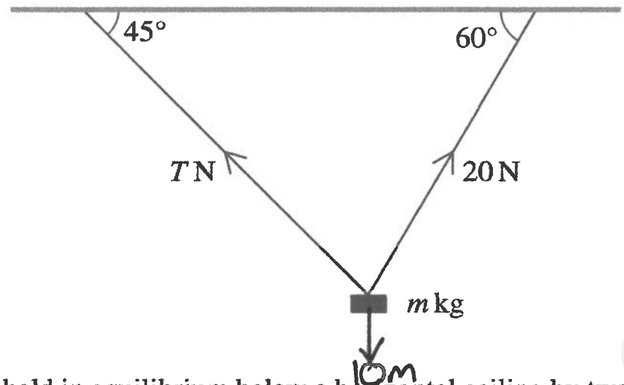
$T = \sqrt{3}$ N

$R(\nearrow)$: $X - 2\sin 30 = 0$

$X = 2\sin 30$

$X = 1$ N

3



A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings, as shown in the diagram. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N.

Find T and m. [5]

$R(\rightarrow)$: $20\cos 60 - T\cos 45 = 0$

$10 - T\cos 45 = 0$

$T\cos 45 = 10$

$T = 10\sqrt{2}$ N

$R(\uparrow)$: $T\sin 45 + 20\sin 60 - 10m = 0$

$10\sqrt{2}\sin 45 + 10\sqrt{3} - 10m = 0$

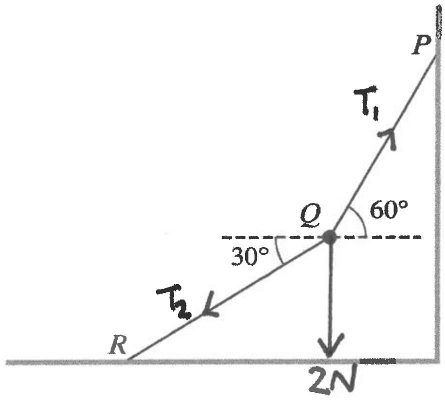
$10 + 10\sqrt{3} - 10m = 0$

$10m = 10 + 10\sqrt{3}$

$m = 1 + \sqrt{3}$

$= 2.73$ kg

3



A particle Q of mass 0.2 kg is held in equilibrium by two light inextensible strings PQ and QR. P is a fixed point on a vertical wall and R is a fixed point on a horizontal floor. The angles which strings PQ and QR make with the horizontal are 60° and 30° respectively (see diagram).

Find the tensions in the two strings. [5]

$R(\rightarrow)$: $T_1 \cos 60 - T_2 \cos 30 = 0$

$T_1 \cos 60 = T_2 \cos 30$

$T_1 = \dfrac{T_2 \cos 30}{\cos 60}$

$T_1 = \sqrt{3}\, T_2$ ①

$R(\uparrow)$: $T_1 \sin 60 - T_2 \sin 30 - 2 = 0$ ②

Sub. ① into ②:

$\sqrt{3}\, T_2 \sin 60 - T_2 \sin 30 - 2 = 0$

$\sqrt{3}\, T_2 \times \frac{\sqrt{3}}{2} - T_2 \times \frac{1}{2} - 2 = 0$

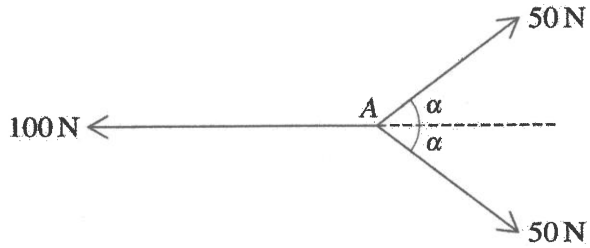
$\frac{3}{2} T_2 - \frac{1}{2} T_2 = 2$

$T_2 = 2\text{N}$

Sub. into ①: $T_1 = \sqrt{3} \times 2$

$= 2\sqrt{3}\text{ N}$

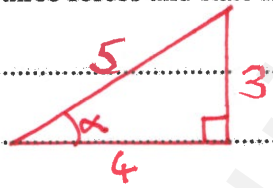
1



Three coplanar forces of magnitudes 100 N, 50 N and 50 N act at a point A, as shown in the diagram. The value of $\cos\alpha$ is $\frac{4}{5}$.

Find the magnitude of the resultant of the three forces and state its direction. [3]

$\cos\alpha = \frac{4}{5} \rightarrow$



$\sin\alpha = \frac{3}{5}$

$\tan\alpha = \frac{3}{4}$

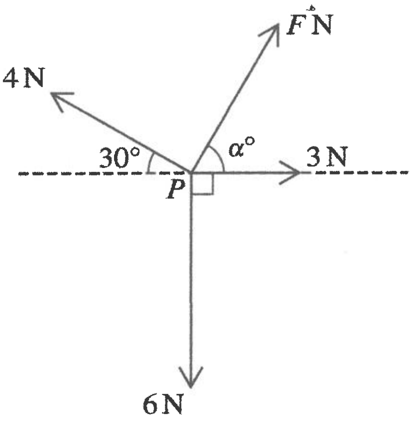
$R(\uparrow)$: $50\sin\alpha - 50\sin\alpha$
$= \underline{0}$

$R(\rightarrow)$: $50\cos\alpha + 50\cos\alpha - 100$
$= 50 \times \frac{4}{5} + 50 \times \frac{4}{5} - 100$

$= 40 + 40 - 100$
$= -20\text{ N}$

$\rightarrow$ magnitude $= \underline{20\text{ N}}$
direction: <u>in direction of 100N force</u>

5



Coplanar forces, of magnitudes F N, 3 N, 6 N and 4 N, act at a point P, as shown in the diagram.

(a) Given that $\alpha = 60$, and that the resultant of the four forces is in the direction of the 3 N force, find F. [3]

Resultant is in direction of 3N force, so resultant in vertical direction = 0:

$R(\uparrow): 4\sin 30 + F\sin 60 - 6 = 0$

$2 + F\sin 60 - 6 = 0$

$F\sin 60 = 4$

$F = \dfrac{4}{\sin 60}$

$F = 4.62\text{N}$

(b) Given instead that the four forces are in equilibrium, find the values of F and α. [5]

$R(\uparrow): 4\sin 30 + F\sin\alpha - 6 = 0$

$F\sin\alpha = 6 - 4\sin 30$

$F\sin\alpha = 6 - 2$

$F\sin\alpha = 4$ (1)

$R(\rightarrow): 3 + F\cos\alpha - 4\cos 30 = 0$

$F\cos\alpha = 4\cos 30 - 3$

$F\cos\alpha = 2\sqrt{3} - 3$ (2)

(1) ÷ (2): $\dfrac{F\sin\alpha}{F\cos\alpha} = \dfrac{4}{2\sqrt{3} - 3}$

$\tan\alpha = \dfrac{4}{2\sqrt{3} - 3}$

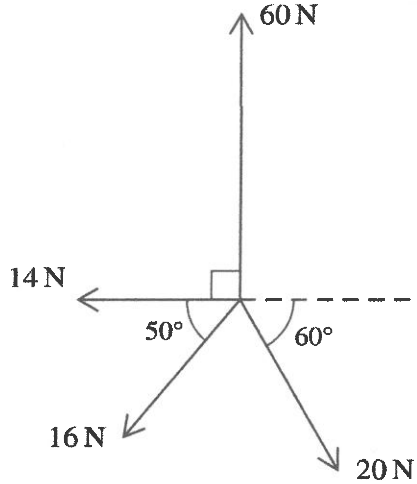
$\alpha = 83.4^\circ$ STO

Sub into (1): $F\sin(83.4) = 4$

$F = \dfrac{4}{\sin(83.4)}$

$= 4.03\text{ N}$

2



Coplanar forces of magnitudes 60 N, 20 N, 16 N and 14 N act at a point in the directions shown in the diagram.

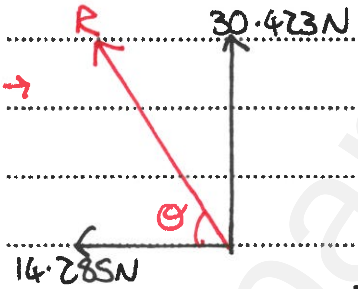
Find the magnitude and direction of the resultant force. [6]

$R(\uparrow)$: $60 - 20\sin 60 - 16\sin 50$

$= \underline{30.423\ N}$ STO

$R(\rightarrow)$: $20\cos 60 - 16\cos 50 - 14$

$= \underline{-14.285\ N}$ STO



$R = \sqrt{30.423^2 + 14.285^2}$

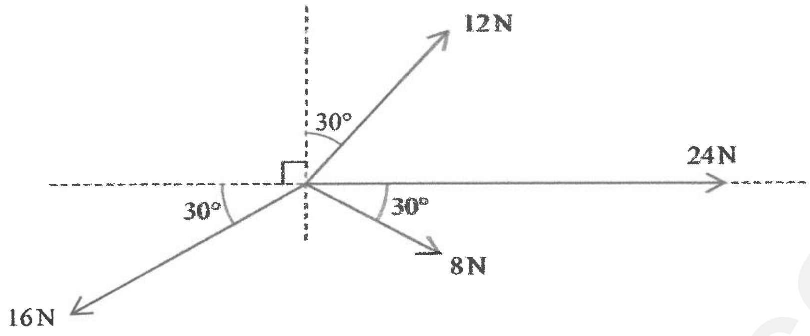
$= \underline{33.6\ N}$

$\tan\theta = \dfrac{30.423}{14.285}$

$\theta = \underline{64.8^\circ \text{ above negative } x \text{ axis}}$

(note mistake in mark scheme)

2



Coplanar forces of magnitudes 16 N, 12 N, 24 N and 8 N act at a point in the directions shown in the diagram.

Find the magnitude and direction of the single additional force acting at the same point which will produce equilibrium. [6]

Find resultant of these four forces:

$R(\uparrow)$: $12\cos 30 - 8\sin 30 - 16\sin 30$

$= -12 + 6\sqrt{3}$ N STO $(= -1.61\text{N}$, ie. 1.61N downwards$)$

$R(\rightarrow)$: $24 + 12\sin 30 + 8\cos 30 - 16\cos 30$

$= 30 - 4\sqrt{3}$ N STO $(= 23.1\text{N}$ to the right$)$

Magnitude: $R = \sqrt{(-1.607...)^2 + (23.07...)^2}$

$= \underline{23.1\text{N}}$

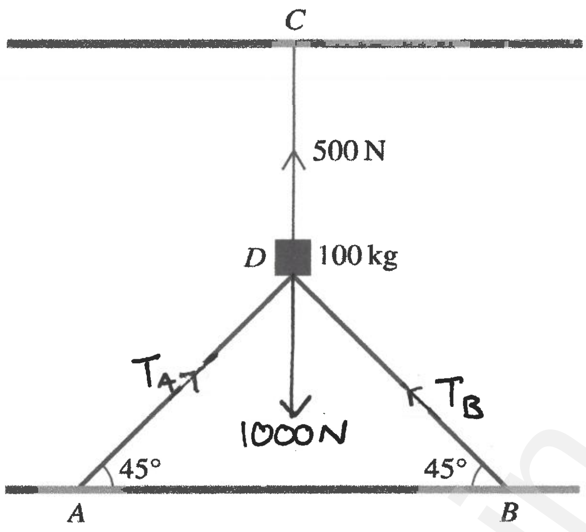
Angle: $\tan\alpha = \dfrac{1.607...}{23.07...}$

$\alpha = \underline{3.99^\circ}$ below positive x-axis.

For equilibrium, we would need a force of the same magnitude as this, but in the opposite direction, so:

$F = \underline{23.1\text{N}}$ $\quad \theta = \underline{3.99^\circ \text{ above negative } x\text{-axis}}$

5



The diagram shows a block D of mass 100 kg supported by two sloping struts AD and BD, each attached at an angle of 45° to fixed points A and B respectively on a horizontal floor. The block is also held in place by a vertical rope CD attached to a fixed point C on a horizontal ceiling. The tension in the rope CD is 500 N and the block rests in equilibrium.

(a) Find the magnitude of the force in each of the struts AD and BD. [3]

$R(\rightarrow)$: $T_B \cos 45 = T_A \cos 45$

$T_B = T_A \; (= T)$

$R(\uparrow)$: $500 - 1000 + T\sin 45 + T\sin 45 = 0$

$-500 + \frac{\sqrt{2}}{2}T + \frac{\sqrt{2}}{2}T = 0$

$-500 + \sqrt{2}T = 0$

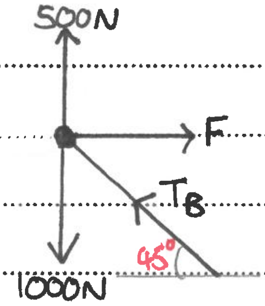
$\sqrt{2}T = 500$

$T = \frac{500}{\sqrt{2}}$

$T = 354\text{ N}$

A horizontal force of magnitude F N is applied to the block in a direction parallel to AB.

(b) Find the value of F for which the magnitude of the force in the strut AD is zero. [3]



$R(\uparrow)$: $500 + T_B \sin 45 - 1000 = 0$

$T_B \sin 45 - 500 = 0$

$T_B \sin 45 = 500$

$T_B = \underline{707.1\text{N}}$ STO

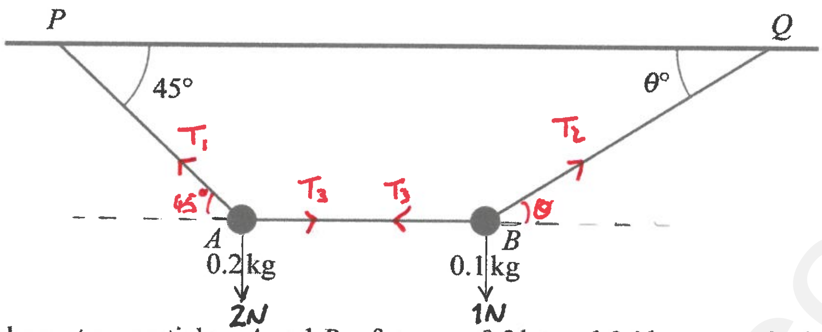
$R(\rightarrow)$: $F - T_B \cos 45 = 0$

$F = T_B \cos 45$

$= 707.1 \times \cos 45$

$= \underline{\underline{500\text{ N}}}$

4



The diagram shows two particles, A and B, of masses 0.2 kg and 0.1 kg respectively. The particles are suspended below a horizontal ceiling by two strings, AP and BQ, attached to fixed points P and Q on the ceiling. The particles are connected by a horizontal string, AB. Angle $APQ = 45°$ and $BQP = \theta°$. Each string is light and inextensible. The particles are in equilibrium.

(a) Find the value of the tension in the string AB. [2]

Particle A:

$R(\uparrow)$: $T_1 \sin 45 - 2 = 0$

$T_1 \sin 45 = 2$

$T_1 = 2\sqrt{2}$ N

$R(\rightarrow)$: $T_3 - T_1 \cos 45 = 0$

$2\sqrt{2}$

$T_3 - 2\sqrt{2} \cos 45 = 0$

$T_3 - 2 = 0$

$T_3 = 2$ N

(b) Find the value of θ and the tension in the string BQ. [4]

Particle B:

$R(\uparrow): \quad T_2 \sin\theta - 1 = 0$

$T_2 \sin\theta = 1 \quad (1)$

$R(\rightarrow): \quad T_2 \cos\theta - T_3 = 0$

$\uparrow 2$

$T_2 \cos\theta - 2 = 0$

$T_2 \cos\theta = 2 \quad (2)$

$(1) \div (2): \quad \frac{\cancel{T_2} \sin\theta}{\cancel{T_2} \cos\theta} = \frac{1}{2}$

$\tan\theta = \frac{1}{2}$

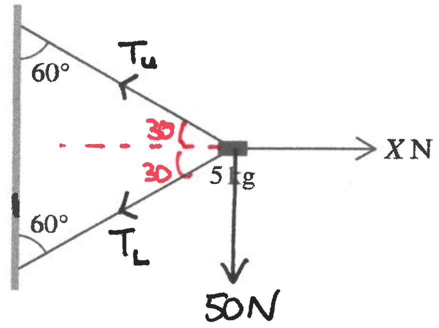
$\theta = 26.6^\circ$ STO

$\rightarrow (1): \quad T_2 \sin(26.56\ldots) = 1$

$T_2 = \frac{1}{\sin(26.56\ldots)}$

$T_2 = 2.24\ N$

6



A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and a horizontal force of magnitude X N, as shown in the diagram. The two strings are both inclined at 60° to the vertical.

(a) Given that $X = 100$, find the tension in the lower string. [4]

$R(\uparrow)$: $T_u \sin 30 - T_L \sin 30 - 50 = 0$

$\frac{1}{2}T_u - \frac{1}{2}T_L = 50$

$T_u - T_L = 100$ ①

$R(\rightarrow)$: $100 - T_u \cos 30 - T_L \cos 30 = 0$

$100 - \frac{\sqrt{3}}{2}T_u - \frac{\sqrt{3}}{2}T_L = 0$

$200 - \sqrt{3}T_u - \sqrt{3}T_L = 0$

$\sqrt{3}T_u + \sqrt{3}T_L = 200$

$T_u + T_L = \frac{200}{\sqrt{3}}$ ②

① + ②: $2T_u = 100 + \frac{200}{\sqrt{3}}$

$2T_u = 215.470$

$T_u = 107.735$

Sub into ①: $T_u - T_L = 100$

$107.735 - T_L = 100$

$-T_L = -7.735$

$T_L = \underline{7.74\text{ N}}$

(b) Find the least value of X for which the block remains in equilibrium in the position shown. [4]

As X decreases, so will T_u and T_L

T_u cannot be zero or there would be nothing to hold the block up, so set $T_L = 0$:

$R(\uparrow)$: $T_u \sin 30 - 50 = 0$

$T_u \sin 30 = 50$

$T_u = \underline{100\ N}$

$R(\rightarrow)$: $X - T_u \cos 30 = 0$

$X - 100 \cos 30 = 0$

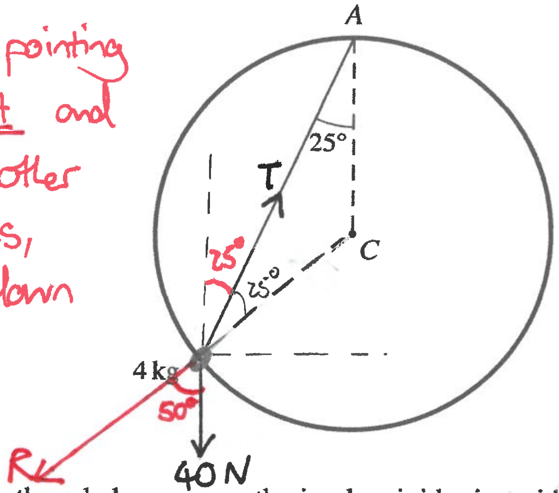
$X = 100 \cos 30$

$= \underline{86.6 N}$

R is perpendicular to surface

3

Since T is pointing up and right and there are no other horizontal forces, R must be down and left.



A ring of mass 4 kg is threaded on a smooth circular rigid wire with centre C. The wire is fixed in a vertical plane and the ring is kept at rest by a light string connected to A, the highest point of the circle. The string makes an angle of 25° to the vertical (see diagram).

Find the tension in the string and the magnitude of the normal reaction of the wire on the ring. [6]

$R(\rightarrow): T\sin 25 - R\sin 50 = 0$

$T\sin 25 = R\sin 50$

$T = \frac{R\sin 50}{\sin 25}$ ①

$R(\uparrow): T\cos 25 - 40 - R\cos 50 = 0$ ②

sub ① into ②:

$\frac{R\sin 50\cos 25}{\sin 25} - 40 - R\cos 50 = 0$

$R\left(\frac{\sin 50\cos 25}{\sin 25} - \cos 50\right) = 40$

$R = 40 \div \left(\frac{\sin 50\cos 25}{\sin 25} - \cos 50\right)$

$R = 40\text{ N}$

sub into ①: $T = \frac{40\sin 50}{\sin 25}$

$T = 72.5\text{ N}$