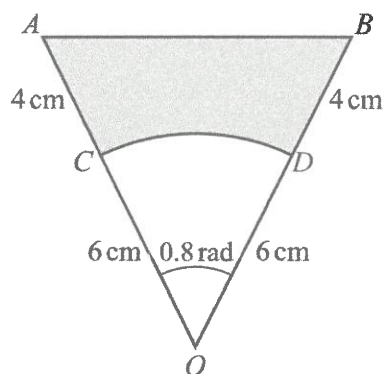


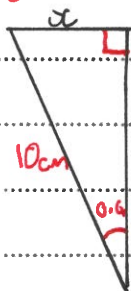


The diagram shows a triangle OAB where $OA = OB = 10$ cm and angle $AOB = 0.8$ radians. Points C and D on OA and OB respectively are such that the arc CD is part of a circle with centre O and radius 6 cm. The shaded region is bounded by the arc CD and the line segments CA , AB and BD .

- (a) Find the perimeter of the shaded region. [3]

Arc CD : Arc length = $r\theta$
 $= 6 \times 0.8$
 $= 4.8$ cm

Line AB :



Triangle split in two

$$\sin(0.4) = \frac{x}{10}$$

$$x = 10 \sin(0.4)$$

$$= 3.894 \dots$$

$$AB = 2 \times 3.894 \dots$$

$$= 7.788 \dots$$

STO

$$\text{Perimeter} = 4.8 + 4 + 4 + 7.788$$

$$= 20.6$$
 cm

- (b) Find the area of the shaded region. [3]

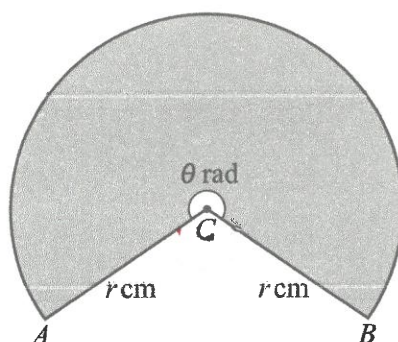
Shaded region = triangle - sector:

Triangle: Area = $\frac{1}{2} ab \sin C$
 $= \frac{1}{2} \times 10 \times 10 \times \sin 0.8$
 $= 35.867 \dots$ STO

Sector: Area = $\frac{1}{2} r^2 \theta$
 $= \frac{1}{2} \times 6^2 \times 0.8$
 $= 14.4$

$$\rightarrow 35.867 - 14.4 = 21.5 \text{ cm}^2$$





The diagram shows a sector of a circle with centre C . The radii CA and CB each have length r cm and the size of the reflex angle ACB is θ radians. The sector, shaded in the diagram, has a perimeter of 65 cm and an area of 225 cm^2 .

(a) Find the values of r and θ .

[4]

Area: $\text{Area} = \frac{1}{2}r^2\theta$

$$225 = \frac{1}{2}r^2\theta$$

$$450 = r^2\theta$$

$$\theta = \frac{450}{r^2} \quad (1)$$

$$2r^2 - 65r + 450 = 0$$

$$r = \frac{-(-65) \pm \sqrt{(-65)^2 - 4(2)(450)}}{2(2)}$$

$$r = 22.5 \text{ or } r = 10$$

$$r = 22.5: \theta = \frac{450}{22.5^2}$$

$$= \frac{8}{9} \times (\theta \text{ is a reflex angle})$$

Perimeter: $\text{Perimeter} = r + r + r\theta$

$$65 = 2r + r\theta \quad (2)$$

$$(1) \rightarrow (2): 65 = 2r + r\left(\frac{450}{r^2}\right)$$

$$65 = 2r + \frac{450}{r}$$

$$65r = 2r^2 + 450$$

$$r = 10: \theta = \frac{450}{10^2}$$

$$= 4.5$$

$$\rightarrow \underline{r = 10}, \underline{\theta = 4.5}$$

(b) Find the area of triangle ACB .

[2]

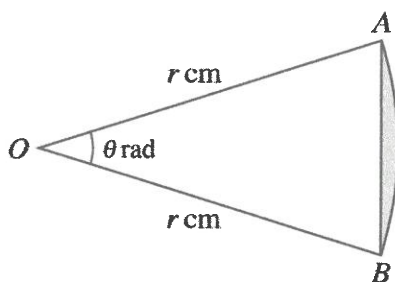
$$\text{Angle } ACB = 2\pi - 4.5$$

$$= 1.7832 \text{ rad}$$

$$\text{Area of triangle} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 10 \times 10 \times \sin(1.7832)$$

$$= \underline{48.9 \text{ cm}^2}$$



The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2 .

- (a) Find the area of the shaded region.

[5]

Arc AB: arc length = $r\theta$

$$9.6 = r\theta$$

$$\theta = \frac{9.6}{r} \quad (1)$$

Sector OAB: Area = $\frac{1}{2}r^2\theta$

$$76.8 = \frac{1}{2}r^2\theta$$

$$153.6 = r^2\theta \quad (2)$$

Sub. (1) into (2):

$$153.6 = r^2 \times \frac{9.6}{r}$$

$$153.6 = 9.6r$$

$$r = 16$$

$$\rightarrow (1): \theta = \frac{9.6}{16} = 0.6$$

Triangle OAB:

$$\text{Area} = \frac{1}{2} \times 16 \times 16 \times \sin(0.6) = 72.3 \text{ cm}^2$$

Area of segment:

$$\text{Area} = 76.8 - 72.3 = \underline{4.53 \text{ cm}^2}$$

- (b) Find the perimeter of the shaded region.

[2]

Line AB: Cosine rule:

$$AB^2 = 16^2 + 16^2 - 2 \times 16 \times 16 \cos(0.6)$$

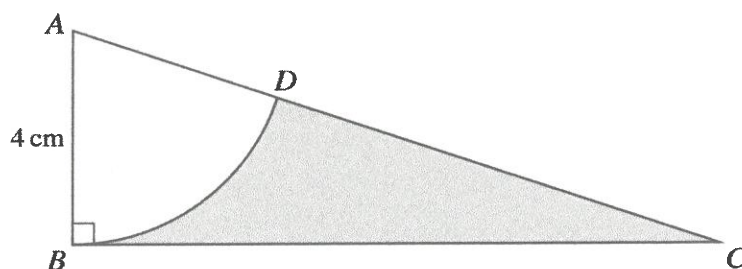
$$= 512 - 512 \cos(0.6)$$

$$AB = \sqrt{512 - 512 \cos(0.6)}$$

$$= 9.46 \text{ cm}$$

$$\rightarrow \text{Perimeter} = 9.6 + 9.46 = \underline{19.1 \text{ cm}}$$

5



The diagram shows a triangle ABC , in which angle $ABC = 90^\circ$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A . The area of the sector is 10 cm^2 .

- (a) Find angle BAD in radians.

[2]

$$\text{Area} = \frac{1}{2} r^2 \theta$$

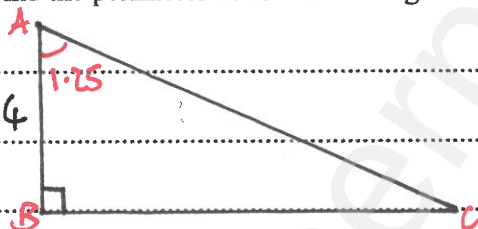
$$10 = \frac{1}{2} \times 4^2 \theta$$

$$10 = 8\theta$$

$$\theta = \underline{1.25}$$

- (b) Find the perimeter of the shaded region.

[4]



$$\text{BC: } \tan(1.25) = \frac{BC}{4}$$

$$BC = 4 \tan(1.25) \\ = 12.0 \text{ STB}$$

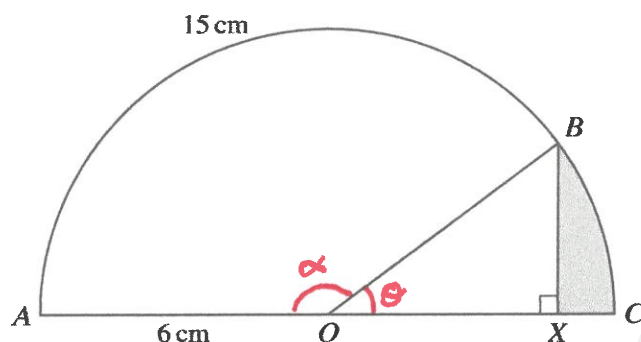
$$\text{AC: } \cos(1.25) = \frac{4}{AC}$$

$$AC = \frac{4}{\cos(1.25)} \\ = 12.7 \text{ STB}$$

$$\begin{aligned} \text{Arc BD arc length} &= r\theta \\ &= 4 \times 1.25 \\ &= 5 \end{aligned}$$

$$\begin{aligned} DC &= AC - 4 \\ &= 12.7 - 4 \\ &= 8.69 \text{ STB} \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 5 + 8.69 + 12.0 \\ &= \underline{25.7 \text{ cm}} \end{aligned}$$



In the diagram, ABC is a semicircle with diameter AC , centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX .

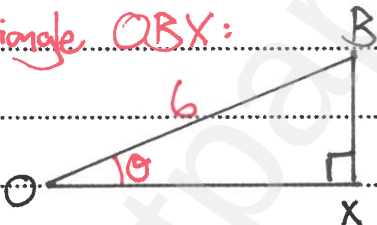
Find the perimeter of the shaded region BXC .

[6]

Find angle α : arc length = $r\alpha$
 $15 = 6 \times \alpha$
 $\alpha = 2.5$

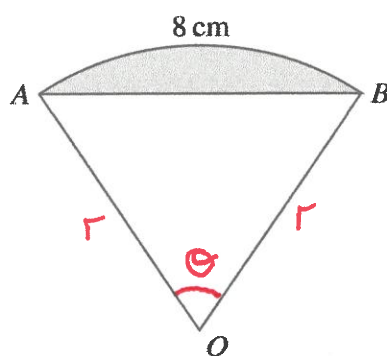
Angle θ : $\theta = \pi - 2.5$
 $= 0.642$ STO

Arc length BC : arc length = $r\theta$
 $= 6 \times 0.642$
 $= 3.85$ STO

Triangle OBX : 
OX: $\cos(0.642) = \frac{OX}{6}$
 $OX = 6 \cos(0.642)$
CX: $CX = 6 - 6 \cos(0.642)$
 $= 1.19$ STO

BX: $\sin(0.642) = \frac{BX}{6}$
 $BX = 6 \sin(0.642)$
 $= 3.59$ STO

Perimeter = $3.85 + 1.19 + 3.59$
 $= 8.63$



The diagram shows a sector OAB of a circle with centre O . The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

- (a) Find the perimeter of the shaded segment.

[4]

Perimeter of sector OAB :

$$\begin{aligned} \text{perimeter} &= r + r + 8 \\ &= 2r + 8 \end{aligned}$$

$$2r + 8 = 20$$

$$2r = 12$$

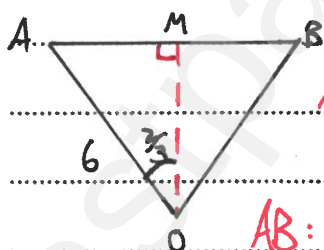
$$r = 6$$

Arc AB :

$$\text{arc length} = r\theta$$

$$8 = 6\theta$$

$$\theta = \frac{4}{3}$$



$$AM: \sin\left(\frac{2}{3}\right) = \frac{AM}{6}$$

$$AM = 6\sin\left(\frac{2}{3}\right)$$

$$\begin{aligned} AB: AB &= 2 \times 6\sin\left(\frac{2}{3}\right) \\ &= 12\sin\left(\frac{2}{3}\right) \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 8 + 12\sin\left(\frac{2}{3}\right) \\ &= \underline{\underline{15.4 \text{ cm}}} \end{aligned}$$

(b) Find the area of the shaded segment.

[2]

Area of sector OAB:

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \times 6^2 \times \frac{4}{3}$$

$$= 24$$

Area of triangle OAB:

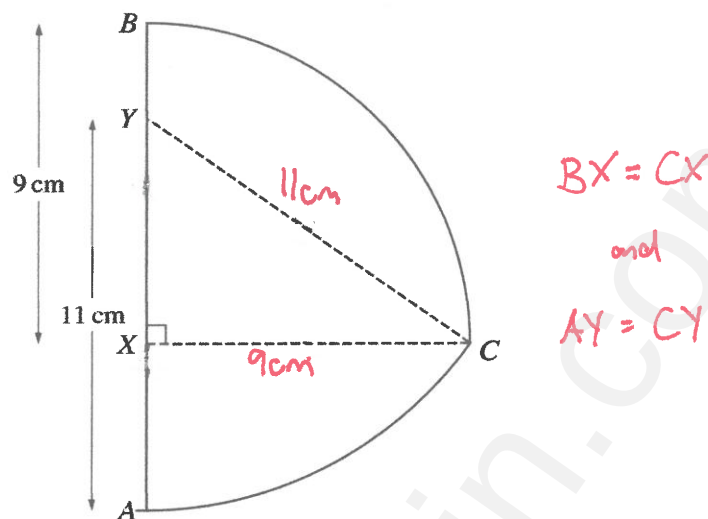
$$\text{Area} = \frac{1}{2} \times 6 \times 6 \times \sin\left(\frac{4}{3}\right)$$

$$= 17.5 \text{ s.t.o.}$$

Area of shaded segment:

$$\text{Area} = 24 - 17.5$$

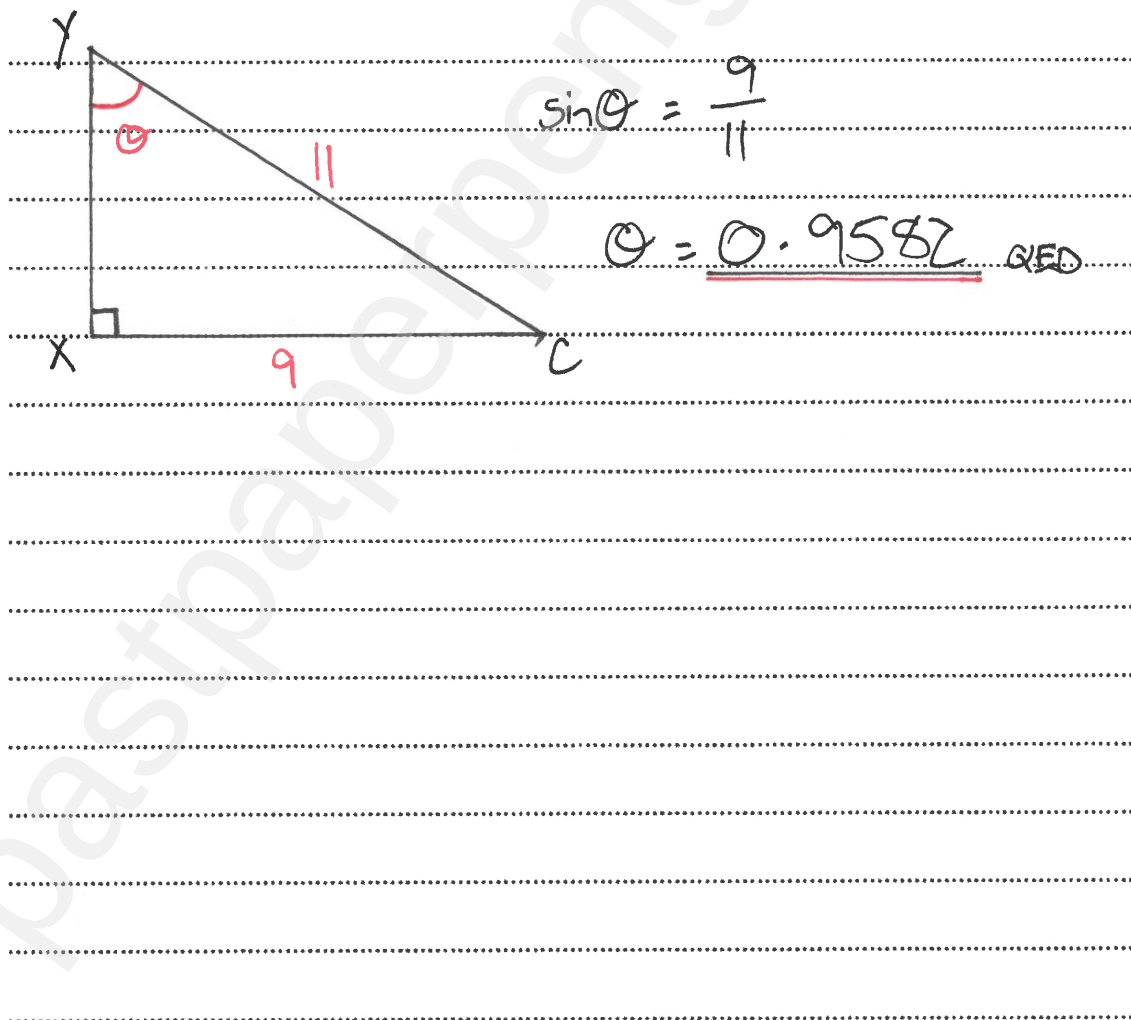
$$= \underline{\underline{6.51 \text{ cm}^2}}$$



In the diagram, X and Y are points on the line AB such that $BX = 9$ cm and $AY = 11$ cm. Arc BC is part of a circle with centre X and radius 9 cm, where CX is perpendicular to AB . Arc AC is part of a circle with centre Y and radius 11 cm.

(a) Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures.

[1]



(b) Find the perimeter of ABC.

[6]

Length XY:

$$\text{Pythagoras: } XY^2 + 9^2 = 11^2$$

$$XY^2 = 11^2 - 9^2$$

$$XY = \sqrt{11^2 - 9^2}$$

$$= \sqrt{40}$$

Length BY: $BY = 9 - \sqrt{40}$

Length AB: $AY + BY = 11 + 9 - \sqrt{40}$

$$= \underline{20 - \sqrt{40}}$$

Arc BC: arc length = $9 \times \frac{\pi}{2}$

$$= \underline{\frac{9\pi}{2}}$$

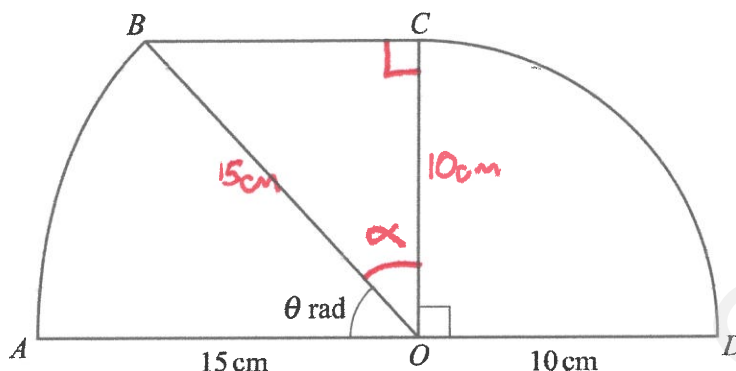
Arc AC: arc length = 11×0.9582

$$= \underline{10.5} \text{ STD}$$

Perimeter = $AB + \text{Arc BC} + \text{Arc AC}$

$$= 20 - \sqrt{40} + \frac{9\pi}{2} + 10.5$$

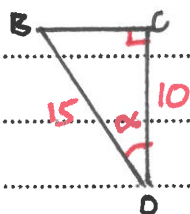
$$= \underline{38.4 \text{ cm}}$$



In the diagram, AOD and BC are two parallel straight lines. Arc AB is part of a circle with centre O and radius 15 cm. Angle $BOA = \theta$ radians. Arc CD is part of a circle with centre O and radius 10 cm. Angle $COD = \frac{1}{2}\pi$ radians.

- (a) Show that $\theta = 0.7297$, correct to 4 decimal places. [1]

$$OB = 15 \text{ cm (radius, so } = OA), OC = 10 \text{ cm (radius, so } = OD)$$



$$\cos \alpha = \frac{10}{15}$$

$$\alpha = 0.84106\dots$$

$$\theta = \frac{\pi}{2} - 0.84106\dots$$

$$= 0.7297 \quad \text{QED}$$

- (b) Find the perimeter and the area of the shape $ABCD$. Give your answers correct to 3 significant figures. [7]

Perimeter: $AB = r\theta$

$$= 15 \times 0.7297$$

$$= 10.9459$$

$$BC = \sqrt{15^2 - 10^2}$$

$$= 5\sqrt{5}$$

$$CD = r\theta$$

$$= 10 \times \frac{\pi}{2}$$

$$= 5\pi$$

$$\text{Perimeter} = 10 + 15 + 10.9459 + 5\sqrt{5} + 5\pi$$

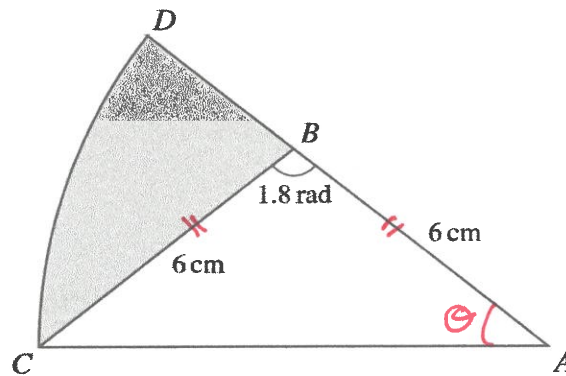
$$= 62.8 \text{ cm}$$

$$\begin{aligned}
 \text{Area: } \triangle OAB &= \frac{1}{2} r^2 \theta \\
 &= \frac{1}{2} \times 15^2 \times 0.7297 \\
 &= 82.0944
 \end{aligned}$$

$$\begin{aligned}
 \triangle OBC &= \frac{1}{2} \times b \times h \\
 &= \frac{1}{2} \times 10 \times 5\sqrt{5} \\
 &= 25\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \triangle OCD &= \frac{1}{2} r^2 \theta \\
 &= \frac{1}{2} \times 10^2 \times \frac{\pi}{2} \\
 &= 25\pi
 \end{aligned}$$

$$\begin{aligned}
 \text{Area} &= 82.0944 + 25\sqrt{5} + 25\pi \\
 &= \underline{217 \text{ cm}^2} \text{ (3sf)}
 \end{aligned}$$



The diagram shows triangle ABC with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc CD is part of a circle with centre A and ABD is a straight line.

- (a) Find the perimeter of the shaded region.

[5]

Cosine rule: $AC^2 = 6^2 + 6^2 - 2(6)(6)\cos(1.8)$
 $= 72 - 72\cos(1.8)$
 $AC = \sqrt{72 - 72\cos(1.8)}$
 $= 9.40$ STO

Angle BAC: $\frac{\pi - 1.8}{2} = 0.671$ STO
isosceles triangle

Arc CD: arc length = $r\theta$
 $= 9.40 \times 0.671$
 $= 6.31$ STO

Length BD: $BD = AD - 6$
 $= 9.40 - 6$
 $= 3.40$

Perimeter = $6 + 6.31 + 3.40$
 $= \underline{\underline{15.7}} \text{ cm}$

(b) Find the area of the shaded region.

[3]

Area of sector ACD:

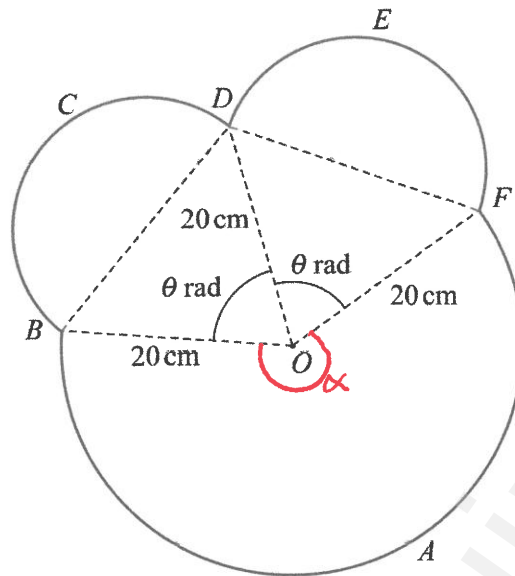
$$\begin{aligned}\text{Area} &= \frac{1}{2} \times (9.40)^2 \times 0.671 \\ &= 29.6\end{aligned}$$

Area of triangle ABC:

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 6 \times 6 \times \sin(1.8) \\ &= 17.5\end{aligned}$$

Area BCD: $\text{Area BCD} = 29.6 - 17.5$

$$= \underline{\underline{12.1 \text{ cm}^2}}$$



The diagram shows a metal plate $ABCDEF$ consisting of five parts. The parts BCD and DEF are semicircles. The part $BAFO$ is a sector of a circle with centre O and radius 20 cm, and D lies on this circle. The parts OBD and ODF are triangles. Angles BOD and DOF are both θ radians.

- (a) Given that $\theta = 1.2$, find the area of the metal plate. Give your answer correct to 3 significant figures. [5]

Area of two triangles OBD and ODF :

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 20^2 \times \sin 1.2 + \frac{1}{2} \times 20^2 \times \sin 1.2 \\ &= 372.816 \end{aligned}$$

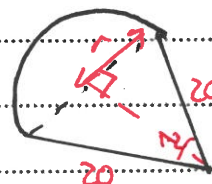
Area of major sector $BAFO$:

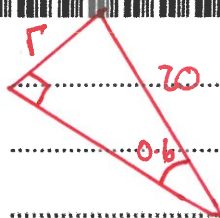
$$\begin{aligned} \alpha &= 2\pi - 1.2 - 1.2 \\ &= 3.883 \dots \text{STO} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 20^2 \times 3.883 \dots \\ &= 776.637 \end{aligned}$$

Area of two semi-circles BCD and DEF :

Find radius of semi-circles first:





$$\sin(0.6) = \frac{r}{20}$$

$$r = 20 \sin(0.6)$$

$$\begin{aligned} \text{Area of two semi-circles} &= \pi \times (20 \sin(0.6))^2 \\ &= 400 \cdot 642 \end{aligned}$$

$$\text{Total Area} = 372.816 + 776.637 + 400 \cdot 642 = \underline{\underline{1550 \text{ cm}^2}}$$

- (b) Given instead that the area of each semicircle is $50\pi \text{ cm}^2$, find the exact perimeter of the metal plate. [5]

$$\text{Area of one semi-circle} = 50\pi:$$

$$\frac{1}{2} \pi r^2 = 50\pi$$

$$\pi r^2 = 100\pi$$

$$r^2 = 100$$

$$r = 10 \text{ cm}$$

$$\text{Circumference of both semi-circles:}$$

$$\begin{aligned} C &= 2 \times \pi \times 10 \\ &= 20\pi \end{aligned}$$

$$\text{Find angle } \theta:$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{10}{20}$$



$$\frac{\theta}{2} = \frac{\pi}{6} \rightarrow \theta = \frac{\pi}{3}$$

$$\text{Perimeter} = 20\pi + \frac{80\pi}{3}$$

$$\text{Find angle } \alpha:$$

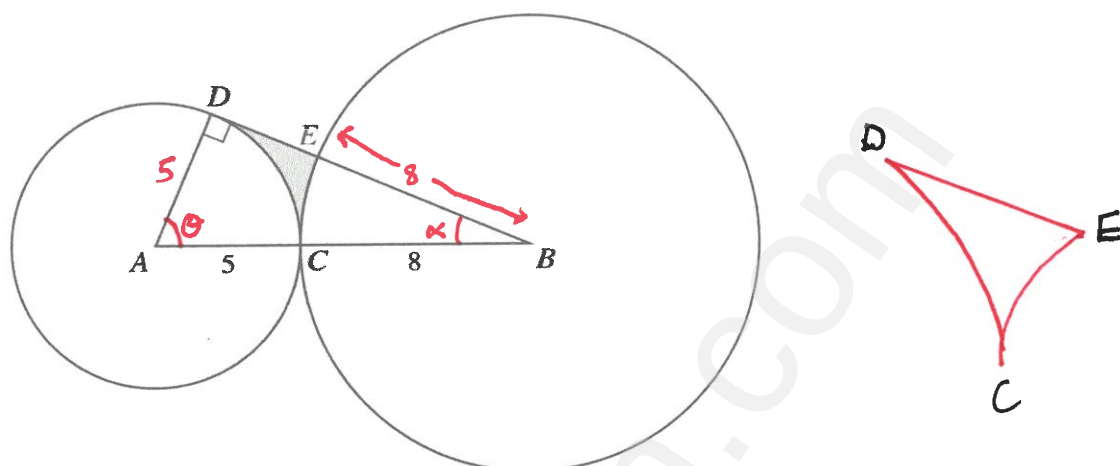
$$\begin{aligned} \alpha &= 2\pi - \frac{\pi}{3} - \frac{\pi}{3} \\ &= \frac{4\pi}{3} \end{aligned}$$

$$= \underline{\underline{\frac{140\pi}{3} \text{ cm}}}$$

$$\text{Arc length BAF:}$$

$$l = 20 \times \frac{4\pi}{3} = \underline{\underline{\frac{80\pi}{3}}}$$





The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line. The tangent at the point D on the smaller circle intersects the larger circle at E and passes through B .

- (a) Find the perimeter of the shaded region.

[5]

Pythagoras: $5^2 + DB^2 = 13^2$
 $DB = \sqrt{13^2 - 5^2}$
 $= 12$

$DE = 12 - 8$
 $= 4$

Angle DAB: $\tan \theta = \frac{12}{5}$
 $\theta = 1.17 \text{ rad}$

Arc CD: $CD = 5 \times 1.17 \dots$
 $= 5.88 \dots$

Angle ABD: $\tan \alpha = \frac{5}{12}$
 $\alpha = 0.396 \dots \text{ rad}$

Arc CE: $CE = 8 \times 0.396 \dots$
 $= 3.15 \dots$

Perimeter = $4 + 5.88 \dots + 3.15 \dots$
 $= 13.0 \text{ cm}$

(b) Find the area of the shaded region.

[3]

$$\text{Area DEC} = \text{Area of triangle ABD} - \text{sector ACD} - \text{sector BCE}$$

$$\text{Area of triangle ABD: Area} = \frac{1}{2} \times 12 \times 5$$

$$= 30$$

$$\text{Area of sector ACD: Area} = \frac{1}{2} \times 5^2 \times (1.17..)$$

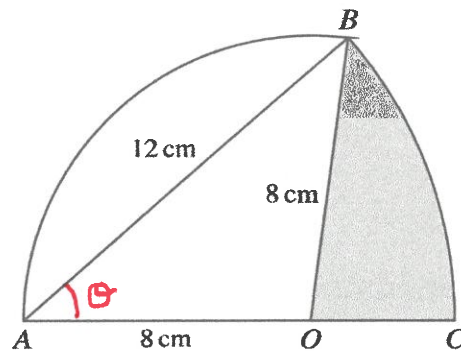
$$= 14.7$$

$$\text{Area of sector BCE: Area} = \frac{1}{2} \times 8^2 \times (0.396..)$$

$$= 12.6$$

$$\text{Area DEC} = 30 - 14.7 - 12.6$$

$$= \underline{2.67}$$



In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

- (a) Find angle BAO in radians.

[2]

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$$8^2 = 8^2 + 12^2 - 2 \times 8 \times 12 \times \cos \theta$$

$$64 = 64 + 144 - 192 \cos \theta$$

$$64 = 208 - 192 \cos \theta$$

$$192 \cos \theta = 144$$

$$\cos \theta = \frac{3}{4}$$

$$\theta = \underline{\underline{0.723}} \text{ STO}$$

(b) Find the area of the shaded region.

[4]

$$\begin{aligned}
 \text{Sector ABC: Area} &= \frac{1}{2} r^2 \theta \\
 &= \frac{1}{2} \times 12^2 \times 0.723 \\
 &= 52.0 \text{ s.t.o.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Triangle AOB: Area} &= \frac{1}{2} \times 8 \times 12 \times \sin(0.723) \\
 &= 31.7 \text{ s.t.o.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Shaded Area: Area} &= 52.0 - 31.7 \\
 &= \underline{20.3 \text{ cm}^2}
 \end{aligned}$$

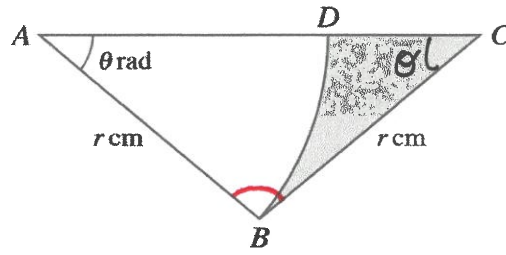
(c) Find the perimeter of the shaded region.

[3]

$$\begin{aligned}
 \text{Arc BC: arc length} &= r\theta \\
 &= 12 \times 0.723 \\
 &= 8.67 \text{ s.t.o.}
 \end{aligned}$$

$$\begin{aligned}
 \text{OC: } OC &= AC - AO \\
 &= 12 - 8 \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 \text{Perimeter} &= BC + OC + BO \\
 &= 8.67 + 4 + 8 \\
 &= \underline{20.7 \text{ cm}}
 \end{aligned}$$



In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A .

- (a) Express the area of the shaded region in terms of r and θ .

[3]

Angle ABC : $\hat{ABC} = \pi - \theta - \theta$
 $= \pi - 2\theta$

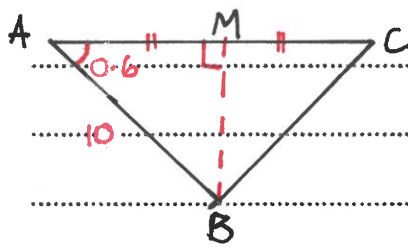
Triangle ABC : Area = $\frac{1}{2} \times r \times r \times \sin(\pi - 2\theta)$
 $= \frac{1}{2} r^2 \sin(\pi - 2\theta)$

Sector ABD : Area = $\frac{1}{2} r^2 \theta$

Shaded Area: Area = $\frac{1}{2} r^2 \sin(\pi - 2\theta) - \frac{1}{2} r^2 \theta$
 $= \frac{1}{2} r^2 (\sin(\pi - 2\theta) - \theta)$

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region.

[4]



$$\text{AM: } \cos(0.6) = \frac{AM}{10}$$

$$AM = 10 \cos(0.6)$$

$$AC = 2 \times 10 \cos(0.6) \\ = 20 \cos(0.6)$$

$$\text{DC: } DC = AC - AD$$

$$= 20 \cos(0.6) - 10$$

$$= \underline{6.51} \text{ s.f.}$$

$$\text{Arc BD: arc length} = r\theta$$

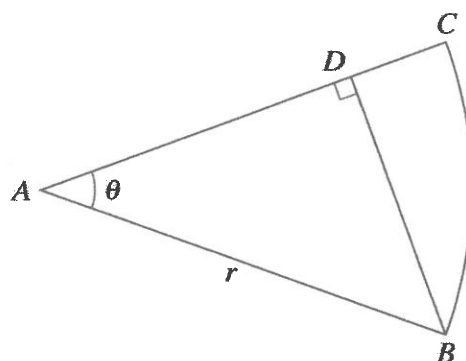
$$= 10 \times 0.6$$

$$= \underline{6}$$

$$\text{Perimeter} = BD + DC + CB$$

$$= 6 + 6.51 + 10$$

$$= \underline{22.5} \text{ cm}$$



The diagram shows a sector ABC of a circle with centre A and radius r . The line BD is perpendicular to AC . Angle CAB is θ radians.

- (a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r . [3]

Sector ABC : area = $\frac{1}{2} \times r^2 \times \frac{\pi}{6}$

$$= \frac{\pi r^2}{12}$$

AD: $\cos\left(\frac{\pi}{6}\right) = \frac{AD}{r}$
 $AD = r \cos\left(\frac{\pi}{6}\right)$

$$= \frac{r\sqrt{3}}{2}$$

Area of triangle ABD :

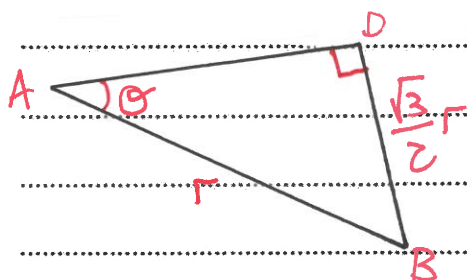
$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times r \times \frac{r\sqrt{3}}{2} \times \sin\left(\frac{\pi}{6}\right) \\ &= \frac{r^2\sqrt{3}}{8} \end{aligned}$$

Area of segment BCD = sector ABC - triangle ABD

$$= \frac{\pi r^2}{12} - \frac{r^2\sqrt{3}}{8}$$

- (b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r . [4]

→ θ is no longer $\frac{\pi}{6}$:



$$\sin \theta = \frac{\frac{\sqrt{3}}{2}r}{r}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

length AD : $\cos\left(\frac{\pi}{3}\right) = \frac{AD}{r}$

$$AD = r \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{r}{2}$$

length CD : $CD = AC - AD$

$$= r - \frac{r}{2}$$

$$= \frac{r}{2}$$

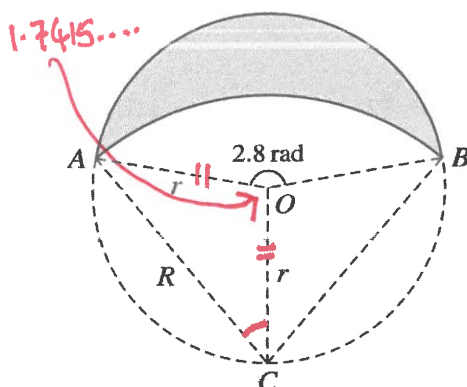
Arc BC : arc length = $r\theta$

$$= r \times \frac{\pi}{3}$$

$$= \frac{\pi r}{3}$$

$$\text{Perimeter} = \frac{\sqrt{3}}{2}r + \frac{r}{2} + \frac{\pi r}{3}$$

10



The diagram shows points A , B and C lying on a circle with centre O and radius r . Angle AOB is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre O and radius r . The lower arc is part of a circle with centre C and radius R .

- (a) State the size of angle ACO in radians.

[1]

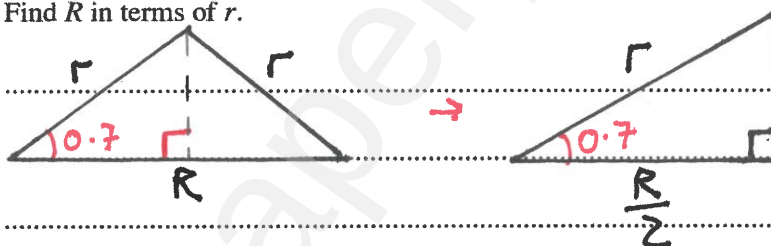
$$\hat{AOC}: \hat{AOC} = \frac{2\pi - 2.8}{2} = \pi - 1.4 \quad (\text{or } 1.7415\dots)$$

$$\hat{ACO}: \hat{ACO} = \frac{\pi - (\pi - 1.4)}{2} \quad (\text{isosceles triangle})$$

$$= \frac{1.4}{2} = \underline{\underline{0.7}}$$

- (b) Find R in terms of r .

[1]



$$\cos(0.7) = \frac{\frac{R}{2}}{r}$$

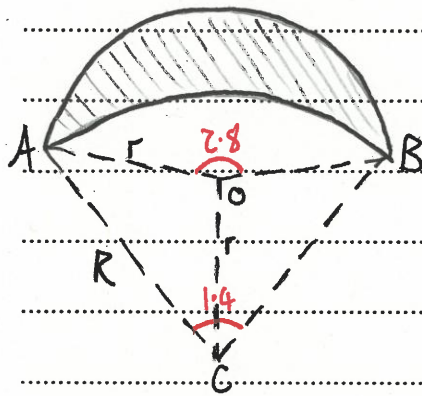
$$\cos(0.7) = \frac{R}{2r}$$

$$R = 2r \cos(0.7)$$

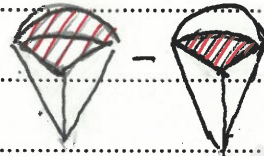
$$= \underline{\underline{1.53r}} \quad \text{STO}$$

(c) Find the area of the shaded region in terms of r .

[7]



Shaded Area =

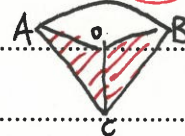
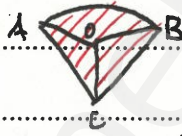


$$= \text{Sector AOB} - \text{Segment AOB}$$

$$\text{Sector AOB: Area} = \frac{1}{2} r^2 \times 2.8$$

$$= 1.4r^2$$

$$\text{Segment AOB} = \text{Sector ACB} - 2 \times \text{triangles}$$



$$\text{Sector ACB: Area} = \frac{1}{2} R^2 \times 1.4$$

$$= 0.7R^2$$

$$= 0.7(1.53r)^2$$

$$= 1.638r^2 \text{ STO}$$

$$1 \text{ triangle (AOC)} = \frac{1}{2} r \times r \times \sin(1.7415...)$$

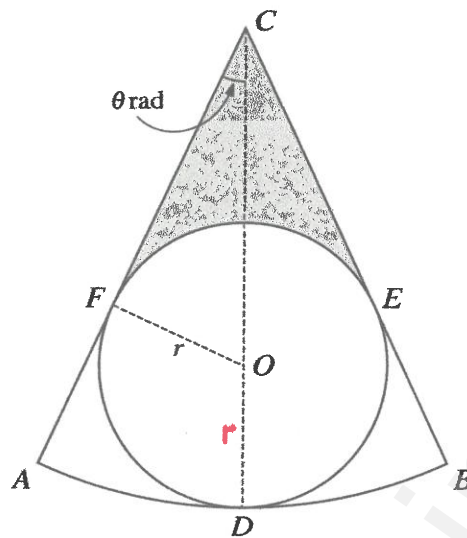
$$= 0.4927r^2 \text{ STO}$$

$$\text{Segment AOB:} = 1.638r^2 - 2 \times 0.4927r^2$$

$$= 0.6525r^2 \text{ STO}$$

$$\text{Shaded Area} = 1.4r^2 - 0.6525r^2$$

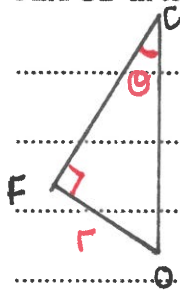
$$= \underline{\underline{0.7475r^2}}$$



The diagram shows a sector CAB which is part of a circle with centre C . A circle with centre O and radius r lies within the sector and touches it at D , E and F , where COD is a straight line and angle ACD is θ radians.

- (a) Find CD in terms of r and $\sin \theta$.

[3]

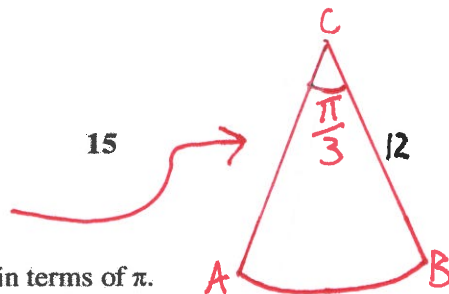


CO: $\sin \theta = \frac{r}{CO}$

$CO = \frac{r}{\sin \theta}$

$CD = \frac{r}{\sin \theta} + r$

It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.



(b) Find the perimeter of sector CAB in terms of π .

[3]

$$CO = \frac{4}{\sin(\frac{\pi}{6})} + 4$$

$$= 12$$

Arc AB: arc length = $12 \times \frac{\pi}{3}$

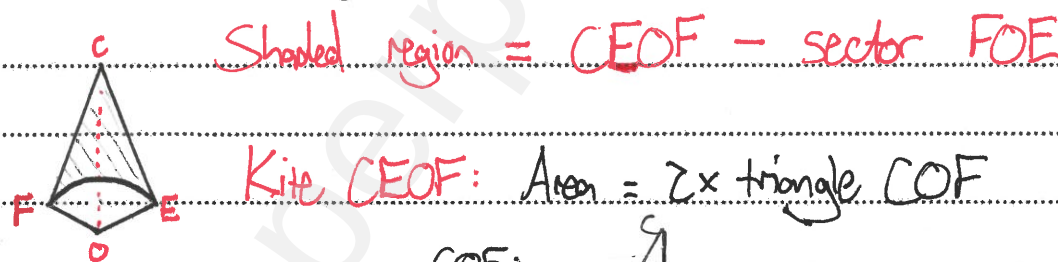
$$= 4\pi$$

$$\text{Perimeter} = 12 + 12 + 4\pi$$

$$= \underline{\underline{24 + 4\pi}}$$

(c) Find the area of the shaded region in terms of π and $\sqrt{3}$.

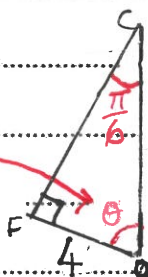
[4]



Shaded region = $CEOF$ - sector FOE

Kite $CEOF$: Area = $2 \times$ triangle COF

COF:



$$CO = \frac{r}{\sin \theta}$$

$$= \frac{4}{\sin(\frac{\pi}{6})}$$

$$= 8$$

$$\theta = \pi - \frac{\pi}{2} - \frac{\pi}{6}$$

$$= \frac{\pi}{3}$$

$$\text{Area} = \frac{1}{2} \times 4 \times 8 \times \sin\left(\frac{\pi}{3}\right)$$

$$= 8\sqrt{3}$$

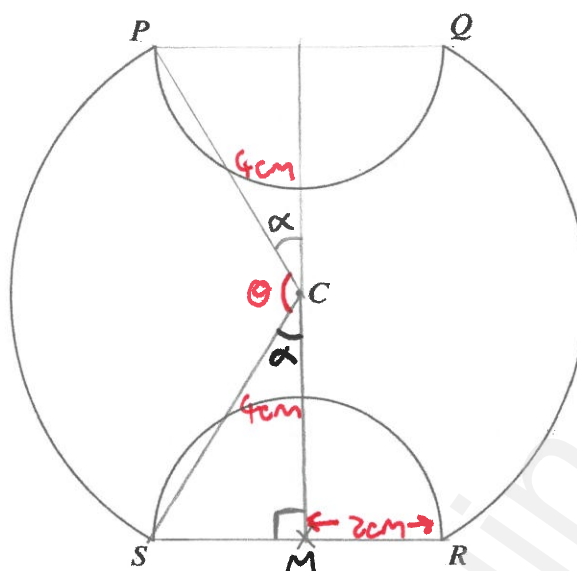
Sector FOE : Area = $\frac{1}{2} \times 4^2 \times \left(\frac{2\pi}{3}\right)$

$$= \frac{16\pi}{3}$$

Shaded Area:

$$= 2(8\sqrt{3}) - \frac{16\pi}{3}$$

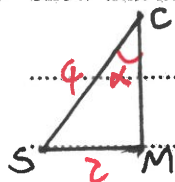
$$= \underline{\underline{16\sqrt{3} - \frac{16\pi}{3}}}$$



The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C . The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

- (a) Show that angle $PCS = \frac{2}{3}\pi$ radians.

[2]



$$\sin \alpha = \frac{2}{4}$$

$$\alpha = \frac{\pi}{6}$$

$$\begin{aligned} \angle C &= \pi - 2\alpha \\ &= \pi - 2\left(\frac{\pi}{6}\right) \\ &= \pi - \frac{\pi}{3} \\ &= \frac{2\pi}{3} \end{aligned}$$

- (b) Find the exact perimeter of the plate.

[3]

Arc PS (and QR): arc length = $4 \times \frac{2\pi}{3}$

$$= \frac{8\pi}{3}$$

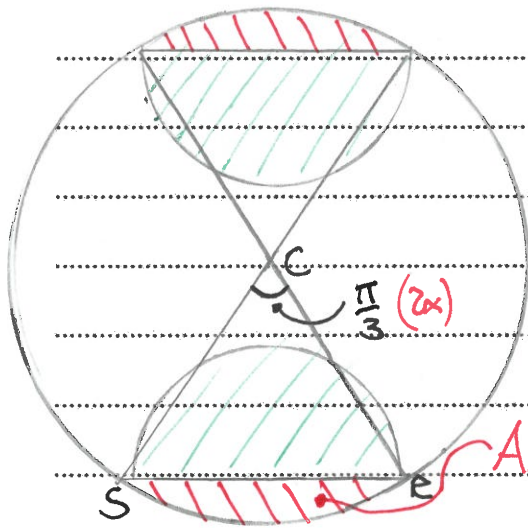
Semi-circle PQ (and RS): $\frac{1}{2} \times \pi \times 4$

$$= 2\pi$$

$$\begin{aligned} \text{Perimeter} &= \frac{8\pi}{3} \times 2 + 2\pi \times 2 \\ &= \frac{16\pi}{3} + 4\pi \\ &= \frac{28\pi}{3} \end{aligned}$$

(c) Show that the area of the plate is $(\frac{20}{3}\pi + 8\sqrt{3}) \text{ cm}^2$.

[5]



Area of Plate =
 Area of large circle
 - Area of two semi-circles
 - Area of two segments

Area of segment:

$$\begin{aligned}\text{Sector CRS} &= \frac{1}{2} \times 4^2 \times \frac{\pi}{3} \\ &= \frac{1}{2} \times 16 \times \frac{\pi}{3} \\ &= \frac{8\pi}{3}\end{aligned}$$

$$\begin{aligned}\text{Triangle CRS} &= \frac{1}{2} \times 4 \times 4 \times \sin\left(\frac{\pi}{3}\right) \\ &= 4\sqrt{3}\end{aligned}$$

$$\text{Area of segment} = \frac{8\pi}{3} - 4\sqrt{3}$$

$$\begin{aligned}\text{Area of one semi-circle} &= \frac{1}{2} \times \pi \times 2^2 \\ &= 2\pi\end{aligned}$$

$$\begin{aligned}\text{Area of large circle} &= \pi \times 4^2 \\ &= 16\pi\end{aligned}$$

$$\text{Area of plate} = 16\pi - 2(2\pi) - 2\left(\frac{8\pi}{3} - 4\sqrt{3}\right)$$

$$= 16\pi - 4\pi - \frac{16\pi}{3} + 8\sqrt{3}$$

$$= \frac{20\pi}{3} + 8\sqrt{3} \quad \text{QED}$$