

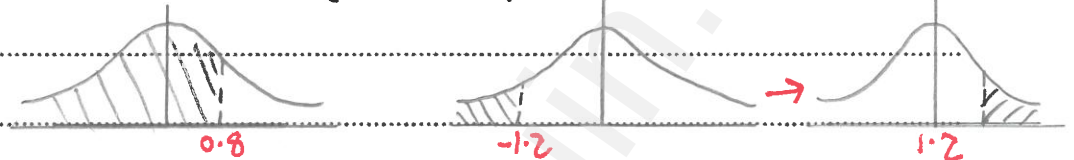
- 1 The times taken to swim 100 metres by members of a large swimming club have a normal distribution with mean 62 seconds and standard deviation 5 seconds.

- (a) Find the probability that a randomly chosen member of the club takes between 56 and 66 seconds to swim 100 metres. [3]

$$P(56 < T < 66) = P\left(\frac{56-62}{5} < Z < \frac{66-62}{5}\right)$$

$$= P(-1.2 < Z < 0.8)$$

$$= P(Z < 0.8) - P(Z < -1.2)$$



$$= \Phi(0.8) - (1 - \Phi(1.2))$$

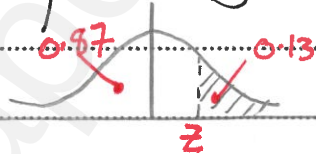
$$= 0.7881 - (1 - 0.8849)$$

$$= \underline{0.673}$$

- (b) 13% of the members of the club take more than t minutes to swim 100 metres. Find the value of t . [3]

$$P(T > t) = 0.13$$

$$P\left(Z > \frac{t-62}{5}\right) = 0.13$$



$$0.87 = \Phi(1.127) \text{ from table}$$

$$\frac{t-62}{5} = 1.127$$

$$t-62 = 5.635$$

$$t = 67.635 \text{ s}$$

$$= \underline{1.13 \text{ minutes}}$$

- 2 The weights of large bags of pasta produced by a company are normally distributed with mean 1.5 kg and standard deviation 0.05 kg.

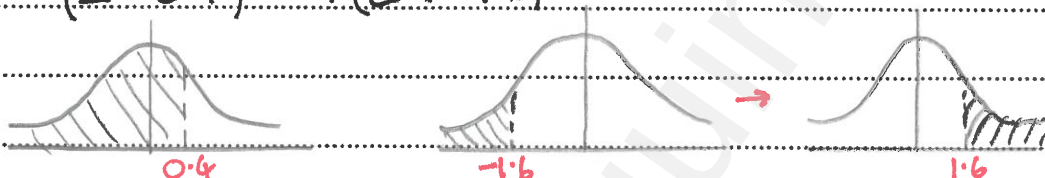
- (a) Find the probability that a randomly chosen large bag of pasta weighs between 1.42 kg and 1.52 kg. [3]

$$P(1.42 < W < 1.52)$$

$$P\left(\frac{1.42 - 1.5}{0.05} < Z < \frac{1.52 - 1.5}{0.05}\right)$$

$$P(-1.6 < Z < 0.4)$$

$$= P(Z < 0.4) - P(Z < -1.6)$$



$$= \Phi(0.4) - (1 - \Phi(1.6))$$

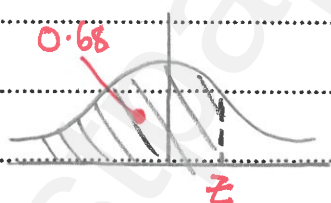
$$= 0.6554 - (1 - 0.9452) = \underline{\underline{0.6006}}$$

The weights of small bags of pasta produced by the company are normally distributed with mean 0.75 kg and standard deviation σ kg. It is found that 68% of these small bags have weight less than 0.9 kg.

- (b) Find the value of σ . [3]

$$P(W < 0.9) = 0.68$$

$$P\left(Z < \frac{0.9 - 0.75}{\sigma}\right) = 0.68$$



$$0.68 = \Phi(0.468) \text{ from table (nearest value)}$$

$$Z = 0.468$$

$$\frac{0.9 - 0.75}{\sigma} = 0.468$$

$$0.15 = 0.468\sigma$$

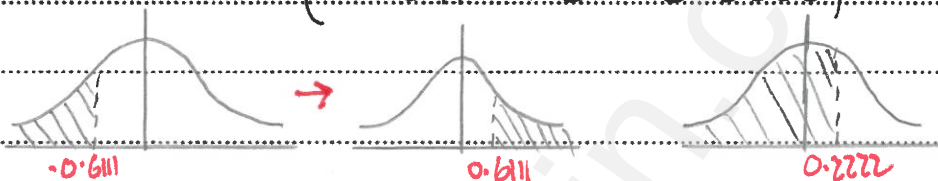
$$\underline{\underline{\sigma = 0.321}}$$

- 3 The time spent by shoppers in a large shopping centre has a normal distribution with mean 96 minutes and standard deviation 18 minutes.

- (a) Find the probability that a shopper chosen at random spends between 85 and 100 minutes in the shopping centre. [3]

$$P(85 < T < 100) = P\left(\frac{85 - 96}{18} < Z < \frac{100 - 96}{18}\right)$$

$$= P(-0.6111 < Z < 0.2222)$$



$$= P(Z < 0.2222) - P(Z < -0.6111)$$

$$= \Phi(0.2222) - (1 - \Phi(0.6111))$$

$$= 0.5879 - (1 - 0.7294)$$

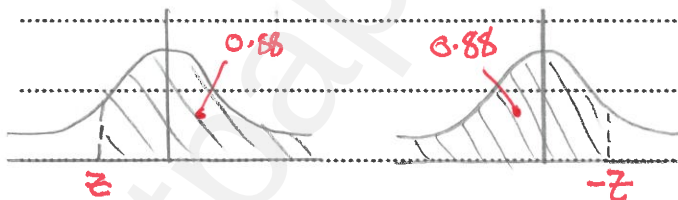
$$= \underline{\underline{0.3173}}$$

88% of shoppers spend more than t minutes in the shopping centre.

- (b) Find the value of t . [3]

$$P(T > t) = 0.88$$

$$P\left(Z > \frac{t - 96}{18}\right) = 0.88$$



$$0.88 = \Phi(1.175)$$

$$\therefore Z = -1.175$$

$$\frac{t - 96}{18} = -1.175$$

$$t - 96 = -21.15$$

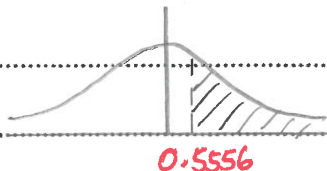
$$t = \underline{\underline{74.85}}$$

- 4 Raj wants to improve his fitness, so every day he goes for a run. The times, in minutes, of his runs have a normal distribution with mean 41.2 and standard deviation 3.6.

(a) Find the probability that on a randomly chosen day Raj runs for more than 43.2 minutes. [3]

$$P(T > 43.2) = P\left(Z > \frac{43.2 - 41.2}{3.6}\right)$$

$$= P(Z > 0.5556)$$



$$= 1 - \Phi(0.5556)$$

$$= 1 - 0.7108$$

$$= \underline{\underline{0.2892}}$$

(b) Find an estimate for the number of days in a year (365 days) on which Raj runs for less than 43.2 minutes. [2]

$$P(T < 43.2) = 1 - 0.2892 \quad (\text{from part (a)})$$

$$= 0.7108$$

$$\text{Number of days} = 0.7108 \times 365$$

$$= 259.442$$

$$\approx \underline{\underline{259 \text{ days}}}$$

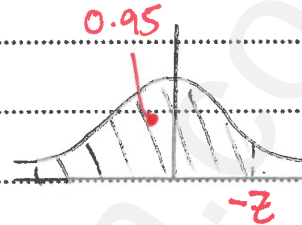
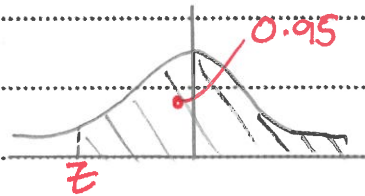
- (c) On 95% of days, Raj runs for more than t minutes.

Find the value of t .

[3]

$$P(T > t) = 0.95$$

$$P\left(Z > \frac{t - 41.2}{3.6}\right) = 0.95$$



$$0.95 = \Phi(1.645)$$

↑ critical value

$$\therefore z = -1.645$$

$$\frac{t - 41.2}{3.6} = -1.645$$

$$t - 41.2 = -5.922$$

$$t = \underline{\underline{35.278}}$$

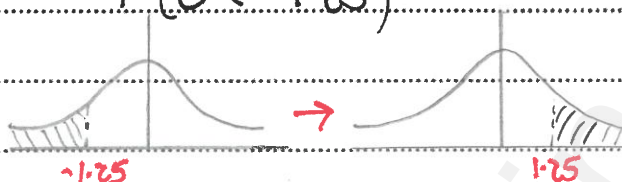
- 4 Trees in the Redian forest are classified as tall, medium or short, according to their height. The heights can be modelled by a normal distribution with mean 40 m and standard deviation 12 m. Trees with a height of less than 25 m are classified as short.

(a) Find the probability that a randomly chosen tree is classified as short.

[3]

$$P(L < 25) = P\left(Z < \frac{25 - 40}{12}\right)$$

$$= P(Z < -1.25)$$



$$= 1 - \Phi(1.25)$$

$$= 1 - 0.8944$$

$$= \underline{0.1056}$$

Of the trees that are classified as tall or medium, one third are tall and two thirds are medium.

- (b) Show that the probability that a randomly chosen tree is classified as tall is 0.298, correct to 3 decimal places.

[2]

$$1 - 0.1056 = 0.8944$$

$$0.8944 \div 3 = \underline{0.298} \text{ (3dp) QED}$$

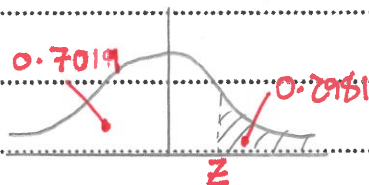
↑ give full answer

(c) Find the height above which trees are classified as tall.

[3]

$$P(L > l) = 0.2981$$

$$P\left(Z > \frac{l-40}{12}\right) = 0.2981$$



$$0.7019 = \Phi(0.53)$$

$$\therefore z = 0.53$$

$$\frac{l-40}{12} = 0.53$$

$$l-40 = 6.36$$

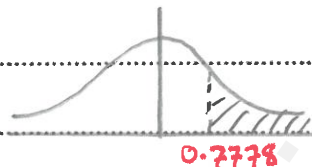
$$l = \underline{46.36}$$

- 5 The time in hours that Davin plays on his games machine each day is normally distributed with mean 3.5 and standard deviation 0.9.

- (a) Find the probability that on a randomly chosen day Davin plays on his games machine for more than 4.2 hours. [3]

$$P(T > 4.2) = P\left(Z > \frac{4.2 - 3.5}{0.9}\right)$$

$$= P(Z > 0.7778)$$



$$= 1 - \Phi(0.778)$$

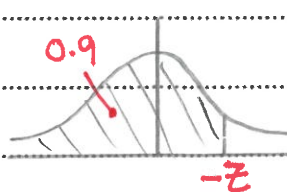
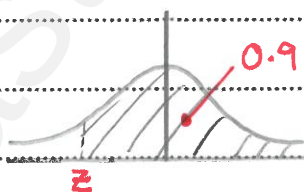
$$= 1 - 0.7818$$

$$= \underline{\underline{0.2182}}$$

- (b) On 90% of days Davin plays on his games machine for more than t hours. Find the value of t . [3]

$$P(T > t) = 0.9$$

$$P\left(Z > \frac{t - 3.5}{0.9}\right) = 0.9$$



$$\underline{\underline{0.9}} = \Phi(1.282)$$

↑ critical value

$$\therefore z = -1.282$$

$$\frac{t - 3.5}{0.9} = -1.282$$

$$t - 3.5 = -1.1538$$

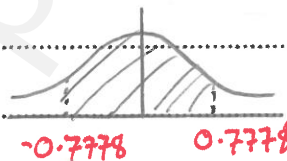
$$t = \underline{2.3462}$$

- (c) Calculate an estimate for the number of days in a year (365 days) on which Davin plays on his games machine for between 2.8 and 4.2 hours. [3]

$$P(2.8 < T < 4.2) = P\left(\frac{2.8 - 3.5}{0.9} < Z < \frac{4.2 - 3.5}{0.9}\right)$$

$$= P(-0.7778 < Z < 0.7778)$$

$$= P(Z < 0.7778) - P(Z < -0.7778)$$



$$= \Phi(0.778) - (1 - \Phi(0.778))$$

$$= 0.7818 - (1 - 0.7818)$$

$$= \underline{0.5636}$$

$$\text{Number of days} = 0.5636 \times 365$$

$$= 205.714$$

$$\approx \underline{206 \text{ days}^*}$$

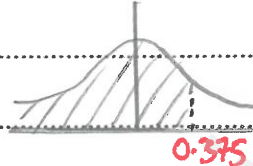
* I disagree with the mark scheme's 205. As this is a probability, 206 is more likely than 205.

- 6 The times taken, in minutes, to complete a particular task by employees at a large company are normally distributed with mean 32.2 and standard deviation 9.6.

- (a) Find the probability that a randomly chosen employee takes more than 28.6 minutes to complete the task. [3]

$$P(T > 28.6) = P(Z > \frac{28.6 - 32.2}{9.6})$$

$$= P(Z > -0.375)$$



$$= \Phi(0.375)$$

$$= \underline{0.6462}$$

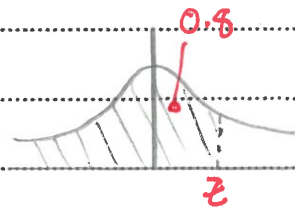
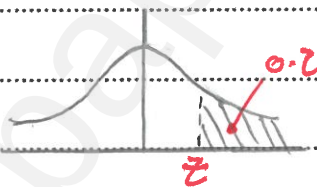
- (b) 20% of employees take longer than t minutes to complete the task.

Find the value of t .

[3]

$$P(T > t) = 0.2$$

$$P(Z > \frac{t - 32.2}{9.6}) = 0.2$$



$$0.8 = \Phi(0.842) \text{ from table}$$

$$\frac{t - 32.2}{9.6} = 0.842$$

$$t - 32.2 = 8.0832$$

$$t = \underline{40.2832}$$

- (c) Find the probability that the time taken to complete the task by a randomly chosen employee differs from the mean by less than 15.0 minutes. [4]

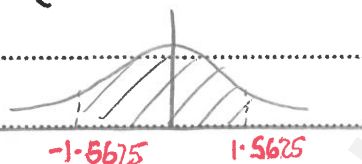
$$32.2 + 15 = 47.2$$

$$32.2 - 15 = 17.2$$

$$P(17.2 < T < 47.2)$$

$$= P\left(\frac{17.2 - 32.2}{9.6} < Z < \frac{47.2 - 32.2}{9.6}\right)$$

$$= P(-1.5625 < Z < 1.5625)$$



$$= \Phi(1.5625) - (1 - \Phi(1.5625))$$

$$= 0.9409 - (1 - 0.9409)$$

$$= \underline{0.8818}$$

↖ between 1.562 and 1.563
in table, so use
middle of
0.9408 and 0.9410

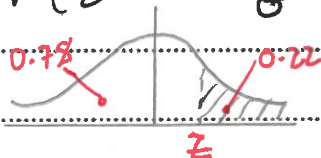
- 3 The weights of apples of a certain variety are normally distributed with mean 82 grams. 22% of these apples have a weight greater than 87 grams.

(a) Find the standard deviation of the weights of these apples.

[3]

$$P(W > 87) = 0.22$$

$$P\left(Z > \frac{87 - 82}{\sigma}\right) = 0.22$$



$$0.78 = \Phi(0.772)$$

$$\therefore z = 0.772$$

$$\frac{87 - 82}{\sigma} = 0.772$$

$$5 = 0.772\sigma$$

$$\sigma = \underline{6.48}$$

← Store

- (b) Find the probability that the weight of a randomly chosen apple of this variety differs from the mean weight by less than 4 grams.

[4]

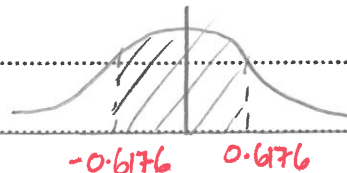
$$82 + 4 = 86$$

$$P(78 < W < 86)$$

$$82 - 4 = 78$$

$$= P\left(\frac{78 - 82}{6.48} < Z < \frac{86 - 82}{6.48}\right)$$

$$= P(-0.6176 < Z < 0.6176)$$



$$= P(Z < 0.6176) - P(Z < -0.6176)$$

$$= \Phi(0.618) - (1 - \Phi(0.618))$$

$$= 0.7317 - (1 - 0.7317)$$

$$= \underline{0.4634}$$

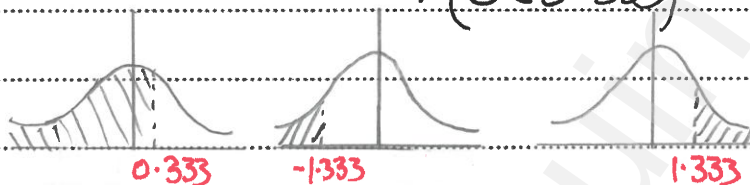
4 The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$.

- (a) Find the probability that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. [3]

$$P(1.98 < W < 2.03) = P\left(\frac{1.98 - 2.02}{0.03} < Z < \frac{2.03 - 2.02}{0.03}\right)$$

$$= P(-1.333 < Z < 0.333)$$

$$= P(Z < 0.333) - P(Z < -1.333)$$



$$= \Phi(0.333) - (1 - \Phi(1.333))$$

$$= 0.6304 - (1 - 0.9087)$$

$$= \underline{\underline{0.5391}}$$

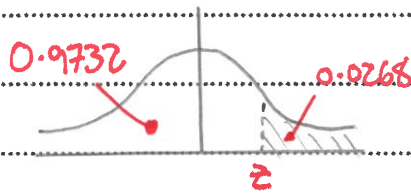
The weights of bags of rice produced by Binders are normally distributed with mean 2.55 kg and standard deviation σ kg. In a random sample of 5000 of these bags, 134 weighed more than 2.6 kg.

(b) Find the value of σ .

[4]

$$P(W > 2.6) = \frac{134}{5000} \\ = 0.0268$$

$$P(W > 2.6) = 0.0268 \\ P\left(Z > \frac{2.6 - 2.55}{\sigma}\right) = 0.0268$$



$$0.9732 = \Phi(1.93) \text{ from table} \\ \therefore z = 1.93$$

$$\frac{2.6 - 2.55}{\sigma} = 1.93$$

$$0.05 = 1.93\sigma$$

$$\sigma = \underline{\underline{0.0259}}$$

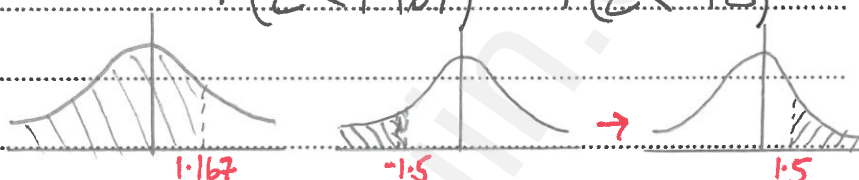
- 4 The weights of male leopards in a particular region are normally distributed with mean 55 kg and standard deviation 6 kg.

- (a) Find the probability that a randomly chosen male leopard from this region weighs between 46 and 62 kg. [4]

$$P(46 < W < 62) = P\left(\frac{46-55}{6} < Z < \frac{62-55}{6}\right)$$

$$= P(-1.5 < Z < 1.167)$$

$$= P(Z < 1.167) - P(Z < -1.5)$$



$$= \Phi(1.167) - (1 - \Phi(1.5))$$

$$= 0.8784 - (1 - 0.9332)$$

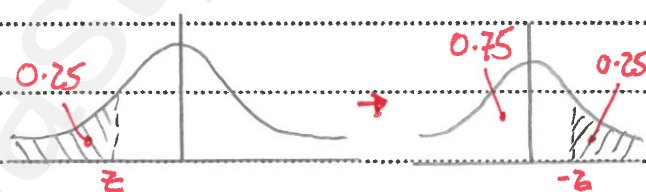
$$= \underline{\underline{0.8116}}$$

The weights of female leopards in this region are normally distributed with mean 42 kg and standard deviation σ kg. It is known that 25% of female leopards in the region weigh less than 36 kg.

- (b) Find the value of σ . [3]

$$P(F < 36) = 0.25$$

$$P\left(Z < \frac{36-42}{\sigma}\right) = 0.25$$



$$\therefore Z = -0.674$$

$$\frac{36-42}{\sigma} = -0.674$$

$$-6 = -0.674\sigma$$

$$\sigma = \underline{\underline{8.90}}$$

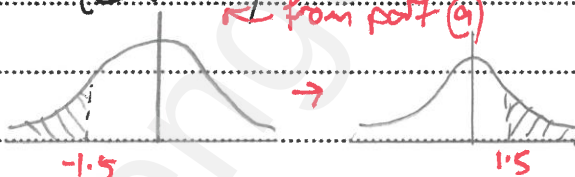
$$0.75 = \Phi(0.674)$$

↑ critical value

The distributions of the weights of male and female leopards are independent of each other. A male leopard and a female leopard are each chosen at random.

- (c) Find the probability that both the weights of these leopards are less than 46 kg. [4]

Male: $P(W < 46) = P(Z < -1.5)$ ← from part (a)



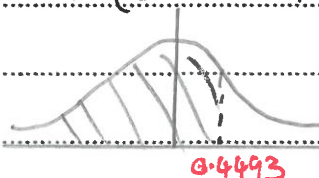
$$= 1 - \Phi(1.5)$$

$$= 1 - 0.9332$$

$$= \underline{0.0668}$$

Female: $P(F < 46) = P\left(Z < \frac{46 - 42}{8.90}\right)$

$$= P(Z < 0.4493)$$



$$= \Phi(0.449)$$

$$= \underline{0.6732}$$

$$P(\text{both weigh} < 46) = 0.0668 \times 0.6732$$

$$= \underline{\underline{0.0450}}$$

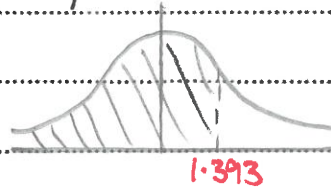
- 6 In a cycling event the times taken to complete a course are modelled by a normal distribution with mean 62.3 minutes and standard deviation 8.4 minutes.

(a) Find the probability that a randomly chosen cyclist has a time less than 74 minutes. [2]

$$P(T < 74)$$

$$P\left(Z < \frac{74 - 62.3}{8.4}\right)$$

$$P(Z < 1.393)$$



$$= \Phi(1.393)$$

$$= \underline{0.9182}$$

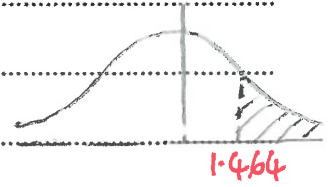
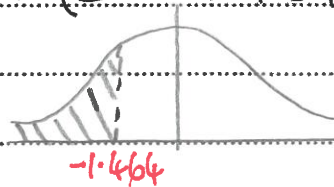
(b) Find the probability that 4 randomly chosen cyclists all have times between 50 and 74 minutes. [4]

$$P(50 < T < 74)$$

$$P\left(\frac{50 - 62.3}{8.4} < Z < \frac{74 - 62.3}{8.4}\right)$$

$$P(-1.464 < Z < 1.393)$$

$$= P(Z < 1.393) - P(Z < -1.464)$$



$$= \Phi(1.393) - (1 - \Phi(1.464))$$

$$= 0.9182 - (1 - 0.9285)$$

$$= \underline{0.8467}$$

↑ y

→ $P(Y_1, Y_2, Y_3, Y_4)$:

$$0.8467 \times 0.8467 \times 0.8467 \times 0.8467$$

$$= 0.8467^4$$

$$= \underline{0.514}$$

In a different cycling event, the times can also be modelled by a normal distribution. 23% of the cyclists have times less than 36 minutes and 10% of the cyclists have times greater than 54 minutes.

(c) Find estimates for the mean and standard deviation of this distribution.

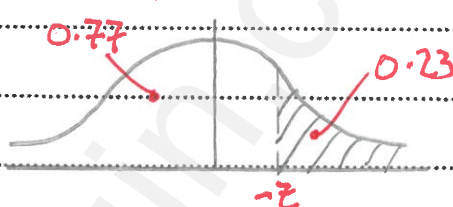
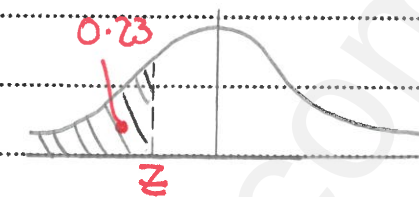
[5]

$$P(T < 36) = 0.23$$

$$P\left(Z < \frac{36 - \mu}{\sigma}\right) = 0.23$$

$$0.77 = \Phi(0.739)$$

$$Z = -0.739$$



$$\frac{36 - \mu}{\sigma} = -0.739$$

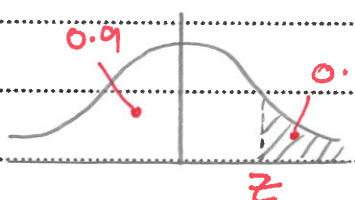
$$36 - \mu = -0.739\sigma \quad (1)$$

$$P(T > 54) = 0.1$$

$$P\left(Z > \frac{54 - \mu}{\sigma}\right) = 0.1$$

$$0.9 = \Phi(1.282)$$

critical value



$$\frac{54 - \mu}{\sigma} = 1.282$$

$$54 - \mu = 1.282\sigma \quad (2)$$

$$(2) - (1): 18 = 2.021\sigma$$

$$\sigma = 8.91 \text{ STD}$$

$$\rightarrow (2): 54 - \mu = 1.282 \times 8.91$$

$$54 - \mu = 11.418$$

$$-\mu = -42.58$$

$$\mu = 42.6$$

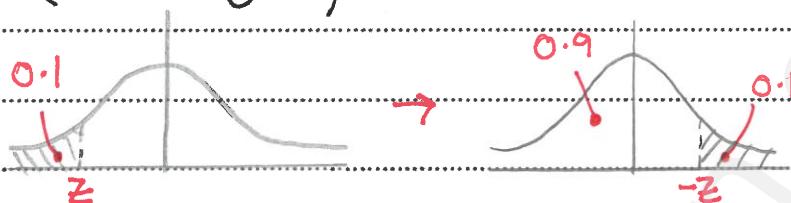
- 6 The mass of grapes sold per day by a large shop can be modelled by a normal distribution with mean 28 kg. On 10% of days less than 16 kg of grapes are sold.

(a) Find the standard deviation of the mass of grapes sold per day.

[3]

$$P(M < 16) = 0.1$$

$$P\left(Z < \frac{16 - 28}{\sigma}\right) = 0.1$$



$$0.9 = \Phi(1.282)$$

↑ critical value

$$z = -1.282$$

$$\frac{16 - 28}{\sigma} = -1.282$$

$$-12 = -1.282\sigma$$

$$\sigma = 9.36$$

The mass of grapes sold on any day is independent of the mass sold on any other day.

(b) 12 days are chosen at random.

Find the probability that less than 16 kg of grapes are sold on more than 2 of these 12 days. [3]

$$M \sim B(12, 0.1)$$

$$P(M > 2) = 1 - (P(0) + P(1) + P(2))$$

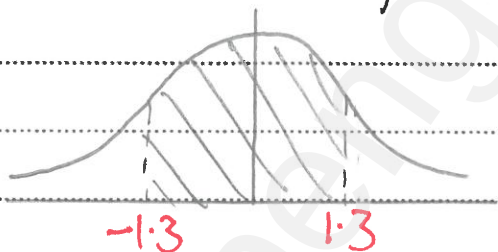
cont.

$$P(M > 2) = 1 - \left({}^{12}C_0 \times 0.1^0 \times 0.9^{12} + {}^{12}C_1 \times 0.1^1 \times 0.9^{11} + {}^{12}C_2 \times 0.1^2 \times 0.9^{10} \right)$$

$$= \underline{\underline{0.111}}$$

- (c) In a random sample of 365 days, on how many days would you expect the mass of grapes sold to be within 1.3 standard deviations of the mean? [4]

$$P(-1.3 < Z < 1.3)$$



$$= \Phi(1.3) - (1 - \Phi(1.3))$$

$$= 0.9032 - (1 - 0.9032)$$

$$= \underline{\underline{0.8064}}$$

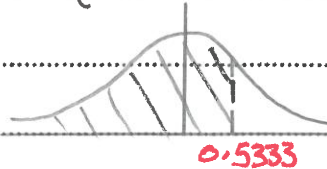
$$365 \times 0.8064 = \underline{\underline{294 \text{ days}}}$$

- 5 The lengths, in cm, of the leaves of a particular type are modelled by the distribution $N(5.2, 1.5^2)$.

- (a) Find the probability that a randomly chosen leaf of this type has length less than 6 cm. [2]

$$P(L < 6) = P\left(Z < \frac{6 - 5.2}{1.5}\right)$$

$$= P(Z < 0.5333)$$



$$= \Phi(0.5333)$$

$$= \underline{0.7029}$$

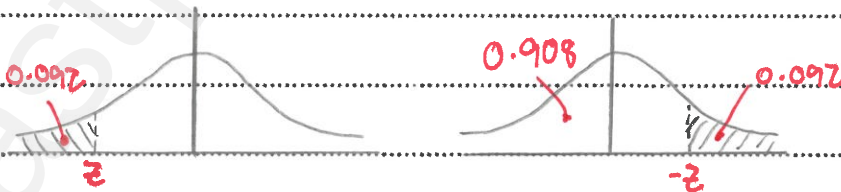
The lengths of the leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3 cm long and 95 are more than 8 cm long.

- (b) Find estimates for the mean and standard deviation of the lengths of leaves of this type. [5]

$$P(X < 3) = \frac{46}{500}$$

$$P(X < 3) = 0.092$$

$$P\left(Z < \frac{3 - \mu}{\sigma}\right) = 0.092$$



$$0.908 = \Phi(1.329)$$

$$\therefore z = -1.329$$

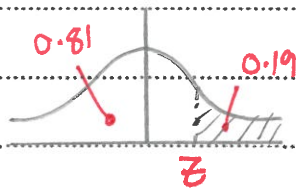
$$\frac{3-\mu}{\sigma} = -1.329$$

$$3-\mu = -1.329\sigma \quad (1)$$

$$P(X > 8) = \frac{95}{500}$$

$$P(X > 8) = 0.19$$

$$P\left(Z > \frac{8-\mu}{\sigma}\right) = 0.19$$



$$0.81 = \Phi(0.878)$$

$$\therefore z = 0.878$$

$$\frac{8-\mu}{\sigma} = 0.878$$

$$8-\mu = 0.878\sigma \quad (2)$$

$$(2)-(1): 5 = 2.207\sigma$$

$$\sigma = 2.27 \leftarrow \text{store}$$

$$\rightarrow (1): 3-\mu = -1.329 \times 2.265$$

$$\mu = 3 + 3.01087$$

$$\mu = \underline{6.01}$$

- (c) In a random sample of 2000 leaves of this second type, how many would the scientist expect to find with lengths more than 1 standard deviation from the mean? [4]

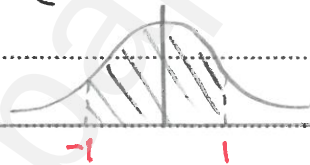
$$6.01 + 2.27 = 8.276 \quad (\text{use unrounded values})$$

$$6.01 - 2.27 = 3.745$$

Within 1 s.d.: $P(3.745 < X < 8.276)$

$$P\left(\frac{3.745-6.01}{2.27} < Z < \frac{8.276-6.01}{2.27}\right)$$

Within 1 s.d.: $P(-1 < Z < 1) \leftarrow \text{can start straight from here}$



$$P(Z < 1) - P(Z < -1)$$

$$= \Phi(1) - (1 - \Phi(1))$$

$$= 0.8413 - (1 - 0.8413)$$

$$= 0.6826$$

$$\text{Outside 1 s.d.} = 1 - 0.6826$$

$$= \underline{0.3174}$$

$$\rightarrow \text{number} = 0.3174 \times 2000$$

$$= 634.8$$

$$\approx \underline{635 \text{ leaves}}$$