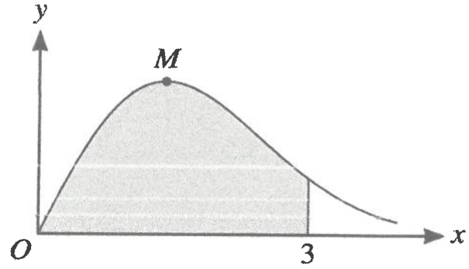
9



The diagram shows the curve $y = \overset{u}{x}\overset{v}{e}^{-\frac{1}{4}x^2}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the exact coordinates of M. [4]

$u = x \qquad v = e^{-\frac{1}{4}x^2}$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = -\frac{1}{2}x e^{-\frac{1}{4}x^2}$

$\frac{dy}{dx} = -\frac{1}{2}x^2 e^{-\frac{1}{4}x^2} + e^{-\frac{1}{4}x^2}$

SP, so $\frac{dy}{dx} = 0$:

$e^{-\frac{1}{4}x^2}\left(-\frac{1}{2}x^2 + 1\right) = 0$

$e^{-\frac{1}{4}x^2} = 0$ or $-\frac{1}{2}x^2 + 1 = 0$

no sol^ns

$\frac{1}{2}x^2 = 1$

$x^2 = 2$

$x = \pm\sqrt{2}$

Point M is to the right of y-axis, so $x = +\sqrt{2}$:

$y = \sqrt{2}\, e^{-\frac{1}{4}(\sqrt{2})^2}$

$= \sqrt{2}\, e^{-\frac{1}{2}}$

$\rightarrow \underline{\left(\sqrt{2}, \sqrt{2}e^{-\frac{1}{2}}\right)}$

2 Find the exact coordinates of the stationary point of the curve $y = \overset{u}{e^{2x}} \overset{v}{\sin 2x}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [5]

$u = e^{2x} \qquad v = \sin 2x$

$\frac{dy}{dx} = 2e^{2x} \qquad \frac{dv}{dx} = 2\cos 2x$

$\frac{dy}{dx} = 2e^{2x}\cos 2x + 2e^{2x}\sin 2x$

$= 2e^{2x}(\cos 2x + \sin 2x)$

SP: $\frac{dy}{dx} = 0$: $2e^{2x}(\cos 2x + \sin 2x) = 0$

$2e^{2x} = 0$ or $\cos 2x + \sin 2x = 0$ ✓

× No sol^s

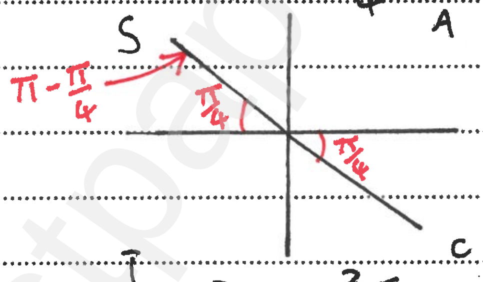
$\cos 2x + \sin 2x = 0$

$\sin 2x = -\cos 2x$

$\frac{\sin 2x}{\cos 2x} = -1$

$\tan 2x = -1$

$2x = -\frac{\pi}{4}$



$2x = \frac{3\pi}{4}$

$x = \frac{3\pi}{8}$

Sub into y: $y = e^{2(\frac{3\pi}{8})}\sin\left(2 \times \frac{3\pi}{8}\right) = e^{\frac{3\pi}{4}}\sin\left(\frac{3\pi}{4}\right)$

$= \frac{\sqrt{2}}{2}e^{\frac{3\pi}{4}}$

→ $\left(\frac{3\pi}{8}, \frac{\sqrt{2}}{2}e^{\frac{3\pi}{4}}\right)$

1 Find the exact coordinates of the points on the curve $y = \frac{x^2}{1-3x}$ at which the gradient of the tangent is equal to 8. [5]

u v

$u = x^2 \qquad v = 1-3x$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = -3$

$$\frac{dy}{dx} = \frac{(1-3x)\times 2x - -3x^2}{(1-3x)^2}$$

$$= \frac{2x - 6x^2 + 3x^2}{(1-3x)^2}$$

$$= \frac{2x - 3x^2}{(1-3x)^2}$$

gradient = 8: $\frac{2x-3x^2}{(1-3x)^2} = 8$

$2x - 3x^2 = 8(1-3x)^2$

$2x - 3x^2 = 8(1-6x+9x^2)$

$2x - 3x^2 = 72x^2 - 48x + 8$

$75x^2 - 50x + 8 = 0$

$$x = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(75)(8)}}{2(75)}$$

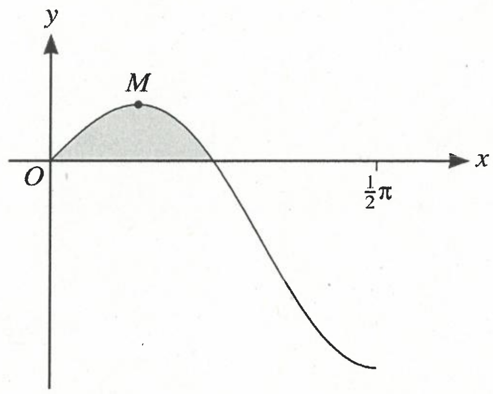
$x = \frac{2}{5}$ or $x = \frac{4}{15}$

$x = \frac{2}{5}$: $y = \frac{(\frac{2}{5})^2}{1-3(\frac{2}{5})} = \frac{\frac{4}{25}}{-\frac{1}{5}} = \frac{-4}{5}$

$x = \frac{4}{15}$: $y = \frac{(\frac{4}{15})^2}{1-3(\frac{4}{15})} = \frac{\frac{16}{225}}{\frac{3}{15}} = \frac{16}{45}$

$\left(\frac{2}{5}, \frac{-4}{5}\right)$ and $\left(\frac{4}{15}, \frac{16}{45}\right)$

11



The diagram shows the curve $y = \sin x \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 significant figures. [6]

$y = \overset{u}{\sin x}\ \overset{v}{\cos 2x}$

$u = \sin x \qquad v = \cos 2x$

$\frac{du}{dx} = \cos x \qquad \frac{dv}{dx} = -2\sin 2x$

$\frac{dy}{dx} = \sin x \times -2\sin 2x + \cos 2x \times \cos x$

$\frac{dy}{dx} = 0$:

$2\sin x\cos x \qquad 2\cos^2 x - 1$

$-2\sin x \sin 2x + \cos x \cos 2x = 0$

$-2\sin x \times 2\sin x\cos x + \cos x(2\cos^2 x - 1) = 0$

$1-\cos^2 x$

$-4\sin^2 x\cos x + 2\cos^3 x - \cos x = 0$

$-4(1-\cos^2 x)\cos x + 2\cos^3 x - \cos x = 0$

$-4\cos x + 4\cos^3 x + 2\cos^3 x - \cos x = 0$

$6\cos^3 x - 5\cos x = 0$

$\cos x(6\cos^2 x - 5) = 0$

$\cos x = 0$ or $6\cos^2 x - 5 = 0$

$x = \frac{\pi}{2}$ $\qquad \cos^2 x = \frac{5}{6}$

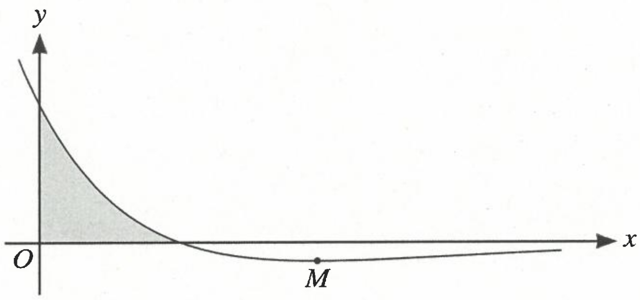
X not the point we want on diagram

$\cos x = \sqrt{\frac{5}{6}}$ or $\cos x = -\sqrt{\frac{5}{6}}$

$x = 0.421 \qquad x = 2.72$ X outside domain

$x = 0.421$

10



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

u v

$y = (2 - x)e^{-\frac{1}{2}x}$

$u = 2 - x \qquad v = e^{-\frac{1}{2}x}$

$\frac{du}{dx} = -1 \qquad \frac{dv}{dx} = -\frac{1}{2}e^{-\frac{1}{2}x}$

$\frac{dy}{dx} = (2-x) \times -\frac{1}{2}e^{-\frac{1}{2}x} + e^{-\frac{1}{2}x} \times -1$

$\frac{dy}{dx} = 0$:

$-\frac{1}{2}(2-x)e^{-\frac{1}{2}x} - e^{-\frac{1}{2}x} = 0$

$\left(-1 + \frac{x}{2}\right)e^{-\frac{1}{2}x} - e^{-\frac{1}{2}x} = 0$

$e^{-\frac{1}{2}x}\left(-1 + \frac{x}{2} - 1\right) = 0$

$e^{-\frac{1}{2}x}\left(-2 + \frac{x}{2}\right) = 0$

$e^{-\frac{1}{2}x} = 0$ or $-2 + \frac{x}{2} = 0$

X not possible

$\frac{x}{2} = 2$

$x = 4$

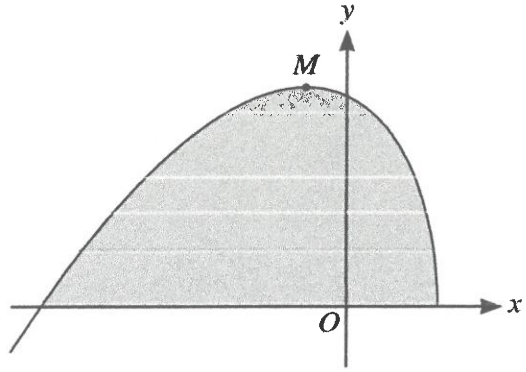
→ original eqⁿ:

$y = (2-4)e^{-\frac{1}{2}\times 4}$

$= -2e^{-2}$

$\rightarrow (4, -2e^{-2})$

10



u v

The diagram shows the curve $y = (x+5)\sqrt{3-2x}$ and its maximum point M.

(a) Find the exact coordinates of M. **[5]**

$u = x + 5 \qquad v = (3-2x)^{\frac{1}{2}}$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{2} \times -2(3-2x)^{-\frac{1}{2}}$

$= -(3-2x)^{-\frac{1}{2}}$

$\frac{dy}{dx} = -(x+5)(3-2x)^{-\frac{1}{2}} + (3-2x)^{\frac{1}{2}}$

SP, so $\frac{dy}{dx} = 0$:

$\frac{-(x+5)}{(3-2x)^{\frac{1}{2}}} + (3-2x)^{\frac{1}{2}} = 0$

$\frac{x+5}{(3-2x)^{\frac{1}{2}}} = (3-2x)^{\frac{1}{2}}$

$x + 5 = 3 - 2x$

$3x = -2$

$x = -\frac{2}{3}$

sub. in to find y:

$y = \left(-\frac{2}{3} + 5\right)\sqrt{3 - 2\left(-\frac{2}{3}\right)}$

$= \frac{13}{3}\sqrt{\frac{13}{3}}$

$= \frac{13\sqrt{39}}{9} \rightarrow \left(\frac{-2}{3}, \frac{13\sqrt{39}}{9}\right)$

4 The equation of a curve is $y = \cos^3 x\sqrt{\sin x}$. It is given that the curve has one stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

$y = \overset{u}{\cos^3 x}\,\overset{v}{\sqrt{\sin x}}$

$u = \cos^3 x \qquad v = (\sin x)^{\frac{1}{2}}$

$\frac{du}{dx} = 3\cos^2 x \times -\sin x \qquad \frac{dv}{dx} = \frac{1}{2}(\sin x)^{-\frac{1}{2}} \times \cos x$

$\frac{dy}{dx} = \frac{1}{2}\cos^4 x(\sin x)^{-\frac{1}{2}} + (\sin x)^{\frac{1}{2}} \times 3\cos^2 x \times -\sin x$

$\frac{dy}{dx} = 0$: $\frac{1}{2}\cos^4 x(\sin x)^{-\frac{1}{2}} - 3\cos^2 x(\sin x)^{\frac{3}{2}} = 0$

$\frac{1}{2}\frac{\cos^4 x}{\sqrt{\sin x}} - 3\cos^2 x(\sin x)^{\frac{3}{2}} = 0$

$\times\sqrt{\sin x} \quad \times\sqrt{\sin x} \quad \times\sqrt{\sin x}$

$\frac{1}{2}\cos^4 x - 3\cos^2 x\sin^2 x = 0$

$1-\cos^2 x$

$\frac{1}{2}\cos^4 x - 3\cos^2 x(1-\cos^2 x) = 0$

$\frac{1}{2}\cos^4 x - 3\cos^2 x + 3\cos^4 x = 0$

$\frac{7}{2}\cos^4 x - 3\cos^2 x = 0$

$\cos^2 x\left(\frac{7}{2}\cos^2 x - 3\right) = 0$

$\cos^2 x = 0$ or $\frac{7}{2}\cos^2 x - 3 = 0$

$\cos x = 0 \qquad \frac{7}{2}\cos^2 x = 3$

$x = \frac{\pi}{2} \qquad \cos^2 x = \frac{6}{7}$

X outside domain

$\cos x = \sqrt{\frac{6}{7}}$ or $\cos x = -\sqrt{\frac{6}{7}}$

$x = 0.388 \qquad x = 2.75$

X outside domain

$x = 0.388$

9 The equation of a curve is $y = x^{-\frac{2}{3}} \ln x$ for $x > 0$. The curve has one stationary point.

(a) Find the exact coordinates of the stationary point. [5]

$$\overset{u}{} \quad \overset{v}{}$$
$$y = x^{-\frac{2}{3}} \ln x$$

$$u = x^{-2/3} \qquad v = \ln x$$
$$\frac{du}{dx} = -\frac{2}{3}x^{-\frac{5}{3}} \qquad \frac{dv}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = x^{-2/3} \times \frac{1}{x} + \ln x \times -\frac{2}{3}x^{-5/3}$$

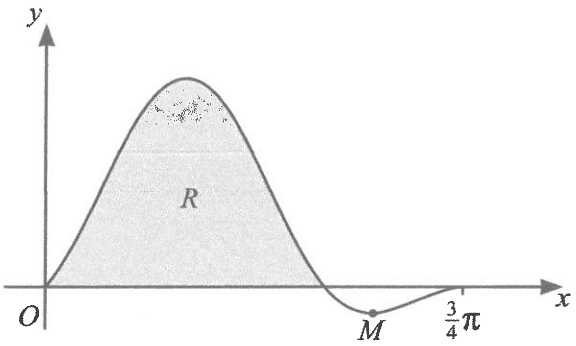
$\frac{dy}{dx} = 0$:
$$x^{-2/3} \times x^{-1} - \frac{2}{3}x^{-5/3}\ln x = 0$$
$$x^{-5/3} - \frac{2}{3}x^{-5/3}\ln x = 0$$
$$x^{-5/3}\left(1 - \frac{2}{3}\ln x\right) = 0$$

$$x^{-5/3} = 0 \qquad \text{or} \qquad 1 - \frac{2}{3}\ln x = 0$$
$$x = 0 \qquad\qquad \frac{2}{3}\ln x = 1$$
× outside domain
$$\ln x = \frac{3}{2}$$
$$\underline{x = e^{3/2}}$$

→ Original eqn:
$$y = \left(e^{\frac{3}{2}}\right)^{-2/3} \ln\left(e^{\frac{3}{2}}\right)$$
$$= e^{-6/6} \times \frac{3}{2}$$
$$= \underline{\frac{3}{2}e^{-1}}$$

$$\rightarrow \left(e^{\frac{3}{2}}, \frac{3}{2}e^{-1}\right)$$

6



The diagram shows the curve $y = \overset{u}{\sin 2x}(\overset{v}{1 + \sin 2x})$, for $0 \leqslant x \leqslant \frac{3}{4}\pi$, and its minimum point M. The shaded region bounded by the curve that lies above the x-axis and the x-axis itself is denoted by R.

(a) Given that the x-coordinate of M lies in the interval $\frac{1}{2}\pi < x < \frac{3}{4}\pi$, find the exact coordinates of M. [4]

$u = \sin 2x \qquad v = 1 + \sin 2x \qquad \frac{\pi}{2} < x < \frac{3\pi}{4}$

$\frac{du}{dx} = 2\cos 2x \qquad \frac{dv}{dx} = 2\cos 2x \qquad \pi < 2x < \frac{3\pi}{2}$

$\frac{dy}{dx} = 2\sin 2x\cos 2x + 2\cos 2x(1 + \sin 2x)$

$= 2\sin 2x\cos 2x + 2\cos 2x + 2\sin 2x\cos 2x$

$= 4\sin 2x\cos 2x + 2\cos 2x$

SP, so $\frac{dy}{dx} = 0$:

$4\sin 2x\cos 2x + 2\cos 2x = 0$

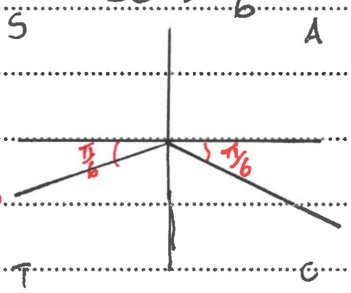
$2\cos 2x(2\sin 2x + 1) = 0$

$2\cos 2x = 0$ or $2\sin 2x + 1 = 0$

$\cos 2x = 0 \qquad 2\sin 2x = -1$

$2x = \frac{\pi}{2} \qquad \sin 2x = \frac{-1}{2}$

$\frac{\pi}{2}$ and others from CAST diagram outside range. $\qquad 2x = \frac{-\pi}{6}$



$\pi + \frac{\pi}{6}$

Sub. $x = \frac{7\pi}{12}$:

$y = \sin\left(2 \times \frac{7\pi}{12}\right)\left(1 + \sin\left(2 \times \frac{7\pi}{12}\right)\right)$

$= \frac{-1}{4}$

$\left(\frac{7\pi}{12}, \frac{-1}{4}\right)$

$2x = \frac{-\pi}{6}, \frac{7\pi}{6}$

outside range $\qquad x = \frac{-\pi}{12}, \frac{7\pi}{12}$ ✓

9 Let $f(x) = \dfrac{1}{(9-x)\sqrt{x}}$.

(a) Find the x-coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

$f(x) = (9-x)^{-1} x^{-\frac{1}{2}}$ u v

$u = (9-x)^{-1}$ $v = x^{-\frac{1}{2}}$

$\frac{du}{dx} = (9-x)^{-2}$ $\frac{dv}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$

$f'(x) = (9-x)^{-1} \times -\frac{1}{2}x^{-\frac{3}{2}} + x^{-\frac{1}{2}} \times (9-x)^{-2}$

$f'(x) = 0$:

$-\frac{1}{2}x^{-3/2}(9-x)^{-1} + x^{-\frac{1}{2}}(9-x)^{-2} = 0$

$\frac{-1}{2x^{\frac{3}{2}}(9-x)} \cdot \frac{\times(9-x)}{\times(9-x)} + \frac{1}{x^{\frac{1}{2}}(9-x)^2} \cdot \frac{\times 2x}{\times 2x} = 0$

$\frac{-1(9-x)}{2x^{\frac{3}{2}}(9-x)^2} + \frac{2x}{2x^{\frac{3}{2}}(9-x)^2} = 0$

$\frac{-1(9-x) + 2x}{2x^{\frac{3}{2}}(9-x)^2} = 0$

$-1(9-x) + 2x = 0$

$-9 + x + 2x = 0$

$-9 + 3x = 0$

$3x = 9$

$\underline{x = 3}$

8 The equation of a curve is $y = e^{-5x}\tan^2 x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

Find the x-coordinates of the stationary points of the curve. Give your answers correct to 3 decimal places where appropriate. [8]

$y = \overset{u}{e^{-5x}} \overset{v}{\tan^2 x}$

$u = e^{-5x}$ $\quad v = \tan^2 x$

$\frac{du}{dx} = -5e^{-5x}$ $\quad \frac{dv}{dx} = 2\tan x \times \sec^2 x$

$\frac{dy}{dx} = 2e^{-5x}\tan x \sec^2 x + \tan^2 x \times -5e^{-5x}$

$\frac{dy}{dx} = 0$: $\quad 1 + \tan^2 x = \sec^2 x$

$2e^{-5x}\tan x \sec^2 x - 5e^{-5x}\tan^2 x = 0$

$2e^{-5x}\tan x(1 + \tan^2 x) - 5e^{-5x}\tan^2 x = 0$

$2e^{-5x}(\tan x + \tan^3 x) - 5e^{-5x}\tan^2 x = 0$

$2e^{-5x}\tan x + 2e^{-5x}\tan^3 x - 5e^{-5x}\tan^2 x = 0$

$e^{-5x}\tan x\,(2 + 2\tan^2 x - 5\tan x) = 0$

$e^{-5x} = 0$ or $\tan x = 0$ or $2\tan^2 x - 5\tan x + 2 = 0$

× not possible $\qquad x = 0$

let $Y = \tan x$: $2Y^2 - 5Y + 2 = 0$

$(2Y - 1)(Y - 2) = 0$

$Y = \frac{1}{2}$ or $Y = 2$

$\tan x = \frac{1}{2}$ or $\tan x = 2$

$x = 0.464$ $\qquad x = 1.107$

$x = 0$, $x = 0.464$, $x = 1.107$

4 The curve with equation $y = e^{2x}(\sin x + 3\cos x)$ has a stationary point in the interval $0 \leqslant x \leqslant \pi$.

(a) Find the x-coordinate of this point, giving your answer correct to 2 decimal places. [4]

$y = \overset{u}{e^{2x}} \overset{v}{(\sin x + 3\cos x)}$

$u = e^{2x}$ $\quad v = \sin x + 3\cos x$

$\frac{du}{dx} = 2e^{2x}$ $\quad \frac{dv}{dx} = \cos x - 3\sin x$

$\frac{dy}{dx} = e^{2x}(\cos x - 3\sin x) + 2e^{2x}(\sin x + 3\cos x)$

$\frac{dy}{dx} = 0$: $e^{2x}\cos x - 3e^{2x}\sin x + 2e^{2x}\sin x + 6e^{2x}\cos x = 0$

$7e^{2x}\cos x - e^{2x}\sin x = 0$

$e^{2x}(7\cos x - \sin x) = 0$

$e^{2x} = 0$ or $7\cos x - \sin x = 0$

X not possible

$\sin x = 7\cos x$

$\frac{\sin x}{\cos x} = 7$

$\tan x = 7$

$\underline{x = 1.43}$

STO

(b) Determine whether the stationary point is a maximum or a minimum. [2]

$\frac{dy}{dx} = \overset{u}{e^{2x}} \overset{v}{(7\cos x - \sin x)}$

$u = e^{2x}$ $\quad v = 7\cos x - \sin x$

$\frac{du}{dx} = 2e^{2x}$ $\quad \frac{dv}{dx} = -7\sin x + \cos x$

$\frac{d^2y}{dx^2} = e^{2x}(-7\sin x + \cos x) + 2e^{2x}(7\cos x - \sin x)$

$= -7e^{2x}\sin x + e^{2x}\cos x + 14e^{2x}\cos x - 2e^{2x}\sin x$

$= 15e^{2x}\cos x - 9e^{2x}\sin x$

when $x = 1.43$, $\frac{d^2y}{dx^2} = -118.27\ldots$ $\therefore$ it's a <u>maximum.</u>

4 The equation of a curve is $y = x\tan^{-1}(\frac{1}{2}x)$.

(a) Find $\frac{dy}{dx}$. [3]

$u \quad v$

$y = x\tan^{-1}\left(\frac{1}{2}x\right)$

$u = x \qquad v = \tan^{-1}\left(\frac{1}{2}x\right)$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{2} \times \frac{1}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{2} \times \frac{1}{1+\frac{x^2}{4}}$

$= \frac{1}{2+\frac{x^2}{2}}$

$\frac{dy}{dx} = x \times \frac{1}{2+\frac{x^2}{2}} + \tan^{-1}\left(\frac{1}{2}x\right)$

$= \frac{x}{2+\frac{x^2}{2}} \begin{smallmatrix}\times 2\\ \times 2\end{smallmatrix} + \tan^{-1}\left(\frac{1}{2}x\right)$

$= \frac{2x}{4+x^2} + \tan^{-1}\left(\frac{1}{2}x\right)$

(b) The tangent to the curve at the point where $x = 2$ meets the y-axis at the point with coordinates $(0, p)$.

Find p. [3]

when $x = 2$, $\quad y = 2 \times \tan^{-1}\left(\frac{1}{2} \times 2\right)$

$= \frac{\pi}{2}$

when $x = 2$, $\frac{dy}{dx} = \frac{2(2)}{4+2^2} + \tan^{-1}\left(\frac{1}{2} \times 2\right)$

$= \frac{1}{2} + \frac{\pi}{4}$

$y - y_1 = m(x - x_1)$

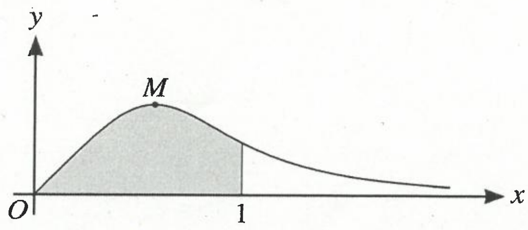
$y - \frac{\pi}{2} = \left(\frac{1}{2} + \frac{\pi}{4}\right)(x - 2)$

when $x = 0$: $\quad y - \frac{\pi}{2} = \left(\frac{1}{2} + \frac{\pi}{4}\right)(-2)$

$y - \frac{\pi}{2} = -1 - \frac{\pi}{2}$

$y = -1$

6



The diagram shows the curve $y = \frac{x}{1 + 3x^4}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [4]

$y = \frac{x}{1 + 3x^4}$ u v

$u = x$ $v = 1 + 3x^4$

$\frac{du}{dx} = 1$ $\frac{dv}{dx} = 12x^3$

$\frac{dy}{dx} = \frac{1(1 + 3x^4) - 12x^4}{(1 + 3x^4)^2}$

$\frac{dy}{dx} = 0$: $\frac{1 + 3x^4 - 12x^4}{(1 + 3x^4)^2} = 0$

$1 + 3x^4 - 12x^4 = 0$

$1 - 9x^4 = 0$

$9x^4 = 1$

$x^4 = \frac{1}{9}$

$x = \sqrt[4]{\frac{1}{9}}$

$x = 0.577$

5 Find the exact coordinates of the stationary points of the curve $y = \frac{e^{3x^2-1}}{1-x^2}$. [6]

u ✓

$u = e^{3x^2-1}$ $v = 1-x^2$

$\frac{du}{dx} = 6xe^{3x^2-1}$ $\frac{dv}{dx} = -2x$

$\frac{dy}{dx} = \frac{(1-x^2)\times 6xe^{3x^2-1} - -2xe^{3x^2-1}}{(1-x^2)^2}$

$= \frac{6xe^{3x^2-1} - 6x^3e^{3x^2-1} + 2xe^{3x^2-1}}{(1-x^2)^2}$

$= \frac{8xe^{3x^2-1} - 6x^3e^{3x^2-1}}{(1-x^2)^2}$

SP, so $\frac{dy}{dx} = 0$:

$\frac{8xe^{3x^2-1} - 6x^3e^{3x^2-1}}{(1-x^2)^2} = 0$

$8xe^{3x^2-1} - 6x^3e^{3x^2-1} = 0$

$2xe^{3x^2-1}(4-3x^2) = 0$

→ $2x = 0$ or $e^{3x^2-1} = 0$ or $4-3x^2 = 0$

$x = 0$ no sol^ns $3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm\frac{2\sqrt{3}}{3}$

Sub: $x=0$: $y = \frac{e^{0-1}}{1-0}$ $x = +\frac{2\sqrt{3}}{3}$: $y = \frac{e^{4-1}}{1-\frac{4}{3}}$ $x = -\frac{2\sqrt{3}}{3}$: $y = \frac{e^{4-1}}{1-\frac{4}{3}}$

$y = e^{-1}$ $y = -3e^3$ $y = -3e^3$

$(0, e^{-1})$, $\left(\frac{2\sqrt{3}}{3}, -3e^3\right)$, $\left(\frac{-2\sqrt{3}}{3}, -3e^3\right)$

5 The equation of a curve is $y = \dfrac{e^{\sin x}}{\cos^2 x}$ for $0 \leqslant x \leqslant 2\pi$.

Find $\dfrac{dy}{dx}$ and hence find the x-coordinates of the stationary points of the curve. [7]

differentiate $e^{\sin x}$:

$y = e^t \qquad t = \sin x$

$\dfrac{dy}{dt} = e^t \qquad \dfrac{dt}{dx} = \cos x$

$\dfrac{dy}{dx} = e^t \cos x$

$= e^{\sin x} \cos x$

differentiate $\cos^2 x$:

$y = t^2 \qquad t = \cos x$

$\dfrac{dy}{dt} = 2t \qquad \dfrac{dt}{dx} = -\sin x$

$\dfrac{dy}{dx} = -2t\sin x$

$= -2\sin x\cos x$

differentiate $y = \dfrac{e^{\sin x}}{\cos^2 x} \begin{matrix} u \\ v \end{matrix}$

$u = e^{\sin x} \qquad V = \cos^2 x$

$\dfrac{du}{dx} = e^{\sin x}\cos x \qquad \dfrac{dv}{dx} = -2\sin x\cos x$

$\dfrac{dy}{dx} = \dfrac{\cos^2 x \times e^{\sin x}\cos x - e^{\sin x} \times -2\sin x\cos x}{(\cos^2 x)^2}$

$= \dfrac{\cos^3 x\, e^{\sin x} + 2\sin x\cos x\, e^{\sin x}}{\cos^4 x}$

SP, so $\dfrac{dy}{dx} = 0$:

$\dfrac{\cos^3 x\, e^{\sin x} + 2\sin x\cos x\, e^{\sin x}}{\cos^4 x} = 0$

$\cos x \, e^{\sin x}(\cos^2 x + 2\sin x) = 0$

$\cos x = 0$ ① or $e^{\sin x} = 0$ ② or $\cos^2 x + 2\sin x = 0$ ③

①: $\cos x = 0$

When $\cos x = 0$, $y = \dfrac{e^{\sin x}}{\cos^2 x}$ is undefined, so this is not a stationary point.

$\cos^2 x = 0$

②: $e^{\sin x} = 0$

No sol^s ($\ln 0$ undefined)

③: $\cos^2 x + 2\sin x = 0$

$1 - \sin^2 x + 2\sin x = 0$

$\sin^2 x - 2\sin x - 1 = 0$

let $Y = \sin x$:

$Y^2 - 2Y - 1 = 0$

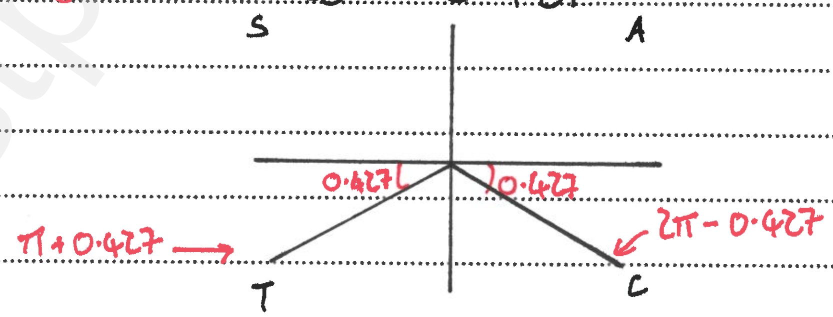
$Y = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$Y = 1 + \sqrt{2}$ or $Y = 1 - \sqrt{2}$

$\sin x = 1 + \sqrt{2}$ ✗ no sol^s

$\sin x = 1 - \sqrt{2}$

$x = -0.427$



$x = 3.57, \ 5.87$