6 The parametric equations of a curve are

$$x = \sqrt{t} + 3, \qquad y = \ln t,$$

for $t > 0$.

(a) Obtain a simplified expression for $\frac{dy}{dx}$ in terms of t. [3]

$x = t^{\frac{1}{2}} + 3$ $\qquad$ $y = \ln t$

$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$ $\qquad$ $\frac{dy}{dt} = \frac{1}{t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{1}{t} \times \frac{1}{\frac{1}{2}t^{-\frac{1}{2}}}$

$= \frac{1}{t} \times 2t^{\frac{1}{2}}$

$= \underline{\underline{2t^{-\frac{1}{2}}}}$

(b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is -2. [3]

If gradient of normal $= -2$, gradient of curve $= \frac{1}{2}$:

$2t^{-\frac{1}{2}} = \frac{1}{2}$

$t^{-\frac{1}{2}} = \frac{1}{4}$ $\qquad$ $x = \sqrt{16} + 3$ $\qquad$ $y = \underline{\ln 16}$

$\qquad = \underline{7}$

$t^{\frac{1}{2}} = 4$

$t = 16$ $\qquad \rightarrow \underline{\underline{(7, \ln 16)}}$

3 The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot\theta$. [5]

$\frac{dx}{d\theta} = 2\sin 2\theta \qquad \frac{dy}{d\theta} = 2 + 2\cos 2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$= (2 + 2\cos 2\theta) \times \frac{1}{2\sin 2\theta}$

$= \frac{2 + 2\cos 2\theta}{2\sin 2\theta}$

$= \frac{1 + \cos 2\theta}{\sin 2\theta}$ $\leftarrow 2\cos^2\theta - 1$, $\leftarrow 2\sin\theta\cos\theta$

$= \frac{1 + 2\cos^2\theta - 1}{2\sin\theta\cos\theta}$

$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot(\theta)$ QED

4 The parametric equations of a curve are

$$x = 1 - \cos\theta, \qquad y = \cos\theta - \tfrac{1}{4}\cos 2\theta.$$

Show that $\dfrac{dy}{dx} = -2\sin^2(\tfrac{1}{2}\theta)$. [5]

$$\frac{dx}{d\theta} = \sin\theta \qquad \frac{dy}{d\theta} = -\sin\theta + \frac{1}{2}\sin 2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= \left(-\sin\theta + \frac{1}{2}\sin 2\theta\right) \times \frac{1}{\sin\theta}$$

$$= \frac{-\sin\theta + \frac{1}{2}\sin 2\theta}{\sin\theta} \quad 2\sin\theta\cos\theta$$

$$= \frac{-\sin\theta + \sin\theta\cos\theta}{\sin\theta}$$

$$= -1 + \cos\theta$$

$$= \cos\theta - 1 \quad \rightarrow \quad \cos 2A = 1 - 2\sin^2 A$$

$$\cos\theta = 1 - 2\sin^2\left(\tfrac{1}{2}\theta\right)$$

$$= 1 - 2\sin^2\left(\tfrac{1}{2}\theta\right) - 1$$

$$= -2\sin^2\left(\tfrac{1}{2}\theta\right) \quad \text{QED}$$

4 The parametric equations of a curve are

$$x = 2t - \tan t, \qquad y = \ln(\sin 2t),$$

for $0 < t < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot t$. [5]

$\frac{dx}{dt} = 2 - \sec^2 t \qquad \frac{dy}{dt} = \frac{2\cos 2t}{\sin 2t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{2\cos 2t}{\sin 2t} \times \frac{1}{2 - \sec^2 t}$

$= \frac{2\cos 2t}{\sin 2t} \times \frac{1}{2 - \frac{1}{\cos^2 t}}$ $\times \cos^2 t$ $\times \cos^2 t$

$= \frac{2\cos 2t}{\sin 2t} \times \frac{\cos^2 t}{2\cos^2 t - 1}$ $= \cos 2t$

$= \frac{2\cancel{\cos 2t}}{\sin 2t} \times \frac{\cos^2 t}{\cancel{\cos 2t}}$

$= \frac{2\cos^2 t}{\sin 2t}$ $2\sin t \cos t$

$= \frac{2\cos^2 t}{2\sin t \cos t}$

$= \frac{2\cos t}{2\sin t}$

$= \cot(t)$ QED

4 The parametric equations of a curve are

$$x = \frac{\cos\theta}{2 - \sin\theta}, \qquad y = \theta + 2\cos\theta.$$

Show that $\frac{dy}{dx} = (2 - \sin\theta)^2$. [5]

$$x = \frac{\cos\theta^{\ u}}{2 - \sin\theta^{\ v}} \qquad y = \theta + 2\cos\theta$$

$$\frac{dy}{d\theta} = 1 - 2\sin\theta$$

$$u = \cos\theta \qquad v = 2 - \sin\theta$$

$$\frac{du}{d\theta} = -\sin\theta \qquad \frac{dv}{d\theta} = -\cos\theta$$

$$\frac{dx}{d\theta} = \frac{(2-\sin\theta)\times -\sin\theta - \cos\theta \times -\cos\theta}{(2-\sin\theta)^2}$$

$$= \frac{-\sin\theta(2-\sin\theta) + \cos^2\theta}{(2-\sin\theta)^2}$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= (1 - 2\sin\theta) \times \frac{(2-\sin\theta)^2}{-\sin\theta(2-\sin\theta) + \cos^2\theta}$$

$$= \frac{(1-2\sin\theta)(2-\sin\theta)^2}{-2\sin\theta + \underbrace{\sin^2\theta + \cos^2\theta}_{=1}}$$

$$= \frac{(1-2\sin\theta)(2-\sin\theta)^2}{-2\sin\theta + 1}$$

$$= \frac{\cancel{(1-2\sin\theta)}(2-\sin\theta)^2}{\cancel{1-2\sin\theta}}$$

$$= (2-\sin\theta)^2$$ QED

2 The parametric equations of a curve are

$$x = (\ln t)^2, \qquad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer. [4]

$x = (\ln t)^2$ $\qquad$ $y = e^{2-t^2}$

$x = u^2$ $\quad u = \ln t$ $\qquad$ $y = e^v$ $\quad v = 2-t^2$

$\frac{dx}{du} = 2u$ $\quad \frac{du}{dt} = \frac{1}{t}$ $\qquad$ $\frac{dy}{dv} = e^v$ $\quad \frac{dv}{dt} = -2t$

$\frac{dx}{dt} = 2u \times \frac{1}{t}$ $\qquad$ $\frac{dy}{dt} = -2te^v$

$= 2\ln t \times \frac{1}{t}$ $\qquad$ $= -2te^{2-t^2}$

$= \frac{2\ln t}{t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= -2te^{2-t^2} \times \frac{t}{2\ln t}$

$= \frac{-2t^2e^{2-t^2}}{2\ln t}$

when $t = e$:

gradient $= \frac{-2e^2 \times e^{2-e^2}}{2\ln e}$

$= \frac{-2e^{4-e^2}}{2}$

$= \underline{-e^{4-e^2}}$

3 The parametric equations of a curve are

$$x = t + \ln(t+2), \qquad y = (t-1)e^{-2t},$$

where $t > -2$.

(a) Express $\frac{dy}{dx}$ in terms of t, simplifying your answer. [5]

$x = t + \ln(t+2)$

$\frac{dx}{dt} = 1 + \frac{1}{t+2}$

$= \frac{t+2}{t+2} + \frac{1}{t+2}$

$= \frac{t+3}{t+2}$

$y = (\overset{u}{t-1})\overset{v}{e^{-2t}}$

$u = t-1 \quad v = e^{-2t}$

$\frac{du}{dt} = 1 \quad \frac{dv}{dt} = -2e^{-2t}$

$\frac{dy}{dt} = -2(t-1)e^{-2t} + e^{-2t}$

$= 2e^{-2t} - 2te^{-2t} + e^{-2t}$

$= 3e^{-2t} - 2te^{-2t}$

$= (3-2t)e^{-2t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= (3-2t)e^{-2t} \times \frac{t+2}{t+3}$

$= \frac{(t+2)(3-2t)}{t+3} e^{-2t}$

(b) Find the exact y-coordinate of the stationary point of the curve. [2]

at SP, $\frac{dy}{dx} = 0$: $\frac{(t+2)(3-2t)}{t+3} e^{-2t} = 0$

$t+2 = 0$ or $3-2t = 0$ or $e^{-2t} = 0$ ✗

$t = -2$ outside permitted values $(t > -2)$

$2t = 3$

$t = \frac{3}{2}$ ✓

$y = \left(\frac{3}{2} - 1\right)e^{-2\times\frac{3}{2}} = 0.5e^{-3}$

6 The parametric equations of a curve are $x = \frac{1}{\cos t}$, $y = \ln \tan t$, where $0 < t < \frac{1}{2}\pi$.

(a) Show that $\frac{dy}{dx} = \frac{\cos t}{\sin^2 t}$. [5]

$x = (\cos t)^{-1}$ $\quad y = \ln(\tan(t))$

$\frac{dx}{dt} = -(\cos t)^{-2} \times -\sin t$ $\quad \frac{dy}{dt} = \frac{\sec^2 t}{\tan t}$

$= \frac{\sin t}{\cos^2 t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{\sec^2 t}{\tan t} \times \frac{\cos^2 t}{\sin t}$

$= \frac{\frac{1}{\cos^2 t}}{\frac{\sin t}{\cos t}}$ $\times \cos^2 t$ $\times \cos^2 t$ $\times \frac{\cos^2 t}{\sin t}$

$= \frac{1}{\sin t \cos t} \times \frac{\cos^2 t}{\sin t}$

$= \frac{\cos^2 t}{\sin^2 t \cos t}$

$= \frac{\cos t}{\sin^2 t}$ QED

(b) Find the equation of the tangent to the curve at the point where $y = 0$. [3]

x coordinate: $y=0$

$\rightarrow \ln(\tan t) = 0$

$\tan t = e^0$

$\tan t = 1$

$t = \tan^{-1}(1)$

$= \frac{\pi}{4}$

$\rightarrow x = \frac{1}{\cos\left(\frac{\pi}{4}\right)}$

$= \frac{1}{\frac{1}{\sqrt{2}}}$ $\times\sqrt{2}$ $\times\sqrt{2}$

$= \underline{\sqrt{2}}$

gradient: $m = \frac{\cos\left(\frac{\pi}{4}\right)}{\sin^2\left(\frac{\pi}{4}\right)}$

$= \frac{\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^2}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}$ $\times 2$ $\times 2$ $= \underline{\sqrt{2}}$

eqn of line: $y - y_1 = m(x - x_1)$

$y - 0 = \sqrt{2}(x - \sqrt{2})$

$\underline{y = \sqrt{2}x - 2}$

5 The parametric equations of a curve are

$$x = te^{2t}, \qquad y = t^2 + t + 3.$$

(a) Show that $\frac{dy}{dx} = e^{-2t}$. [3]

$x = \overset{u}{t}\,\overset{v}{e^{2t}}$ $\qquad y = t^2 + t + 3$

$u = t \qquad v = e^{2t}$

$\frac{du}{dt} = 1 \qquad \frac{dv}{dt} = 2e^{2t} \qquad \frac{dy}{dt} = 2t + 1$

$\frac{dx}{dt} = 2te^{2t} + e^{2t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= (2t+1) \times \frac{1}{2te^{2t} + e^{2t}}$

$= \frac{2t+1}{2te^{2t} + e^{2t}}$

$= \frac{\cancel{2t+1}}{e^{2t}(\cancel{2t+1})}$

$= \frac{1}{e^{2t}}$

$= \underline{e^{-2t}}$ QED

(b) Hence show that the normal to the curve, where $t = -1$, passes through the point $\left(0,\ 3 - \frac{1}{e^4}\right)$. [3]

gradient at $t=-1$:

$m = e^{-2(-1)}$

$= e^2$

gradient of normal:

$m_{norm} = \frac{-1}{e^2}$

Coordinates when $t=-1$:

$x = -1 \times e^{-2}$

$= -e^{-2}$

$= \frac{-1}{e^2}$

$y = (-1)^2 + -1 + 3$

$= 1 - 1 + 3$

$= 3$

eqn of normal:

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-1}{e^2}\left(x - -\frac{1}{e^2}\right)$

$y - 3 = \frac{-x}{e^2} - \frac{1}{e^4}$

Sub. $x = 0$:

$y - 3 = 0 - \frac{1}{e^4}$

$y = 3 - \frac{1}{e^4}$

So it goes through $\left(0, 3 - \frac{1}{e^4}\right)$ QED

6 The parametric equations of a curve are

$$x = \ln(2 + 3t), \qquad y = \frac{t}{2 + 3t}.$$

(a) Show that the gradient of the curve is always positive. [5]

$x = \ln(2+3t)$ $\qquad y = \frac{t}{2+3t}$ $\;u$ $\;v$

$\frac{dx}{dt} = \frac{3}{2+3t}$ $\qquad u = t \quad v = 2+3t$

$\frac{du}{dt} = 1 \quad \frac{dv}{dt} = 3$

$$\frac{dy}{dt} = \frac{(2+3t)\times 1 - 3t}{(2+3t)^2}$$

$$= \frac{2}{(2+3t)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{2}{(2+3t)^2} \times \frac{2+3t}{3}$$

$$= \frac{2}{3(2+3t)}$$

$x = \ln(2+3t) \leftarrow$ can't have ln of a negative number, so x only exists when $2+3t$ is positive.

If $(2+3t)$ is always positive, so is $\frac{2}{3(2+3t)}$.

(b) Find the equation of the tangent to the curve at the point where it intersects the y-axis. [3]

At y-axis, $x=0$:

$$0 = \ln(2+3t)$$

$$2+3t = e^0$$

$$2+3t = 1$$

$$3t = -1$$

$$t = \frac{-1}{3}$$

y-coordinate:

$$y = \frac{\frac{-1}{3}}{2+3(\frac{-1}{3})}$$

$$= \frac{\frac{-1}{3}}{2+-1}$$

$$= \frac{\frac{-1}{3}}{1}$$

$$= \frac{-1}{3}$$

gradient:

$$m = \frac{2}{3(2+3(\frac{-1}{3}))}$$

$$= \frac{2}{3(2-1)}$$

$$= \frac{2}{3}$$

eqn of line:

$$y - y_1 = m(x - x_1)$$

$$y - -\tfrac{1}{3} = \tfrac{2}{3}(x - 0)$$

$$y + \tfrac{1}{3} = \tfrac{2}{3}x$$

$$\underline{y = \tfrac{2}{3}x - \tfrac{1}{3}}$$

7 The parametric equations of a curve are

$$x = 3\sin 2t, \qquad y = \tan t + \cot t,$$

for $0 < t < \frac{1}{2}\pi$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{-2}{3\sin^2 2t}$. [5]

$x = 3\sin 2t$ $\qquad$ $y = \tan t + \cot t$

$\dfrac{dx}{dt} = 6\cos 2t$ $\qquad$ $\dfrac{dy}{dt} = \sec^2 t - \operatorname{cosec}^2 t$

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$= (\sec^2 t - \operatorname{cosec}^2 t) \times \dfrac{1}{6\cos 2t}$

$= \dfrac{\sec^2 t - \operatorname{cosec}^2 t}{6\cos 2t}$

$= \dfrac{\frac{1}{\cos^2 t} - \frac{1}{\sin^2 t}}{6\cos 2t}$ $\quad \times\cos^2 t$, $\times\cos^2 t$

$= \dfrac{1 - \frac{\cos^2 t}{\sin^2 t}}{6\cos^2 t\cos 2t}$ $\quad \times\sin^2 t$, $\times\sin^2 t$

$= \dfrac{\sin^2 t - \cos^2 t}{6\sin^2 t\cos^2 t\cos 2t}$ $\leftarrow$ $\cos^2\theta - \sin^2\theta = \cos 2\theta$

$\sin^2 t - \cos^2 t = -\cos 2t$

$= \dfrac{-\cancel{\cos 2t}}{6\sin^2 t\cos^2 t\cancel{\cos 2t}}$

$= \dfrac{-1}{6\sin^2 t\cos^2 t}$

$= \dfrac{-1}{6(\sin t\cos t)^2}$ $\leftarrow$ $\sin 2t = 2\sin t\cos t$

$\frac{1}{2}\sin 2t = \sin t\cos t$

$= \dfrac{-1}{6(\frac{1}{2}\sin 2t)^2} = \dfrac{-1}{6\times\frac{1}{4}\sin^2 2t}$ $\times 4$ $= \dfrac{-4}{6\sin^2 2t} = \dfrac{-2}{3\sin^2 2t}$ QED

(b) Find the equation of the normal to the curve at the point where $t = \frac{1}{4}\pi$. Give your answer in the form $py + qx + r = 0$, where p, q and r are integers. [3]

x and y-coords at $t = \frac{\pi}{4}$:

x: $x = 3\sin\left(2 \times \frac{\pi}{4}\right)$

$= 3$

y: $y = \tan\left(\frac{\pi}{4}\right) + \frac{1}{\tan\left(\frac{\pi}{4}\right)}$

$= 1 + 1$

$= 2$

$\rightarrow (3, 2)$

gradient at $t = \frac{\pi}{4}$:

$m = \frac{-2}{3\sin^2\left(2 \times \frac{\pi}{4}\right)}$

$= \frac{-2}{3}$

gradient of normal:

$m_{norm} = \frac{3}{2}$

Eqn of normal:

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{3}{2}(x - 3)$

$y - 2 = \frac{3}{2}x - \frac{9}{2}$

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

$2y - 4 = 3x - 9$

$2y - 3x + 5 = 0$

8 The parametric equations of a curve are

$$x = \tan^2 2t, \quad y = \cos 2t,$$

for $0 < t < \frac{1}{4}\pi$.

(a) Show that $\frac{dy}{dx} = -\frac{1}{2}\cos^3 2t$. [4]

$x = \tan^2 2t$

Chain rule: $x = u^2 \quad u = \tan 2t$

$\frac{dx}{du} = 2u \quad \frac{du}{dt} = 2\sec^2 2t$

$\frac{dx}{dt} = \frac{dx}{du} \times \frac{du}{dt}$

$= 2u \times 2\sec^2 2t$

$= 2\tan 2t \times 2\sec^2 2t$

$= \underline{4\tan 2t \sec^2 2t}$

$y = \cos 2t$

$\underline{\frac{dy}{dt} = -2\sin 2t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= -2\sin 2t \times \frac{1}{4\tan 2t \sec^2 2t}$

$= \frac{-\sin 2t}{2\tan 2t \sec^2 2t}$

$= \frac{-\sin 2t}{2\tan 2t \times \frac{1}{\cos^2 2t}}$ $\times \cos^2 2t$ $\times \cos^2 2t$

$= \frac{-\sin 2t \cos^2 2t}{2\tan 2t}$

$= \frac{-\sin 2t \cos^2 2t}{2 \times \frac{\sin 2t}{\cos 2t}}$ $\times \cos 2t$ $\times \cos 2t$

$= \frac{-\cancel{\sin 2t}\cos^3 2t}{2\cancel{\sin 2t}}$

$= \frac{-\cos^3 2t}{2}$

$= \underline{-\frac{1}{2}\cos^3 2t}$ QED

(b) Hence find the equation of the normal to the curve at the point where $t = \frac{1}{8}\pi$. Give your answer in the form $y = mx + c$. [4]

Find x and y-coords at $t = \frac{\pi}{8}$:

$x = \tan^2\left(\frac{2\pi}{8}\right)$

$= 1$

$y = \cos\left(\frac{2\pi}{8}\right)$

$= \frac{\sqrt{2}}{2}$

$\rightarrow$ coordinates of point are: $\left(1, \frac{\sqrt{2}}{2}\right)$

Find gradient at $t = \frac{\pi}{8}$:

$m = \frac{-1}{2}\cos^3\left(\frac{2\pi}{8}\right)$

$m = \frac{-\sqrt{2}}{8}$

Gradient of Normal:

$m_{perp} = \frac{8}{\sqrt{2}} \begin{matrix}\times\sqrt{2}\\ \times\sqrt{2}\end{matrix}$

$= \frac{8\sqrt{2}}{2} = 4\sqrt{2}$

Equation of Normal:

$y - y_1 = m(x - x_1)$

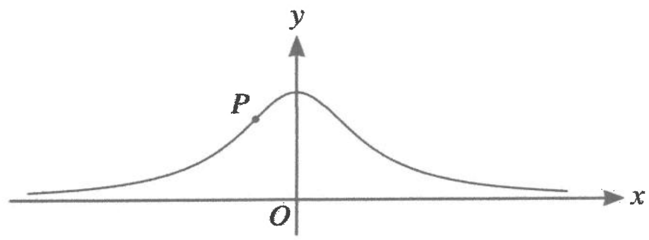
$y - \frac{\sqrt{2}}{2} = 4\sqrt{2}(x - 1)$

$y - \frac{\sqrt{2}}{2} = 4\sqrt{2}x - 4\sqrt{2}$

$y = 4\sqrt{2}x - 4\sqrt{2} + \frac{\sqrt{2}}{2}$

$y = 4\sqrt{2}x - \frac{7\sqrt{2}}{2}$

5



The diagram shows the curve with parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(a) Show that the gradient of the curve at the point with parameter θ is $-2\sin\theta\cos^3\theta$. [3]

$\frac{dx}{d\theta} = \sec^2\theta \qquad \frac{dy}{d\theta} = 2\cos\theta \times -\sin\theta$

$= -2\sin\theta\cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$= -2\sin\theta\cos\theta \times \frac{1}{\sec^2\theta}$

$= -2\sin\theta\cos\theta \times \cos^2\theta$

$= \underline{-2\sin\theta\cos^3\theta}$ QED

The gradient of the curve has its maximum value at the point P.

(b) Find the exact value of the x-coordinate of P. [4]

To find max. of gradient, need to differentiate the gradient and set = 0:

$$\frac{d}{d\theta}\left(-2\sin\theta\cos^3\theta\right) = 0$$

$u = -2\sin\theta \qquad v = \cos^3\theta$

$\frac{du}{d\theta} = -2\cos\theta \qquad \frac{dv}{d\theta} = 3\cos^2\theta \times -\sin\theta$

$= -3\sin\theta\cos^2\theta$

$\rightarrow (-2\sin\theta)(-3\sin\theta\cos^2\theta) + (\cos^3\theta)(-2\cos\theta) = 0$

$6\sin^2\theta\cos^2\theta - 2\cos^4\theta = 0$

$2\cos^2\theta(3\sin^2\theta - \cos^2\theta) = 0$

$2\cos^2\theta = 0$ or $3\sin^2\theta - \cos^2\theta = 0$

$\cos^2\theta = 0 \qquad 3\sin^2\theta = \cos^2\theta$

$\cos\theta = \frac{\pi}{2} \qquad \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{3}$

outside domain

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \frac{1}{\sqrt{3}}$ or $\tan\theta = -\frac{1}{\sqrt{3}}$

$x = \tan\theta$:

$x = \frac{1}{\sqrt{3}}$ or $\underline{x = \frac{-1}{\sqrt{3}}}$

wrong side of y-axis