

The diagram shows a sector of a circle, centre O , where $OB = OC = 15$ cm. The size of angle BOC is $\frac{2}{5}\pi$ radians. Points A and D on the lines OB and OC respectively are joined by an arc AD of a circle with centre O . The shaded region is bounded by the arcs AD and BC and by the straight lines AB and DC . It is given that the area of the shaded region is $\frac{209}{5}\pi$ cm².

Find the perimeter of the shaded region. Give your answer in terms of π .

[5]

$$\text{Area} = \frac{209}{5}\pi :$$

big sector

unshaded sector

$$\frac{209}{5}\pi = \frac{1}{2} \times 15^2 \times \frac{2}{5}\pi - \frac{1}{2} \times r^2 \times \frac{2}{5}\pi$$

$$\frac{209}{5}\pi = 45\pi - \frac{1}{5}r^2\pi$$

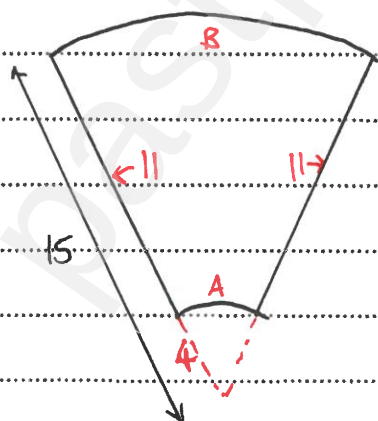
$$\frac{209}{5} = 45 - \frac{1}{5}r^2$$

$$209 = 225 - r^2$$

$$r^2 = 16$$

$$r = 4$$

Perimeter:



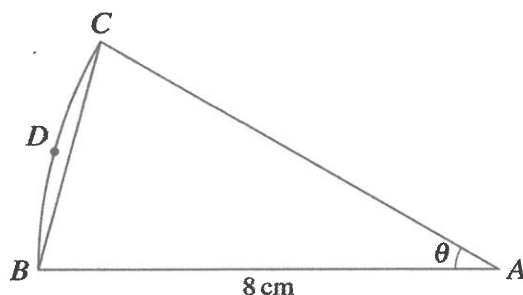
$$= A + B + 11 + 11$$

$$= 4 \times \theta + 15 \times \theta + 22$$

$$= 4 \times \frac{2}{5}\pi + 15 \times \frac{2}{5}\pi + 22$$

$$= \frac{8}{5}\pi + 6\pi + 22$$

$$= 22 + \frac{38}{5}\pi \text{ cm}$$



The diagram shows a sector ABC of a circle with centre A and radius 8 cm . The area of the sector is $\frac{16}{3}\pi\text{ cm}^2$. The point D lies on the arc BC .

Find the perimeter of the segment BCD .

[4]

Find angle θ :

$$A_{\text{sector}} = \frac{1}{2} r^2 \theta$$

$$\frac{16\pi}{3} = \frac{1}{2} \times 8^2 \times \theta$$

$$\frac{16\pi}{3} = 32\theta$$

$$\theta = \frac{\pi}{6}$$

Arc BC : Arc length = $r\theta$

$$= 8 \times \frac{\pi}{6}$$

$$= \frac{4\pi}{3}$$

Line BC : Cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$BC^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \cos\left(\frac{\pi}{6}\right)$$

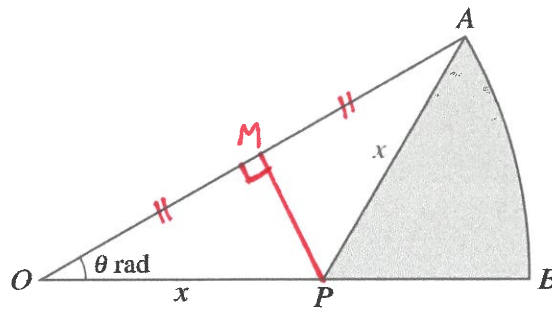
$$= 128 - 128 \times \frac{\sqrt{3}}{2}$$

$$= 128 - 64\sqrt{3}$$

$$BC = \sqrt{128 - 64\sqrt{3}}$$

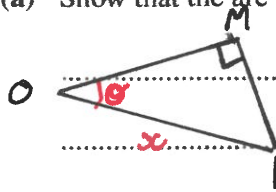
$$\text{Perimeter} = \frac{4\pi}{3} + \sqrt{128 - 64\sqrt{3}}$$

$$= \underline{\underline{8.33\text{ cm}}}$$



The diagram shows a sector OAB of a circle with centre O . Angle $AOB = \theta$ radians and $OP = AP = x$.

- (a) Show that the arc length AB is $2x\theta \cos \theta$. [2]



$$\cos \theta = \frac{OM}{x}$$

$$OM = x \cos \theta$$

$$OA = 2x \cos \theta$$

$$\text{arc length} = r\theta$$

$$= 2x \cos \theta \times \theta$$

$$= 2x\theta \cos \theta \quad \text{QED}$$

- (b) Find the area of the shaded region APB in terms of x and θ . [4]

Sector OAB: Area = $\frac{1}{2} \times (2x \cos \theta)^2 \times \theta$

$$= \frac{1}{2} \times 4x^2 \cos^2 \theta \times \theta$$

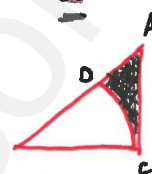
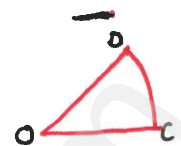
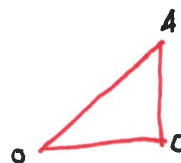
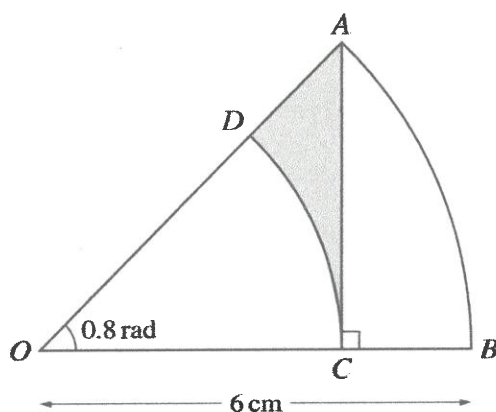
$$= 2x^2 \theta \cos^2 \theta$$

Triangle OAP: Area = $\frac{1}{2} \times x \times (2x \cos \theta) \times \sin \theta$

$$= x^2 \sin \theta \cos \theta$$

Area ABP: $2x^2 \theta \cos^2 \theta - x^2 \sin \theta \cos \theta$

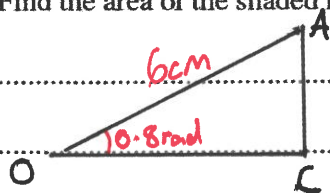
$$\underline{x^2 \cos \theta (2\theta \cos \theta - \sin \theta)}$$



The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB . The arc CD is part of a circle with centre O , where D lies on OA .

Find the area of the shaded region.

[6]



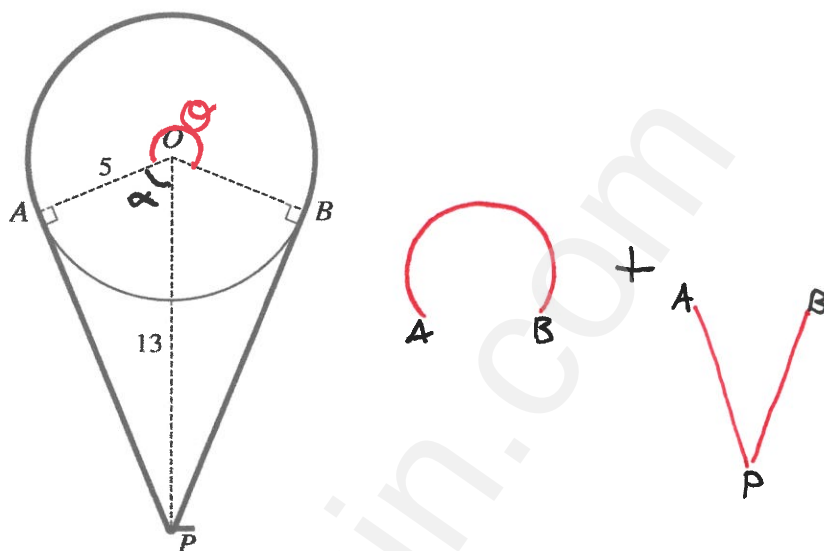
$$\cos(0.8) = \frac{OC}{6}$$

$$OC = 6 \cos(0.8) \\ = 4.18... \quad \text{STO}$$

Area of triangle OAC : $\text{Area} = \frac{1}{2}(4.18...)(6) \sin(0.8) \\ = 8.996... \quad \text{STO}$

Area of sector OCD : $\text{Area} = \frac{1}{2} \times (4.18...)^2 \times 0.8 \\ = 6.989... \quad \text{STO}$

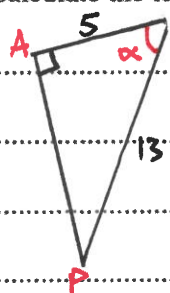
$$OCA - OCD = 8.996... - 6.989... \\ = \underline{\underline{2.01}}$$



The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O . Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord.

[6]



Angle AOP: $\cos \alpha = \frac{5}{13}$

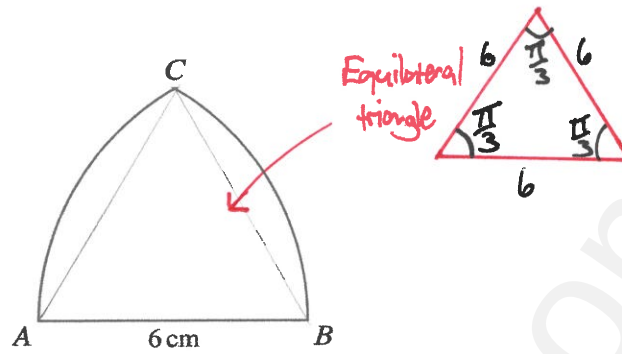
$\alpha = 1.18$ STO

Length AP: $13^2 = 5^2 + AP^2$
 $AP = \sqrt{13^2 - 5^2}$
 $= 12$

Major arc AB: $\theta = 2\pi - (2 \times 1.18)$
 $= 3.93$ STO

arc length $= r\theta$
 $= 5 \times 3.93$
 $= 19.7$

Length of cord $= AP + BP + \text{arc}_{AB}$
 $= 12 + 12 + 19.7$
 $= 43.7 \text{ cm}$



The diagram shows a metal plate ABC in which the sides are the straight line AB and the arcs AC and BC . The line AB has length 6 cm. The arc AC is part of a circle with centre B and radius 6 cm, and the arc BC is part of a circle with centre A and radius 6 cm.

- (a) Find the perimeter of the plate, giving your answer in terms of π .

[3]

Arc AC (and BC):

$$\text{arc length} = r\theta$$

$$= 6 \times \frac{\pi}{3}$$

$$= 2\pi$$

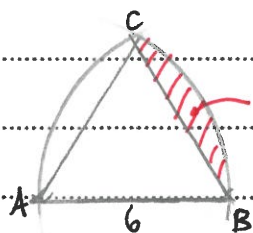
$$\text{Perimeter} = AB + BC + CA$$

$$= 6 + 2\pi + 2\pi$$

$$= \underline{4\pi + 6} \text{ cm}$$

(b) Find the area of the plate, giving your answer in terms of π and $\sqrt{3}$.

[4]



Area of segment = Area of sector - Area of triangle

Area of sector ABC:

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} \\ &= 6\pi \end{aligned}$$

Area of triangle ABC:

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 6 \times 6 \times \sin\left(\frac{\pi}{3}\right) \\ &= 9\sqrt{3} \end{aligned}$$

Area of segment:

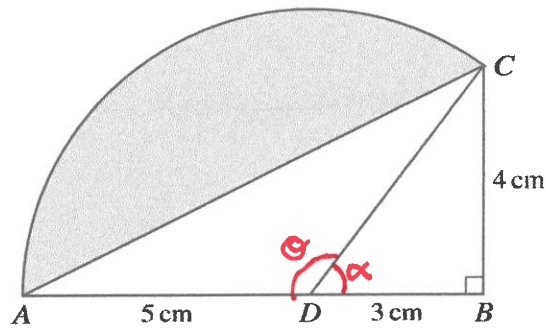
$$\text{Area} = 6\pi - 9\sqrt{3}$$

Area of plate = Triangle ABC + 2 × segment

$$= 9\sqrt{3} + 2(6\pi - 9\sqrt{3})$$

$$= 9\sqrt{3} + 12\pi - 18\sqrt{3}$$

$$= \underline{12\pi - 9\sqrt{3}} \text{ cm}^2$$

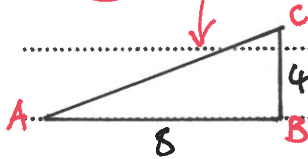


The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D .

- (a) Find the perimeter of the shaded region.

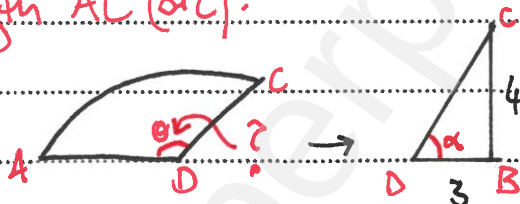
[5]

Length AC (line):



$$\begin{aligned} AC^2 &= 8^2 + 4^2 \\ AC &= \sqrt{8^2 + 4^2} \\ &= 4\sqrt{5} \end{aligned}$$

Length AC (arc):



angle BDC :

$$\tan \alpha = \frac{4}{3}$$

$$\alpha = 0.927 \text{ rad}$$

$$\begin{aligned} \theta &= \pi - 0.927 \\ &= 2.21 \end{aligned}$$

$$\text{arc } AC = r \times \theta$$

$$= 5 \times 2.21$$

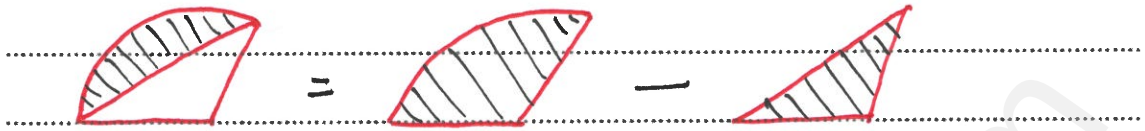
$$= 11.1$$

$$\text{Perimeter} = 4\sqrt{5} + 11.1$$

$$= 20.0 \text{ cm}$$

(b) Find the area of the shaded region.

[3]



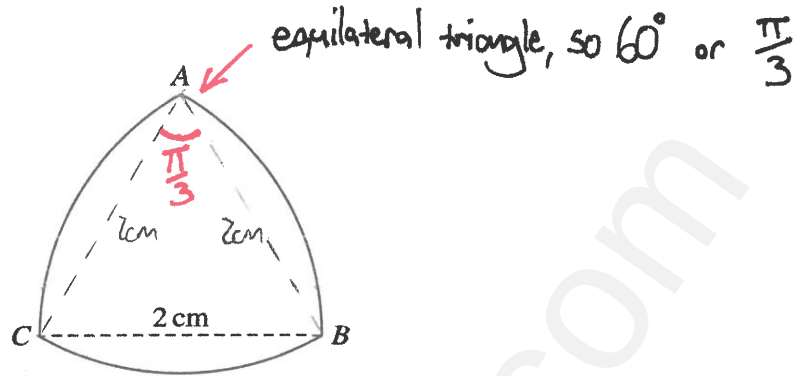
Area of sector ACD:

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} r^2 \theta \\
 &= \frac{1}{2} (5)^2 \times 2.21 \\
 &= 27.7 \text{ STO}
 \end{aligned}$$

Area of triangle ACD:

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} ab \sin C \\
 &= \frac{1}{2} \times 5 \times 5 \times \sin(2.21) \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of segment} &= 27.7 - 10 \\
 &= \underline{17.7 \text{ cm}^2}
 \end{aligned}$$



The diagram shows the shape of a coin. The three arcs AB , BC and CA are parts of circles with centres C , A and B respectively. ABC is an equilateral triangle with sides of length 2 cm.

- (a) Find the perimeter of the coin.

[2]

$$\begin{aligned} \text{Arc } BC &= r\theta \\ &= 2 \times \frac{\pi}{3} \\ \text{Perimeter} &= 3 \times \left(2 \times \frac{\pi}{3}\right) \\ &= \underline{\underline{2\pi}} \end{aligned}$$

- (b) Find the area of the face ABC of the coin, giving the answer in terms of π and $\sqrt{3}$.

[4]

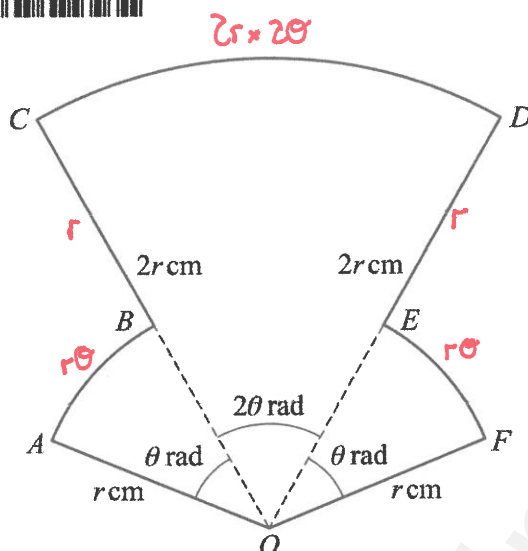
$$\begin{aligned} \text{Area of triangle } ABC: \\ &= \frac{1}{2} \times 2 \times 2 \times \sin\left(\frac{\pi}{3}\right) \\ &= 2 \times \frac{\sqrt{3}}{2} \\ &= \underline{\underline{\sqrt{3}}} \end{aligned}$$

Area of segment CB :

$$\begin{aligned} &= \text{Area of sector} - \text{Area of triangle} \\ &= \frac{1}{2} r^2 \theta - \sqrt{3} \\ &= \frac{1}{2} \times 2^2 \times \frac{\pi}{3} - \sqrt{3} \\ &= \underline{\underline{\frac{2\pi}{3} - \sqrt{3}}} \end{aligned}$$

Area of coin = triangle + 3 × segments:

$$\begin{aligned} &= \sqrt{3} + 3\left(\frac{2\pi}{3} - \sqrt{3}\right) \\ &= \sqrt{3} + 2\pi - 3\sqrt{3} = \underline{\underline{2\pi - 2\sqrt{3}}} \end{aligned}$$



The diagram shows a metal plate $OABCDEF$ consisting of sectors of two circles, each with centre O . The radii of sectors AOB and EOF are r cm and the radius of sector COD is $2r$ cm. Angle $AOB = \text{angle } EOF = \theta$ radians and angle $COD = 2\theta$ radians.

It is given that the perimeter of the plate is 14 cm and the area of the plate is 10 cm^2 .

Given that $r > \frac{3}{2}$ and $\theta < \frac{3}{4}$, find the values of r and θ .

[6]

Perimeter:

$$r + r\theta + r + 2r \times 2\theta + r + r\theta + r = 14$$

$$4r + 2r\theta + 4r\theta = 14$$

$$4r + 6r\theta = 14$$

$$2r + 3r\theta = 7 \quad (1)$$

Area:

2 small sectors big sector

$$\frac{1}{2}r^2\theta \times 2 + \frac{1}{2}(2r)^2 \times 2\theta = 10$$

$$r^2\theta + \frac{1}{2} \times 4r^2 \times 2\theta = 10$$

$$r^2\theta + 4r^2\theta = 10$$

$$5r^2\theta = 10$$

$$r^2\theta = 2$$

$$\theta = \frac{2}{r^2} \quad (2)$$



Sub. ② into ①:

$$2r + 3r\left(\frac{2}{r^2}\right) = 7$$

$$2r + \frac{6}{r} = 7$$

$\times r$ $\times r$ $\times r$

$$2r^2 + 6 = 7r$$

$$2r^2 - 7r + 6 = 0$$

$$(2r - 3)(r - 2) = 0$$

$$2r - 3 = 0 \quad \text{or} \quad \underline{r = 2}$$

$$r = 1.5$$

$$r = 1.5$$

$$\times r > \frac{3}{2}$$

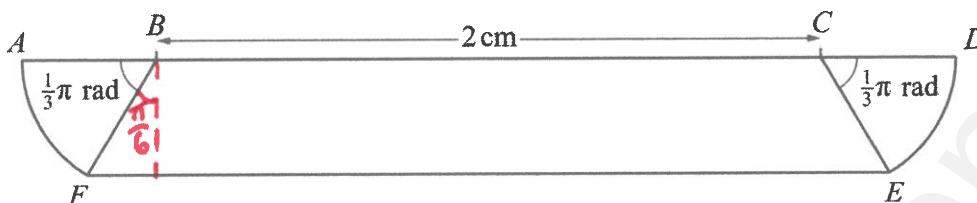
→ ②: $r = 2$:

$$\textcircled{1} = \frac{2}{2^2}$$

$$= \frac{2}{4}$$

$$\underline{\underline{\textcircled{1} = \frac{1}{2}}}$$





The diagram shows a symmetrical plate $ABCDEF$. The line $ABCD$ is straight and the length of BC is 2 cm. Each of the two sectors ABF and DCE is of radius r cm and each of the angles ABF and DCE is equal to $\frac{1}{3}\pi$ radians.

(a) It is given that $r = 0.4$ cm.

(i) Show that the length $EF = 2.4$ cm. [2]

$$\sin\left(\frac{\pi}{6}\right) = \frac{x}{0.4}$$

$$x = 0.4 \sin\left(\frac{\pi}{6}\right) = 0.2$$

$$EF = 2 + 0.2 + 0.2 = \underline{\underline{2.4 \text{ cm}}}$$

(ii) Find the area of the plate. Give your answer correct to 3 significant figures. [4]

$$y^2 + 0.2^2 = 0.4^2$$

$$y^2 + 0.04 = 0.16$$

$$y^2 = 0.12$$

$$y = \frac{\sqrt{3}}{5}$$

$$\begin{aligned} \text{Area} &= 2 \times \text{sectors} + 2 \times \text{triangles} + \text{rectangle} \\ &= 2 \times \frac{1}{2} r^2 \theta + 2 \times \frac{b \times h}{2} + 2 \times \frac{\sqrt{3}}{5} \\ &= 2 \times \frac{1}{2} \times 0.4^2 \times \frac{\pi}{3} + 2 \times \frac{0.2 \times \frac{\sqrt{3}}{5}}{2} + \frac{2\sqrt{3}}{5} \\ &= \frac{4\pi}{75} + \frac{\sqrt{3}}{25} + \frac{2\sqrt{3}}{5} \\ &= \underline{\underline{0.930}} \text{ (3sf)} \end{aligned}$$

- (b) It is given instead that the perimeter of the plate is 6 cm.

Find the value of r . Give your answer correct to 3 significant figures.

$$\text{Perimeter} = \overset{\text{top}}{\underbrace{2 + 2r}_{AB + CD}} + \overset{\text{bottom}}{\underbrace{2 \times \text{arc length} + 2 \times r \sin\left(\frac{\pi}{6}\right)}_{AF + DE}} + 2 \quad [4]$$

$$= 2 + 2r + 2 \times r \theta + 2r \times \frac{1}{2} + 2$$

$$= 2 + 2r + 2 \times r \left(\frac{\pi}{3}\right) + r + 2$$

$$= 4 + 3r + \frac{2\pi r}{3}$$

Perimeter = 6:

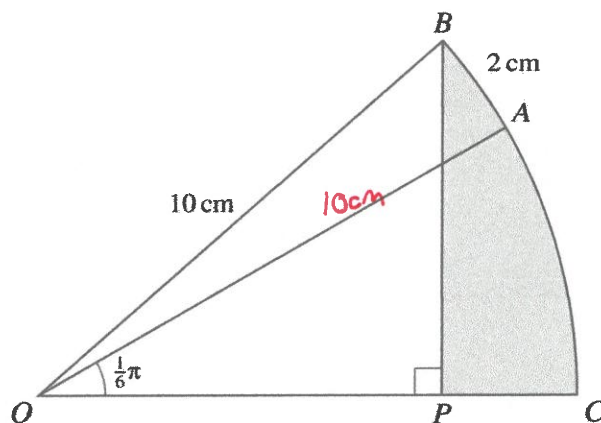
$$4 + 3r + \frac{2\pi r}{3} = 6$$

$$3r + \frac{2\pi r}{3} = 2$$

$$r \left(3 + \frac{2\pi}{3}\right) = 2$$

$$r = \frac{2}{3 + \frac{2\pi}{3}}$$

$$= \underline{\underline{0.393}}$$



The diagram shows a sector $OBAC$ of a circle with centre O and radius 10 cm. The point P lies on OC and BP is perpendicular to OC . Angle $AOC = \frac{1}{6}\pi$ and the length of the arc AB is 2 cm.

- (a) Find the angle BOC .

[2]

Arc AC: arc length = $r\theta$
 $= 10 \times \frac{\pi}{6}$
 $= \frac{5\pi}{3}$

Arc BC: arc length = $AC + 2$
 $= \frac{5\pi}{3} + 2$

arc length = $r\theta$

$\frac{5\pi}{3} + 2 = 10\theta$

$\theta = \frac{\frac{5\pi}{3} + 2}{10}$

$= \frac{5\pi + 6}{30}$

or = 0.724

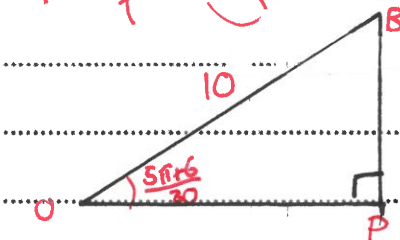
- (b) Hence find the area of the shaded region BPC giving your answer correct to 3 significant figures. [4]

Area of sector OBC :

$$\text{Area} = \frac{1}{2} \times 10^2 \times \left(\frac{5\pi + 6}{30} \right)$$

$$= 36.2 \text{ STO}$$

Area of triangle OBP :



OP: $\cos\left(\frac{5\pi + 6}{30}\right) = \frac{OP}{10}$

$$OP = 7.49 \text{ STO}$$

BP: $\sin\left(\frac{5\pi + 6}{30}\right) = \frac{BP}{10}$

$$BP = 6.62 \text{ STO}$$

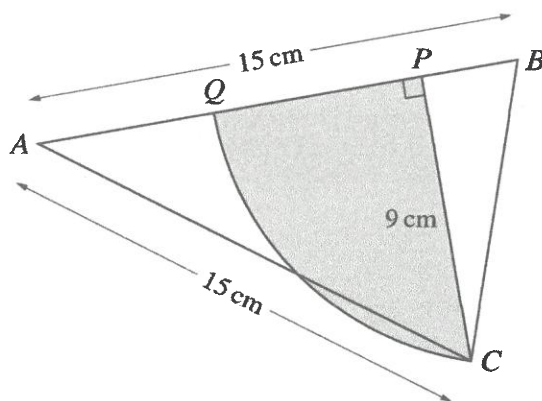
$$\text{Area} = \frac{1}{2} \times 7.49 \times 6.62$$

$$= 24.8$$

Area BPC = sector OBC - triangle OBP

$$= 36.2 - 24.8$$

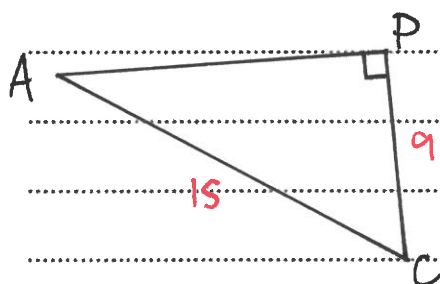
$$= \underline{11.4 \text{ cm}^2}$$



In the diagram the lengths of AB and AC are both 15 cm. The point P is the foot of the perpendicular from C to AB . The length $CP = 9$ cm. An arc of a circle with centre B passes through C and meets AB at Q .

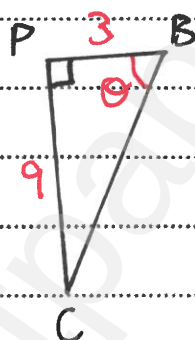
(a) Show that angle $ABC = 1.25$ radians, correct to 3 significant figures.

[2]



Pythagoras: $9^2 + AP^2 = 15^2$
 $AP^2 = 15^2 - 9^2$
 $AP = \sqrt{15^2 - 9^2}$
 $= 12$

$PB = 15 - 12$
 $= 3$



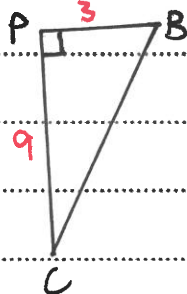
$\tan \theta = \frac{9}{3}$

$\theta = 1.25$ QED
 STD

- (b) Calculate the area of the shaded region which is bounded by the arc CQ and the lines CP and PQ . [4]

Shaded Region = Sector BCQ - triangle BPC

Radius of sector = BC :

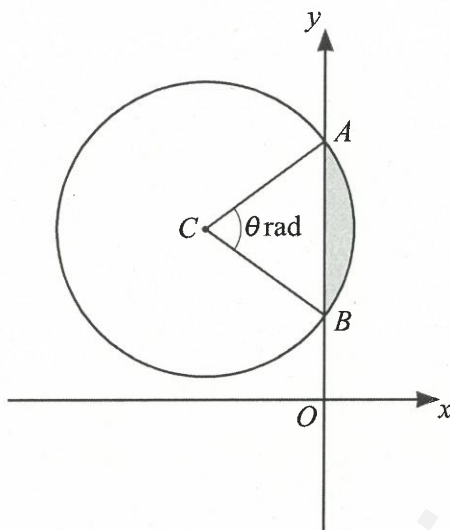


$$\begin{aligned} \text{Pythagoras: } BC^2 &= 3^2 + 9^2 \\ &= 9 + 81 \\ BC &= \sqrt{90} \\ &= 3\sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{Area of sector: } \text{area} &= \frac{1}{2} \times (3\sqrt{10})^2 \times 1.25 \\ &= 56.2 \end{aligned}$$

$$\begin{aligned} \text{Area of triangle: } \text{area} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 3 \times 9 \\ &= 13.5 \end{aligned}$$

$$\begin{aligned} \text{Shaded area} &= 56.2 - 13.5 \\ &= \underline{\underline{42.7 \text{ cm}^2}} \end{aligned}$$



The diagram shows the circle with centre $C(-4, 5)$ and radius $\sqrt{20}$ units. The circle intersects the y -axis at the points A and B . The size of angle ACB is θ radians.

- (a) Find the equation of the tangent to the circle at the point $(-6, 9)$. [3]

$$\text{Gradient of radius} = \frac{9 - 5}{-6 - (-4)} = \frac{4}{-2} = -2$$

$$\text{Gradient of tangent} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 9 = \frac{1}{2}(x - (-6))$$

$$y - 9 = \frac{1}{2}x + 3$$

$$y = \frac{1}{2}x + 12$$

- (b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. [2]

$$C: (-4, 5), r = \sqrt{20}$$

$$(x + 4)^2 + (y - 5)^2 = (\sqrt{20})^2$$

$$x^2 + 8x + 16 + y^2 - 10y + 25 = 20$$

$$x^2 + y^2 + 8x - 10y + 41 = 20$$

$$x^2 + y^2 + 8x - 10y + 21 = 0$$

- (c) Find the value of
- θ
- correct to 4 significant figures.

[3]

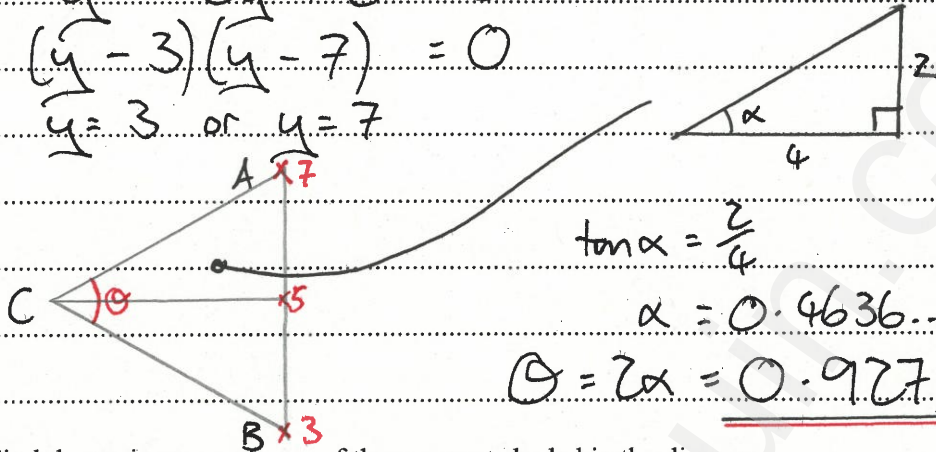
Find A and B by substituting $x=0$:

$$0^2 + y^2 + 0 - 10y + 21 = 0$$

$$y^2 - 10y + 21 = 0$$

$$(y-3)(y-7) = 0$$

$$y=3 \text{ or } y=7$$



$$\tan \alpha = \frac{2}{4}$$

$$\alpha = 0.4636...$$

$$\theta = 2\alpha = 0.9273$$

510

- (d) Find the perimeter and area of the segment shaded in the diagram.

[4]

Perimeter:

$$\text{Arc length} = r\theta$$

$$= \sqrt{20} \times 0.9273$$

$$\text{Perimeter} = 4 + \sqrt{20} \times 0.9273$$

$$= \underline{8.15}$$

Area:

$$\text{Area of sector ACB} = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}(\sqrt{20})^2 \times 0.9273$$

$$= 9.27295...$$

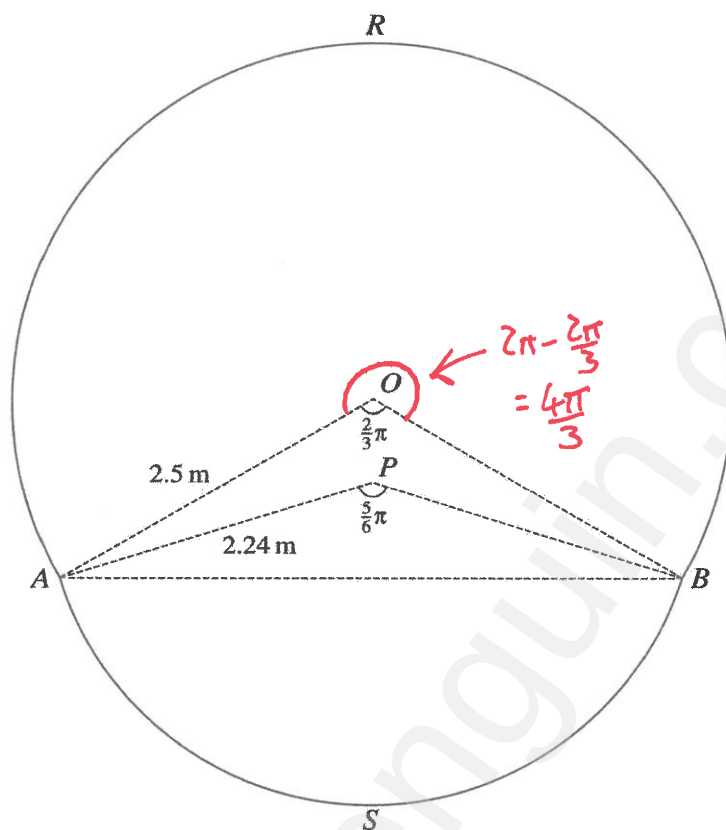
$$\text{Area of triangle ACB} = \frac{1}{2} \times r \times r \times \sin \theta$$

$$= \frac{1}{2}(\sqrt{20})^2 \times \sin(0.9273)$$

$$= \underline{8}$$

$$\text{Shaded Area} = 9.27295 - 8$$

$$= \underline{1.27}$$



The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of radius 2.5 m, with centre O , a sector $PASB$ of another circle of radius 2.24 m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$.

- (a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places.

[3]

$$\text{Arc ASB: arc length} = 2.24 \times \frac{5\pi}{6}$$

$$= \frac{28\pi}{15}$$

$$\text{Arc ARB: arc length} = 2.5 \times \frac{4\pi}{3}$$

$$= \frac{10\pi}{3}$$

$$\text{Perimeter} = \frac{28\pi}{15} + \frac{10\pi}{3}$$

$$= \underline{\underline{16.34 \text{ m}}}$$

- (b) Find the difference in area of the two triangles AOB and APB , giving your answer correct to 2 decimal places. [2]

$$\text{Triangle } AOB: \text{ area} = \frac{1}{2} \times 2.5 \times 2.5 \times \sin\left(\frac{2\pi}{3}\right)$$

$$= 2.71 \text{ STO}$$

$$\text{Triangle } APB: \text{ area} = \frac{1}{2} \times 2.24 \times 2.24 \times \sin\left(\frac{5\pi}{6}\right)$$

$$= 1.2544$$

$$\text{Difference} = 2.71 - 1.2544$$

$$= \underline{1.45}$$

- (c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]

$$RASB = \text{Area of major sector } OARB + \text{sector } PASB + \text{difference } (1.45)$$

$$\text{Major sector } OARB: \text{ area} = \frac{1}{2} \times 2.5^2 \times \frac{4\pi}{3}$$

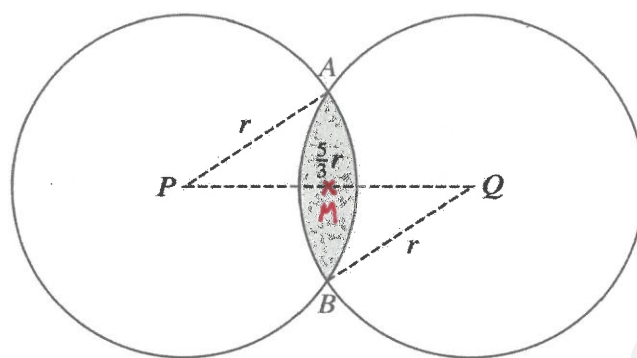
$$= \frac{25\pi}{6}$$

$$\text{Sector } PASB: \text{ area} = \frac{1}{2} \times 2.24^2 \times \frac{5\pi}{6}$$

$$= 6.57 \text{ STO}$$

$$\text{Total area} = \frac{25\pi}{6} + 6.57 + 1.45$$

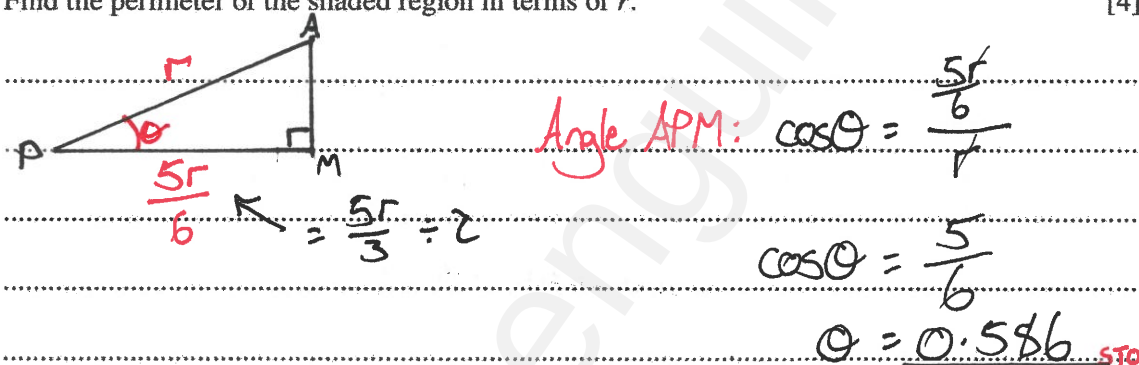
$$= \underline{21.1 \text{ m}^2}$$



The diagram shows two identical circles intersecting at points A and B and with centres at P and Q . The radius of each circle is r and the distance PQ is $\frac{5}{3}r$.

- (a) Find the perimeter of the shaded region in terms of r .

[4]



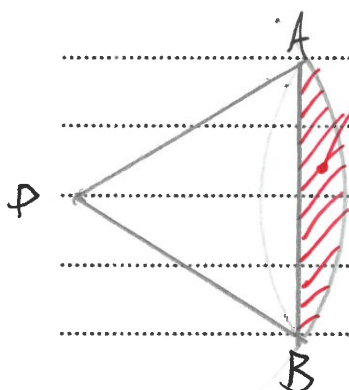
Angle APB: $\hat{APB} = 2 \times 0.586$
 $= 1.17 \text{ STO}$

Arc AB: arc length $= r \times 1.17$
 $= 1.17r$

Perimeter $= 2 \times \text{Arc AB}$
 $= 2 \times 1.17r$
 $= \underline{\underline{2.34r}}$

(b) Find the area of the shaded region in terms of r .

[3]



Area of segment = Sector PAB - Triangle PAB

Sector PAB: Area = $\frac{1}{2} \times r^2 \times 1.17$

Triangle PAB: Area = $\frac{1}{2} \times r \times r \times \sin(1.17)$

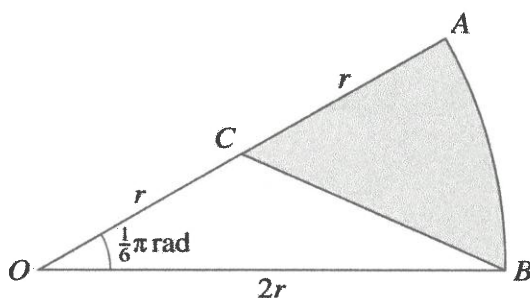
Segment: Area = $\frac{1}{2} r^2 \times 1.17 - \frac{1}{2} r^2 \sin(1.17)$
 $= \frac{1}{2} r^2 (1.17 - \sin(1.17))$

Shaded area = $2 \times \text{segment}$

$$= 2 \times \frac{1}{2} r^2 (1.17 - \sin(1.17))$$

$$= r^2 (1.17 - \sin(1.17))$$

$$= \underline{\underline{0.250 r^2}}$$



In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA .

- (a) Show that the exact length of BC is $r\sqrt{5-2\sqrt{3}}$. [2]

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\begin{aligned} BC^2 &= r^2 + (2r)^2 - 2 \times r \times 2r \times \cos\left(\frac{\pi}{6}\right) \\ &= r^2 + 4r^2 - 4r^2 \times \frac{\sqrt{3}}{2} \\ &= 5r^2 - 2\sqrt{3}r^2 \\ &= r^2(5 - 2\sqrt{3}) \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{r^2(5 - 2\sqrt{3})} \\ &= \sqrt{r^2} \times \sqrt{5 - 2\sqrt{3}} \\ &= \underline{r\sqrt{5 - 2\sqrt{3}}} \quad \text{QED} \end{aligned}$$

- (b) Find the exact perimeter of the shaded region.

[2]

$$\text{Arc AB: arc length} = 2r \times \frac{\pi}{6} \\ = \frac{\pi r}{3}$$

$$\text{Perimeter} = \underline{\underline{r + \frac{\pi r}{3} + r\sqrt{5} - 2\sqrt{3}}}$$

- (c) Find the exact area of the shaded region.

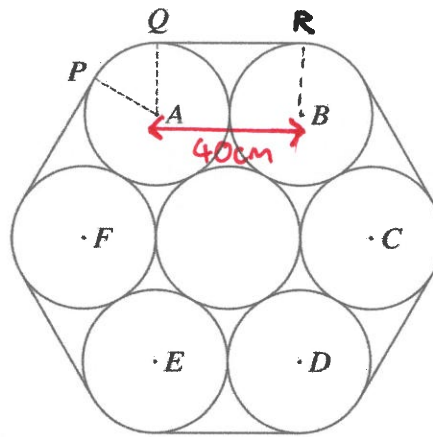
[3]

$$\text{Area ABC} = \text{Sector OAB} - \text{Triangle OBC}$$

$$\text{Sector OAB: Area} = \frac{1}{2} \times (2r)^2 \times \frac{\pi}{6} \\ = \frac{1}{2} \times 4r^2 \times \frac{\pi}{6} \\ = \frac{\pi r^2}{3}$$

$$\text{Triangle OBC: Area} = \frac{1}{2} \times r \times 2r \times \sin\left(\frac{\pi}{6}\right) \\ = r^2 \sin\left(\frac{\pi}{6}\right) \\ = \frac{r^2}{2}$$

$$\text{Area ABC} = \underline{\underline{\frac{\pi r^2}{3} - \frac{r^2}{2}}}$$



The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

- (a) Show that angle $PAQ = \frac{1}{3}\pi$ radians.

[2]

$$PAQ = 2\pi - \frac{2\pi}{3} - \frac{\pi}{2} - \frac{\pi}{2}$$

$$= \frac{\pi}{3}$$

$\frac{2\pi}{3}$ ← interior angle of regular hexagon

- (b) Find the length of the rope.

[4]

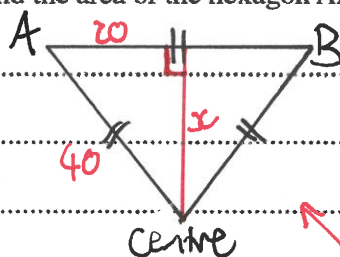
Distance AB: $AB = 2 \times \text{radius}$
 $= 40 \text{ cm}$

Arc PQ: arc length = $r\theta$
 $= 20 \times \frac{\pi}{3}$
 $= \frac{20\pi}{3}$

Total length = $6\left(40 + \frac{20\pi}{3}\right)$
 $= \underline{240 + 40\pi} \text{ cm}$

- (c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$.

[2]



$$x: x^2 + 20^2 = 40^2$$

$$x^2 = 40^2 - 20^2$$

$$x = \sqrt{40^2 - 20^2}$$

$$= 20\sqrt{3}$$

$$\text{Area of whole triangle: } \frac{1}{2} \times 40 \times 20\sqrt{3}$$

$$= 400\sqrt{3}$$

$$\text{Area of hexagon: } 6 \times 400\sqrt{3}$$

$$= \underline{\underline{2400\sqrt{3}}}$$

- (d) Find the area of the complete region enclosed by the rope.

[3]

Area of sector APQ:

$$\text{area} = \frac{1}{2} \times 20^2 \times \frac{\pi}{3}$$

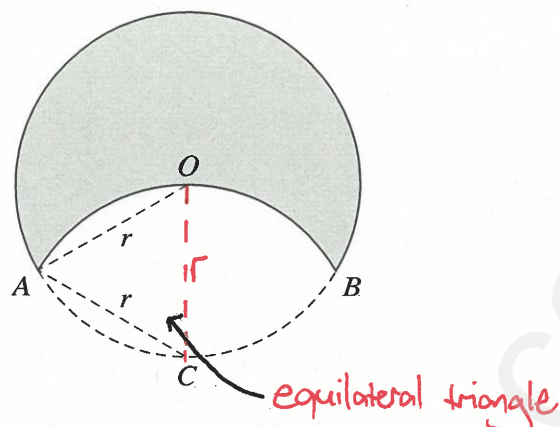
$$= \frac{200\pi}{3}$$

$$\text{Area of rectangle ABRQ: } 20 \times 40 = 800$$

$$\text{Total Area} = 6 \times \text{sectors} + 6 \times \text{rectangles} + \text{Hexagon}$$

$$= 6 \left(\frac{200\pi}{3} \right) + 6(800) + 2400\sqrt{3}$$

$$= \underline{\underline{400\pi + 4800 + 2400\sqrt{3}}}$$



The diagram shows a motif formed by the major arc AB of a circle with radius r and centre O , and the minor arc AOB of a circle, also with radius r but with centre C . The point C lies on the circle with centre O .

- (a) Given that angle $ACB = k\pi$ radians, state the value of the fraction k . [1]

ACO is an equilateral triangle, so $ACO = 60^\circ = \frac{\pi}{3}$ radians

$$ACB = 2 \times ACO$$

$$= \frac{2\pi}{3} \text{ radians}$$

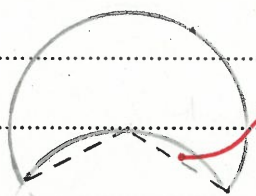
$$\Rightarrow \underline{k = \frac{2}{3}}$$

- (b) State the perimeter of the shaded motif in terms of π and r . [1]

Arc AOB is the same as arc ACB , so the perimeter is the same as the circumference of a circle: $2\pi r$

(c) Find the area of the shaded motif, giving your answer in terms of π , r and $\sqrt{3}$.

[5]



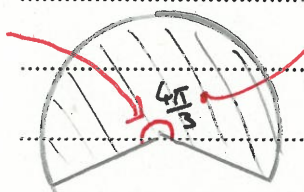
Area of tiny segment:

$$= \text{Area of sector} - \text{Area of triangle}$$

$$= \frac{1}{2} r^2 \left(\frac{\pi}{3} \right) - \frac{1}{2} \times r \times r \times \sin\left(\frac{\pi}{3}\right)$$

$$= \frac{\pi}{6} r^2 - \frac{\sqrt{3}}{4} r^2$$

$$2\pi - \frac{2\pi}{3}$$



Area of major sector:

$$= \frac{1}{2} r^2 \times \frac{4\pi}{3}$$

$$= \frac{2\pi}{3} r^2$$

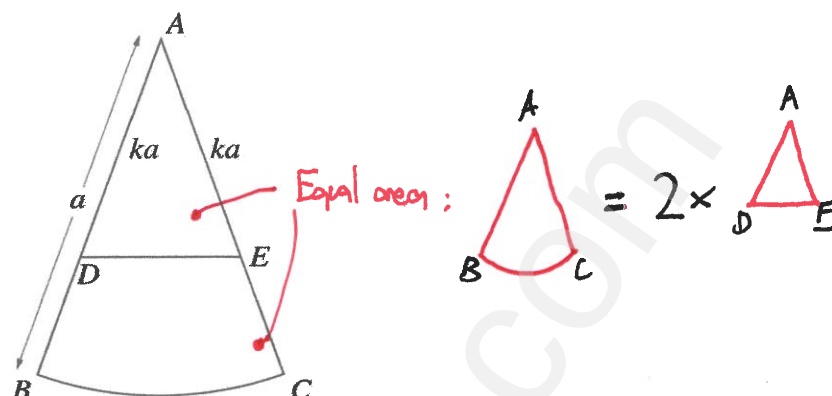
Shaded motif = Major sector - 2 x segments

$$= \frac{2\pi}{3} r^2 - 2 \left(\frac{\pi}{6} r^2 - \frac{\sqrt{3}}{4} r^2 \right)$$

$$= \frac{2\pi}{3} r^2 - \frac{\pi}{3} r^2 + \frac{\sqrt{3}}{2} r^2$$

$$= \frac{\pi}{3} r^2 + \frac{\sqrt{3}}{2} r^2$$

$$= \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2} \right) r^2$$



The diagram shows a sector ABC which is part of a circle of radius a . The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

- (a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

Area of sector ABC : $\text{Area} = \frac{1}{2} \times a^2 \times \frac{\pi}{6}$
 $= \frac{\pi a^2}{12}$

Area of triangle ADE : $\text{Area} = \frac{1}{2} \times ka \times ka \times \sin\left(\frac{\pi}{6}\right)$
 $= \frac{1}{4} k^2 a^2$

Sector $ABC = 2 \times \text{Triangle } ADE$:

$$\frac{\pi a^2}{12} = 2 \times \frac{k^2 a^2}{4}$$

$$\frac{\pi a^2}{12} = \frac{k^2 a^2}{2}$$

$$2\pi a^2 = 12k^2 a^2$$

$$\pi a^2 = 6k^2 a^2$$

$$\pi = 6k^2$$

$$k^2 = \frac{\pi}{6}$$

$$k = \sqrt{\frac{\pi}{6}}$$

$$= 0.7236$$

(can't be negative because it's a length and a is positive)

- (b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\frac{\theta}{\sin \theta} > 1$.

Find the set of possible values of k .

[3]

Area of sector ABC:
$$\text{Area} = \frac{1}{2} \times a^2 \times \theta$$

$$= \frac{a^2 \theta}{2}$$

Area of triangle ADE:
$$\text{Area} = \frac{1}{2} \times ka \times ka \times \sin \theta$$

$$= \frac{k^2 a^2 \sin \theta}{2}$$

Sector ABC = 2 × Triangle ADE:

$$\frac{a^2 \theta}{2} = 2 \times \frac{k^2 a^2 \sin \theta}{2}$$

$$\frac{a^2 \theta}{2} = k^2 a^2 \sin \theta$$

$$\theta = 2k^2 \sin \theta$$

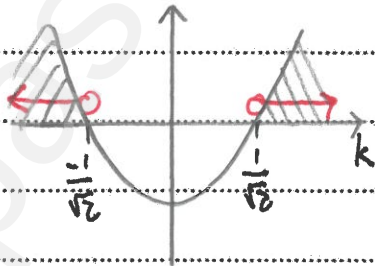
$$\theta = 2k^2 \sin \theta$$

$$\frac{\theta}{\sin \theta} = 2k^2$$

$$\frac{\theta}{\sin \theta} > 1, \text{ so: } 2k^2 > 1$$

$$k^2 > \frac{1}{2} \text{ or: } (k + \frac{1}{\sqrt{2}})(k - \frac{1}{\sqrt{2}}) > 0$$

critical values: $k = -\frac{1}{\sqrt{2}}, k = \frac{1}{\sqrt{2}}$



k must be positive, so $k > \frac{1}{\sqrt{2}}$ only,
but also $k < 1$ (given in question) so:

$$\frac{1}{\sqrt{2}} < k < 1$$