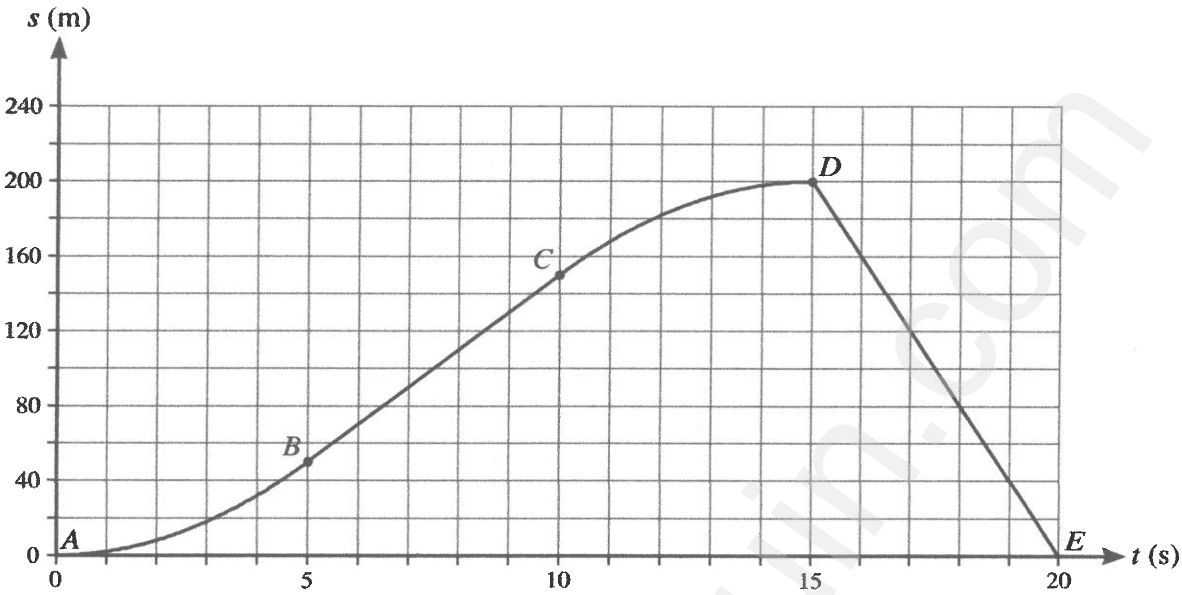
3



The displacement of a particle moving in a straight line is s metres at time t seconds after leaving a fixed point O. The particle starts from rest and passes through points P, Q and R, at times $t = 5$, $t = 10$ and $t = 15$ respectively, and returns to O at time $t = 20$. The distances OP, OQ and OR are 50 m, 150 m and 200 m respectively.

The diagram shows a displacement-time graph which models the motion of the particle from $t = 0$ to $t = 20$. The graph consists of two curved segments AB and CD and two straight line segments BC and DE.

(a) Find the speed of the particle between $t = 5$ and $t = 10$. [1]

Speed is constant between B and C, so no acceleration, so we can use:

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{100}{5}$$

$$= \underline{20\,\text{ms}^{-1}}$$

(b) Find the acceleration of the particle between $t = 0$ and $t = 5$, given that it is constant. [2]

$s = 50$ | $s = ut + \frac{1}{2}at^2$
$u = 0$ | $50 = 0 + \frac{1}{2} \times a \times 25$
$v =$ | $50 = 12.5a$
$a =$ | $\underline{a = 4\text{ms}^{-2}}$
$t = 5$

(c) Find the average speed of the particle during its motion. [2]

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

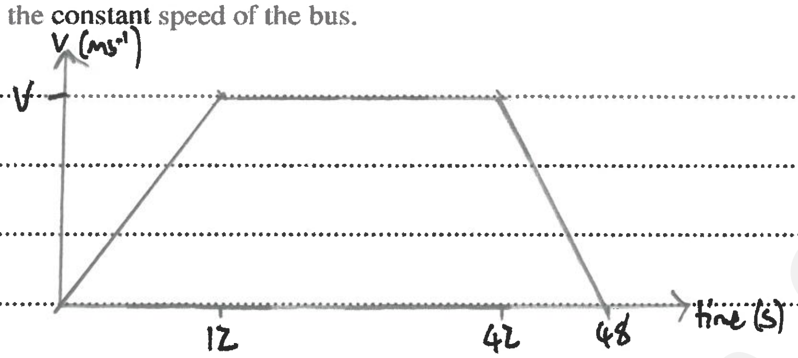
$$\text{distance} = \underset{A \to D}{200} + \underset{D \to E}{200}$$

$$\text{av. speed} = \frac{400}{20}$$

$$= \underline{20\text{ms}^{-1}}$$

1 A bus moves from rest with constant acceleration for 12 s. It then moves with constant speed for 30 s before decelerating uniformly to rest in a further 6 s. The total distance travelled is 585 m.

(a) Find the constant speed of the bus. [2]



distance = area:

$585 = \frac{1}{2}(30 + 48) \times V$

$585 = \frac{1}{2} \times 78 \times V$

$585 = 39V$

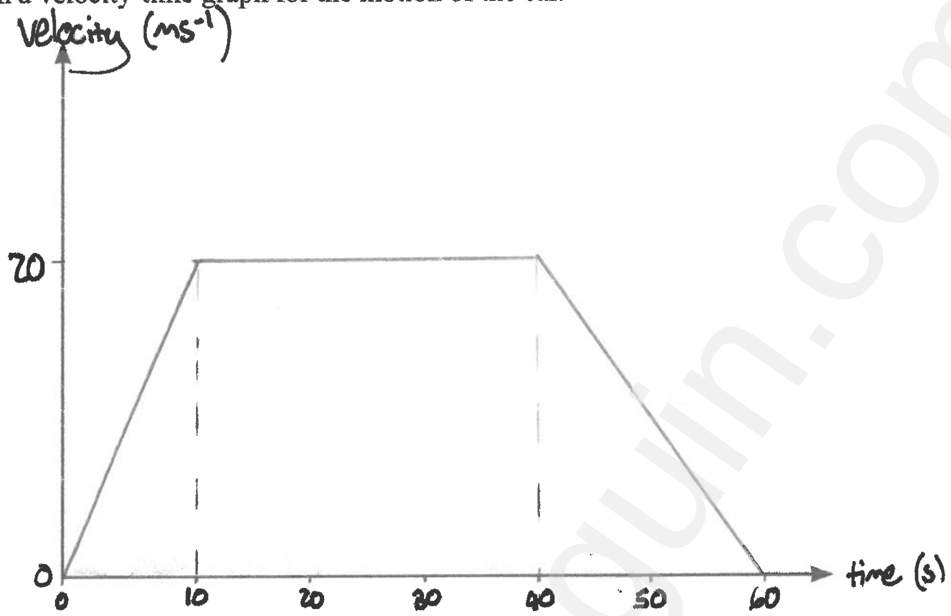
$V = 15 ms^{-1}$

(b) Find the magnitude of the deceleration. [1]

s =	$V = u + at$
u = 15	$0 = 15 + 6a$
v = 0	$6a = -15$
a =	$a = -2.5$
t = 6	magnitude of a = $2.5 ms^{-2}$

1 A car starts from rest and accelerates at $2\,\mathrm{m\,s^{-2}}$ for $10\,\mathrm{s}$. It then travels at a constant speed for $30\,\mathrm{s}$. The car then uniformly decelerates to rest over a period of $20\,\mathrm{s}$.

(a) Sketch a velocity-time graph for the motion of the car. [2]



(b) Find the total distance travelled by the car. [2]

distance = area

$= \frac{1}{2}(30+60)\times 20$

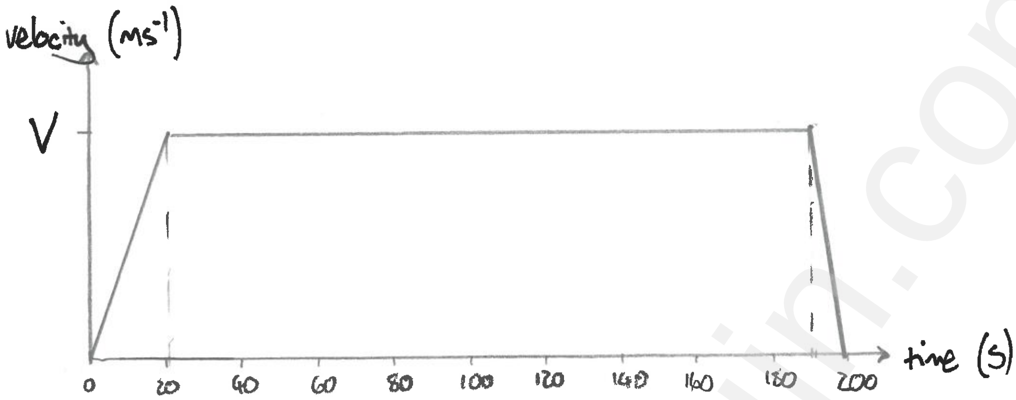
$= \frac{1}{2}\times 90\times 20$

$= 10\times 90$

$= 900\,\mathrm{m}$

1 A tram starts from rest and moves with uniform acceleration for 20 s. The tram then travels at a constant speed, $V\,\mathrm{m\,s^{-1}}$, for 170 s before being brought to rest with a uniform deceleration of magnitude twice that of the acceleration. The total distance travelled by the tram is 2.775 km.

(a) Sketch a velocity-time graph for the motion, stating the total time for which the tram is moving. [2]



Total time tram is moving = 200s

(b) Find V. [2]

Area = 2775 m

$\frac{1}{2}(170 + 200) \times V = 2775$

$185V = 2775$

$V = 15\,\mathrm{ms^{-1}}$

(c) Find the magnitude of the acceleration. [2]

s =	$v = u + at$
u = 0	$15 = 0 + 20a$
v = 15	$a = 0.75\,\mathrm{ms^{-2}}$
a =	
t = 20	

1 A car starts from rest and moves in a straight line with constant acceleration for a distance of 200 m, reaching a speed of $25\,\text{m s}^{-1}$. The car then travels at this speed for 400 m, before decelerating uniformly to rest over a period of 5 s.

(a) Find the time for which the car is accelerating. [2]

$s = 200$ | $s = \frac{1}{2}(u+v)t$

$u = 0$ | $200 = \frac{1}{2}(25) \times t$

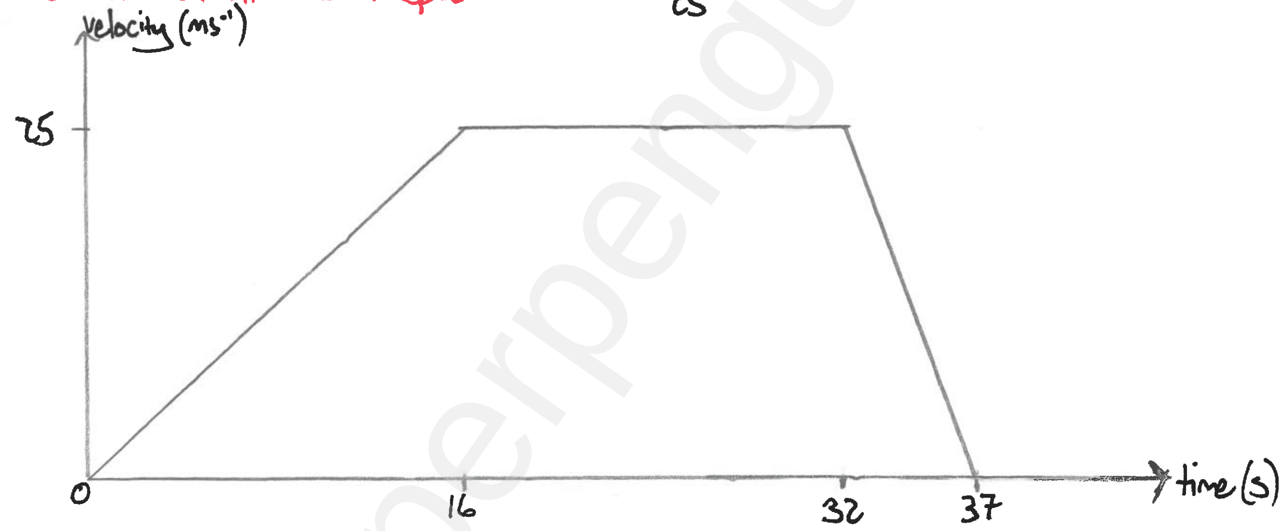
$v = 25$ | $200 = 12.5t$

$a =$ | $t = 16\text{s}$

$t =$

(b) Sketch the velocity–time graph for the motion of the car, showing the key points. [2]

Time travelled at constant speed: $\text{time} = \frac{400}{25} = 16\text{s}$



(c) Find the average speed of the car during its motion. [2]

$\text{Total distance} = \frac{1}{2}(16+37) \times 25$

$= \frac{1}{2} \times 53 \times 25$

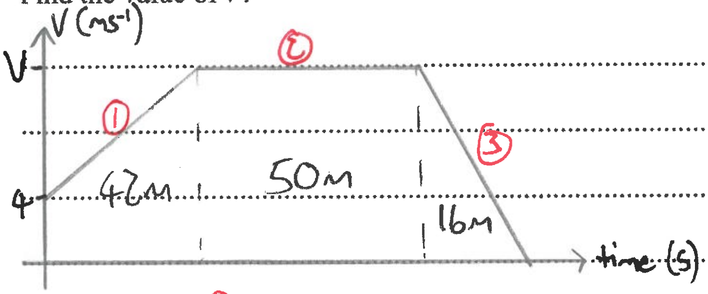
$= 662.5\text{m}$

$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$

$= \frac{662.5}{37} = 17.9\,\text{ms}^{-1}$

2 A cyclist is travelling along a straight horizontal road at a speed of $4\,\mathrm{m\,s^{-1}}$ when she passes a point O. She accelerates at a constant rate for a distance of 42 m, reaching a speed of $V\,\mathrm{m\,s^{-1}}$. She maintains the speed of $V\,\mathrm{m\,s^{-1}}$ for 50 m and then decelerates at $2\,\mathrm{m\,s^{-2}}$ before coming to rest. The distance travelled while decelerating is 16 m.

(a) Find the value of V. [2]



Using section ③:

$s = 16$	$v^2 = u^2 + 2as$
$u:$	$0^2 = u^2 + 2(-2)(16)$
$v: 0$	$0 = u^2 - 64$
$a: -2$	$u^2 = 64$
$t:$	$u = 8\,\mathrm{ms^{-1}}$

(b) Find the total time for which she is in motion from the instant that she passes O. [3]

①: Area $= \frac{1}{2}(4+8) \times t$

$42 = \frac{1}{2} \times 12 \times t$

$42 = 6t$

$t = 7\,\mathrm{s}$

② Area $= 8 \times t$

$50 = 8t$

$t = 6.25\,\mathrm{s}$

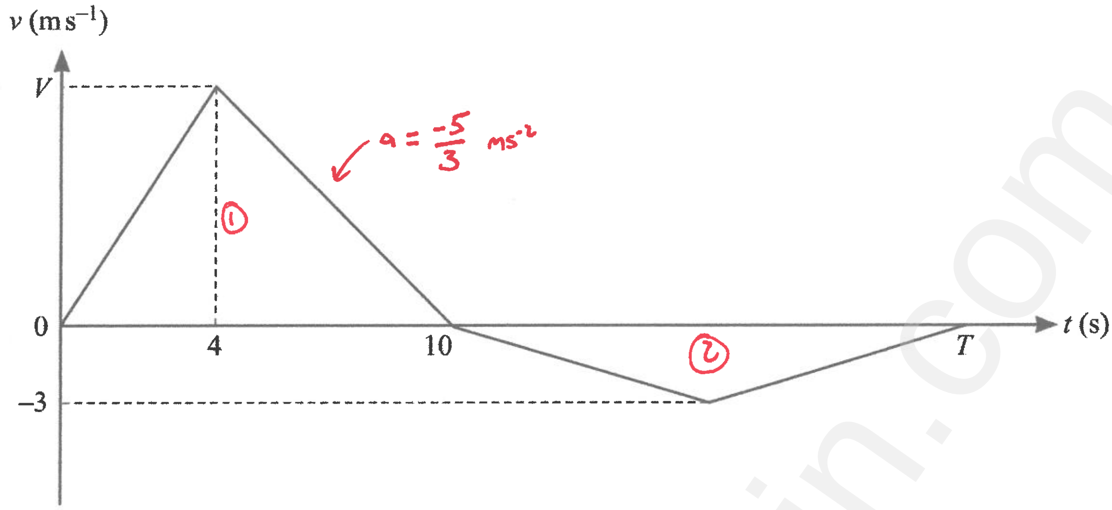
③ Area $= \frac{1}{2} \times 8 \times t$

$16 = 4t$

$t = 4\,\mathrm{s}$

Total $= 17.25\,\mathrm{s}$

1



The velocity of a particle moving in a straight line at time t seconds after leaving a fixed point O is v m s^{-1}. The diagram shows a velocity-time graph which models the motion of the particle from $t = 0$ to $t = T$. The graph consists of four straight line segments. The particle accelerates from rest to a speed of V m s^{-1} over a period of 4 s, and then decelerates at $\frac{5}{3}$ m s^{-2} to instantaneous rest over a period of 6 s. The particle then travels back towards O, reaching a maximum speed of 3 m s^{-1} before coming to rest at time $t = T$.

(a) Find the value of V. [2]

4s→10s:

s:

u:

$v = 0$

$a = \frac{-5}{3}$

$t = 6$

$V = u + at$

$0 = u + \frac{-5}{3} \times 6$

$0 = u - 10$

$u = 10$ ms^{-1}

(b) Given that the total distance travelled by the particle from $t = 0$ to $t = T$ is 68 m, find the value of T. [3]

①: Area $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 10 \times 10$

$= 50$ m

Area ② $= 68 - 50$

$= 18$ m

$\frac{1}{2} \times b \times h = 18$

$\frac{1}{2} \times b \times 3 = 18$ $\times 2$

$b \times 3 = 36$ $\div 3$

$b = 12$

$T = 10 + 12$

$= 22$ s

6 An elevator is pulled vertically upwards by a cable. The elevator accelerates at $0.4\,\text{m}\,\text{s}^{-2}$ for 5 s, then travels at constant speed for 25 s. The elevator then decelerates at $0.2\,\text{m}\,\text{s}^{-2}$ until it comes to rest.

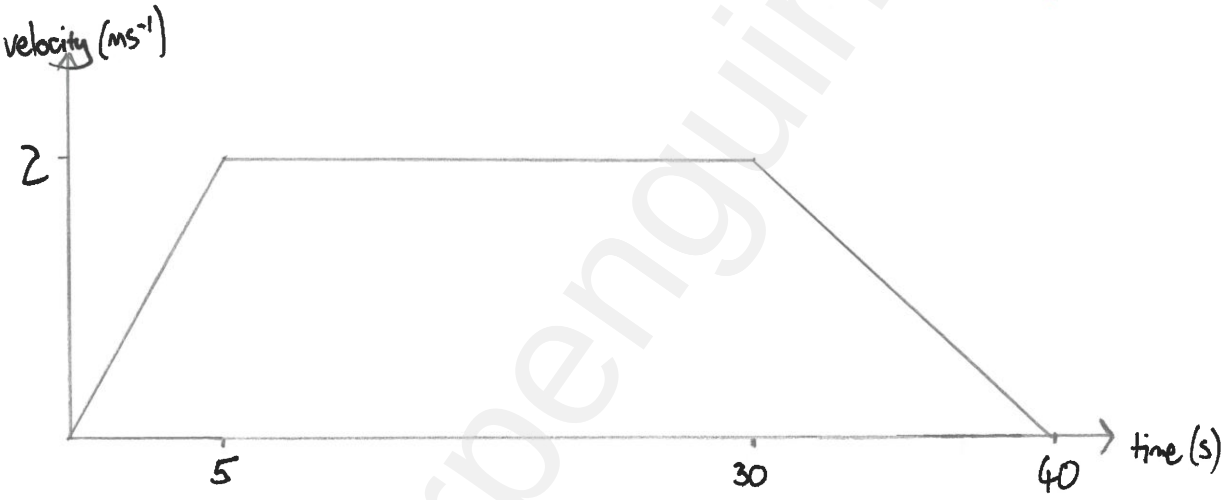
(a) Find the greatest speed of the elevator and hence draw a velocity-time graph for the motion of the elevator. [3]

$s =$
$u = 0$
$v =$
$a = 0.4$
$t = 5$

$v = u + at$
$= 0 + 0.4 \times 5$
$= \underline{\underline{2\,\text{ms}^{-1}}}$

deceleration:
$s =$
$u = 2$
$v = 0$
$a = -0.2$
$t =$

$v = u + at$
$0 = 2 + (-0.2)t$
$-2 = -0.2t$
$\underline{t = 10\text{s}}$



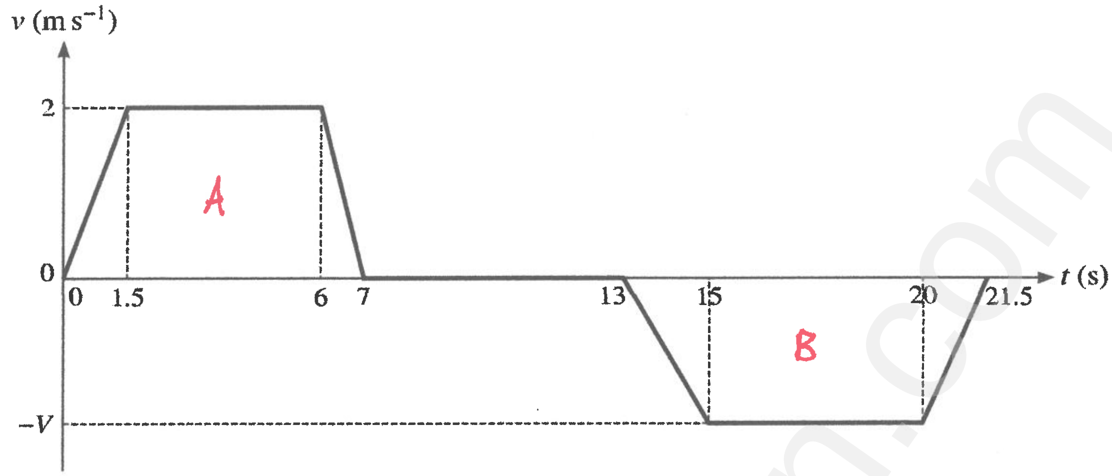
(b) Find the total distance travelled by the elevator. [2]

distance = area = $\frac{1}{2}(25 + 40) \times 2$

$= \frac{1}{2} \times 65 \times 2$

$= \underline{\underline{65\text{m}}}$

4



An elevator moves vertically, supported by a cable. The diagram shows a velocity-time graph which models the motion of the elevator. The graph consists of 7 straight line segments.

The elevator accelerates upwards from rest to a speed of 2 m s^{-1} over a period of 1.5 s and then travels at this speed for 4.5 s, before decelerating to rest over a period of 1 s.

The elevator then remains at rest for 6 s, before accelerating to a speed of V m s^{-1} downwards over a period of 2 s. The elevator travels at this speed for a period of 5 s, before decelerating to rest over a period of 1.5 s.

(a) Find the acceleration of the elevator during the first 1.5 s. [1]

$s =$ | $v = u + at$

$u = 0$ | $2 = 0 + 1.5a$

$v = 2$ | $a = \frac{2}{1.5}$

$a =$ | $= \frac{4}{3}\ \text{ms}^{-2}$

$t = 1.5$

(b) Given that the elevator starts and finishes its journey on the ground floor, find V. [2]

If elevator starts and finishes in same place, displacements A and B are equal: Area A = Area B

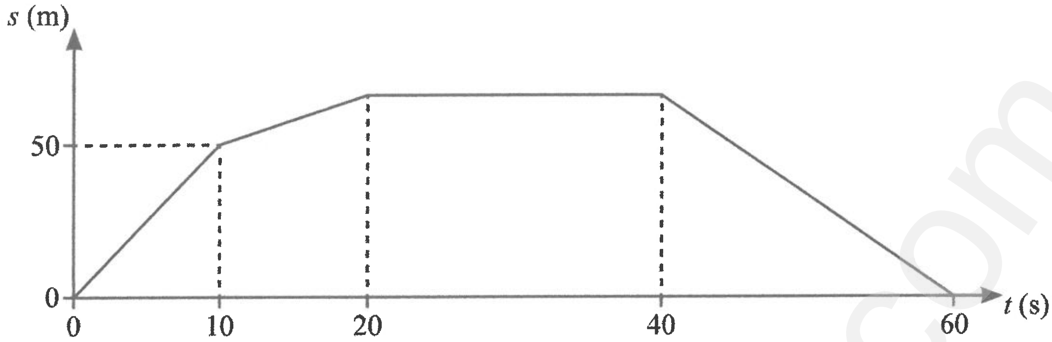
$\frac{1}{2}(4.5 + 7) \times 2 = \frac{1}{2}(5 + 8.5) \times V$

$\frac{1}{2}(11.5) \times 2 = \frac{1}{2}(13.5) \times V$

$11.5 = 6.75V$

$V = 1.70\ \text{ms}^{-1}$

1



The displacement of a particle at time t s after leaving a fixed point O is s m. The diagram shows a displacement-time graph which models the motion of the particle. The graph consists of 4 straight line segments. The particle travels 50 m in the first 10 s, then travels at $2\,\text{m s}^{-1}$ for a period of 10 s. The particle then comes to rest for a period of 20 s, before returning to its starting point when $t = 60$.

(a) Find the velocity of the particle during the last 20 s of its motion. [2]

displacement from 0 → 10s = 50m

displacement from 10 → 20s: distance = speed × time (constant speed)

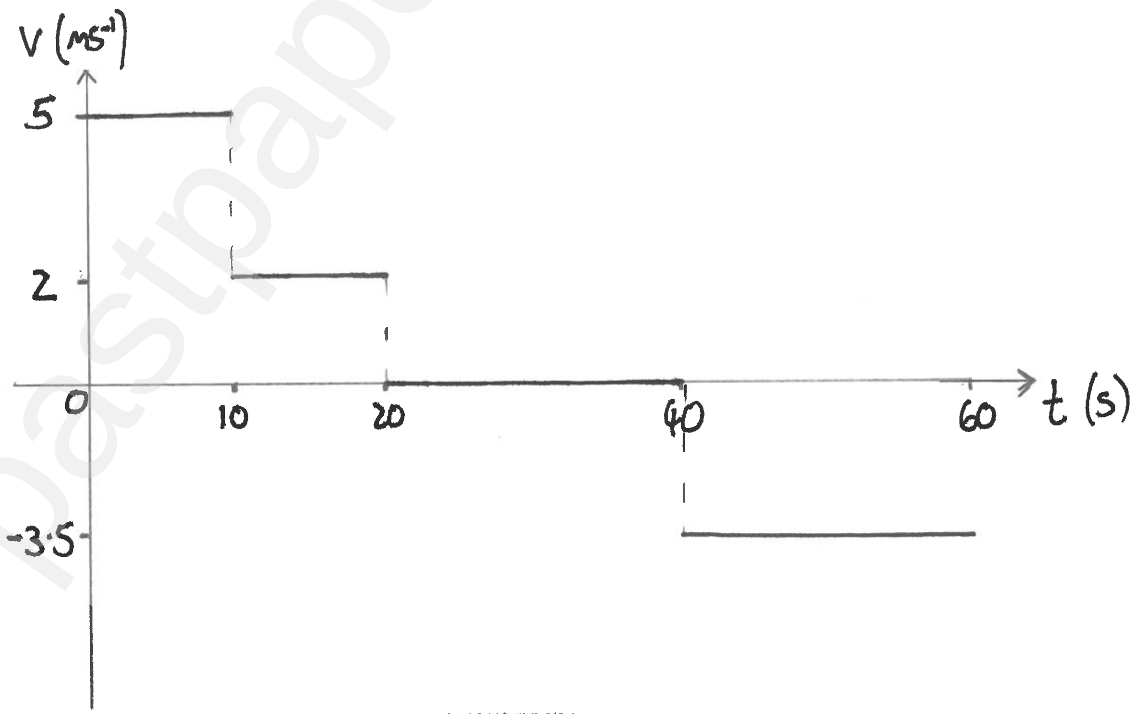
$= 2 \times 10$

$= 20\text{m}$

from 40 → 60s: Velocity $= \dfrac{\text{displacement}}{\text{time}}$

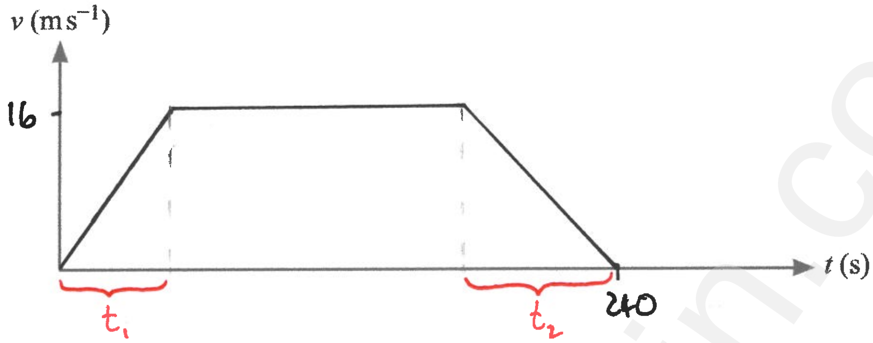
$= \dfrac{-70}{20} = -3.5\,\text{ms}^{-1}$

(b) Sketch a velocity-time graph for the motion of the particle from $t = 0$ to $t = 60$. [3]



4 A bus travels between two stops, A and B. The bus starts from rest at A and accelerates at a constant rate of $a\,\text{m s}^{-2}$ until it reaches a speed of $16\,\text{m s}^{-1}$. It then travels at this constant speed before decelerating at a constant rate of $0.75a\,\text{m s}^{-2}$, coming to rest at B. The total time for the journey is 240 s.

(a) Sketch the velocity-time graph for the bus's journey from A to B. [1]



(b) Find an expression, in terms of a, for the length of time that the bus is travelling with constant speed. [2]

t_1: $v = u + at$ $\quad$ t_2: $v = u + at$

$16 = 0 + at_1$ $\quad$ $0 = 16 + (-0.75a)t_2$

$t_1 = \frac{16}{a}$ $\quad$ $0.75at_2 = 16 \rightarrow t_2 = \frac{64}{3a}$

Time at constant speed: $240 - \frac{16}{a}\frac{\times 3}{\times 3} - \frac{64}{3a}$

$= 240 - \frac{112}{3a}$

(c) Given that the distance from A to B is 3000 m, find the value of a. [3]

Area $= \frac{1}{2}(a+b) \times h$

$= \frac{1}{2}\left(240 + 240 - \frac{112}{3a}\right) \times 16$

$= 8\left(480 - \frac{112}{3a}\right)$

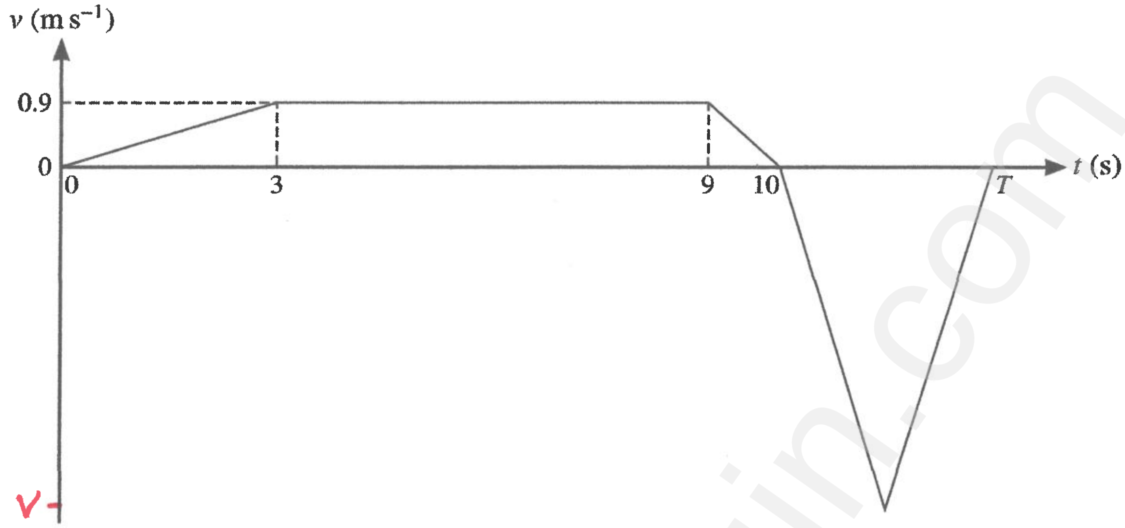
$3000 = 3840 - \frac{896}{3a}$

$\frac{896}{3a} = 840$

$896 = 2520a$

$a = \frac{16}{45}\ \text{ms}^{-2}$

4



The velocity of a particle at time t s after leaving a fixed point O is $v \text{ m s}^{-1}$. The diagram shows a velocity-time graph which models the motion of the particle. The graph consists of 5 straight line segments. The particle accelerates to a speed of 0.9 m s^{-1} in a period of 3 s, then travels at constant speed for 6 s, and then comes instantaneously to rest 1 s later. The particle then moves back and returns to rest at O at time T s.

(a) Find the distance travelled by the particle in the first 10 s of its motion. [2]

distance = area $= \frac{1}{2}(6+10) \times 0.9$

$= \frac{1}{2} \times 16 \times 0.9$

$= \underline{7.2\text{m}}$

(b) Given that $T = 12$, find the minimum velocity of the particle. [2]

Minimum velocity = lowest point on graph.
Since particle returns to starting point, area of triangle = area of trapezium in part (a):

$\text{Area} = \frac{1}{2} \times b \times h$

$7.2 = \frac{1}{2} \times 2 \times h$

$h = 7.2$

$\rightarrow v = -7.2\ \text{ms}^{-1}$ (negative velocity)

(c) Given instead that the greatest speed of the particle is $3\ \text{m s}^{-1}$, find the value of T and hence find the average speed of the particle for the whole of the motion. [4]

$\text{Area} = \frac{1}{2} \times b \times h$

$7.2 = \frac{1}{2} \times b \times 3$

$7.2 = 1.5b$

$b = 4.8$

so $T = 10 + 4.8$

$= 14.8$

$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$

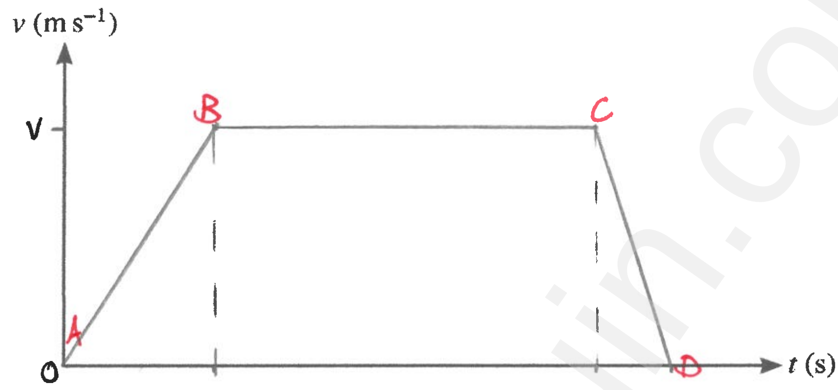
$= \frac{2 \times 7.2}{14.8}$

$= \frac{14.4}{14.8}$

$= 0.973\ \text{ms}^{-1}$

4 A car starts from rest and moves in a straight line with constant acceleration $a\,\mathrm{m\,s^{-2}}$ for a distance of 50 m. The car then travels with constant velocity for 500 m for a period of 25 s, before decelerating to rest. The magnitude of this deceleration is $2a\,\mathrm{m\,s^{-2}}$.

(a) Sketch the velocity-time graph for the motion of the car. [1]



(b) Find the value of a. [3]

during constant velocity section (BC): speed $= \frac{500}{25}$

$= 20\,\mathrm{ms^{-1}}$

$s = 50$	$V^2 = u^2 + 2as$
$u = 0$	$20^2 = 0^2 + 2 \times a \times 50$
$v = 20$	$400 = 100a$
$a =$	$a = 4\,\mathrm{ms^{-2}}$
$t =$	

(c) Find the total time for which the car is in motion. [3]

AB:

$s = 50$	$V = u + at$
$u = 0$	$20 = 0 + 4t$
$V = 20$	$t = 5\,\mathrm{s}$
$a = 4$	
$t =$	

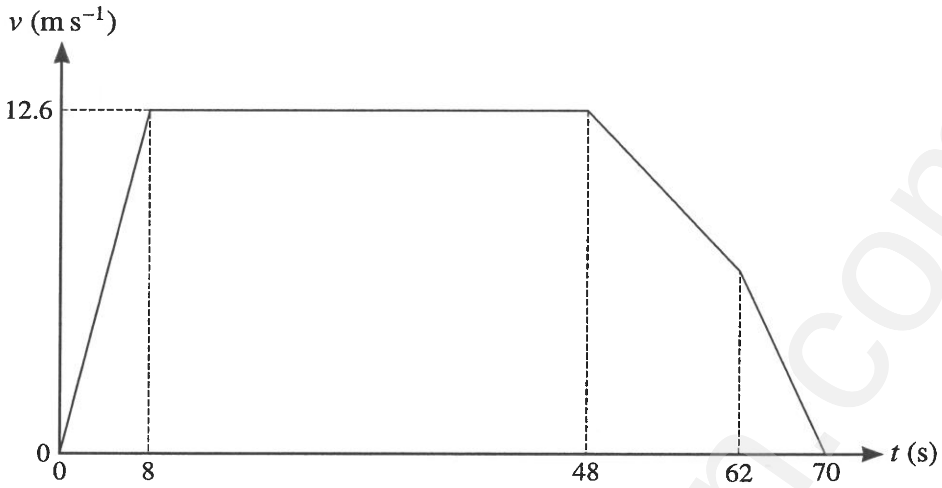
CD:

$s =$	$V = u + at$
$u = 20$	$0 = 20 - 8t$
$V = 0$	$8t = 20$
$a = -8$	$t = 2.5\,\mathrm{s}$
$t =$	

total time $= 5 + 25 + 2.5$

$= 32.5\,\mathrm{s}$

3



The diagram shows the velocity-time graph for the motion of a bus. The bus starts from rest and accelerates uniformly for 8 seconds until it reaches a speed of $12.6\,\mathrm{m\,s^{-1}}$. The bus maintains this speed for 40 seconds. It then decelerates uniformly in two stages. Between 48 and 62 seconds the bus decelerates at $a\,\mathrm{m\,s^{-2}}$ and between 62 and 70 seconds it decelerates at $2a\,\mathrm{m\,s^{-2}}$ until coming to rest.

(a) Find the distance covered by the bus in the first 8 seconds. [1]

distance = area : $\frac{1}{2} \times 8 \times 12.6$

$= \underline{50.4\text{m}}$

(b) Find the value of a. [3]

48→62s: $s =$ | $v = u + at$

$u = 12.6$ | $= 12.6 + -a(14)$

$v =$ | $= 12.6 - 14a$

$a = -a$

$t = 14$

Continued ..

62→70s: $s =$

$u = 12.6 - 14a$

$v = 0$

$a = -2a$

$t = 8$

$v = u + at$

$0 = 12.6 - 14a + (-2a) \times 8$

$0 = 12.6 - 14a - 16a$

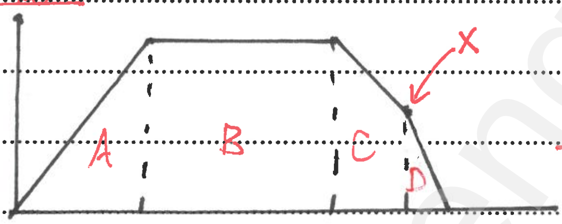
$30a = 12.6$

$a = 0.42 \text{ ms}^{-2}$

(c) Find the average speed of the bus for the whole journey. [4]

average speed $= \dfrac{\text{total distance}}{\text{total time}}$

distance:



$A = 50.4 \text{ m}$ (part (a))

$B: 40 \times 12.6 = 504 \text{ m}$

velocity at X:

$v = 12.6 - 14(0.42)$ (from part (b))

$= 6.72 \text{ ms}^{-1}$

C:

$\frac{1}{2}(12.6 + 6.72) \times 14 = 135.24 \text{ m}$

D:

$\frac{1}{2}(8)(6.72) = 26.88$

Total distance: $50.4 + 504 + 135.24 + 26.88 = 716.52$

average speed $= \dfrac{716.52}{70} = 10.236 \text{ ms}^{-1}$

4 Two cyclists, Isabella and Maria, are having a race. They both travel along a straight road with constant acceleration, starting from rest at point A.

Isabella accelerates for 5 s at a constant rate $a\,\mathrm{m\,s^{-2}}$. She then travels at the constant speed she has reached for 10 s, before decelerating to rest at a constant rate over a period of 5 s.

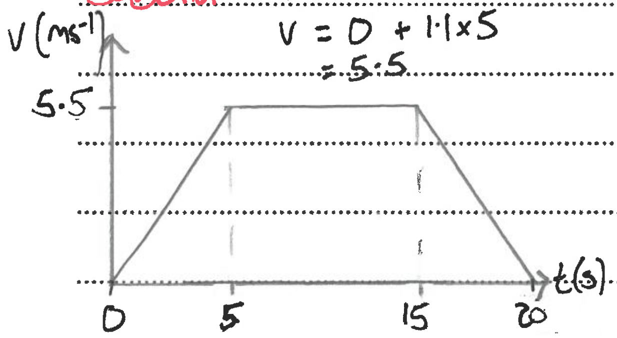
Maria accelerates at a constant rate, reaching a speed of $5\,\mathrm{m\,s^{-1}}$ in a distance of 27.5 m. She then maintains this speed for a period of 10 s, before decelerating to rest at a constant rate over a period of 5 s.

(a) Given that $a = 1.1$, find which cyclist travels further. [5]

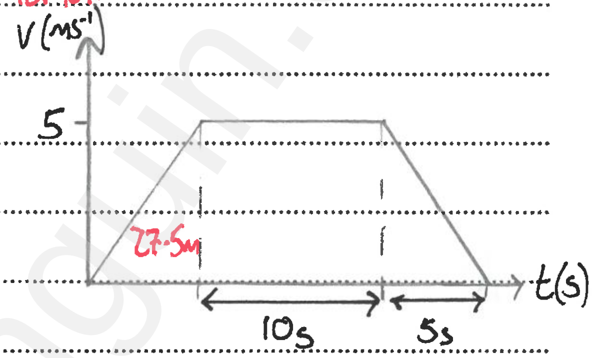
Isabella: $v = u + at$

$v = 0 + 1.1 \times 5$

$= 5.5$



Maria:



Area $= \frac{1}{2}(10+20) \times 5.5$ Area $= 27.5 + 10 \times 5 + \frac{1}{2} \times 5 \times 5$

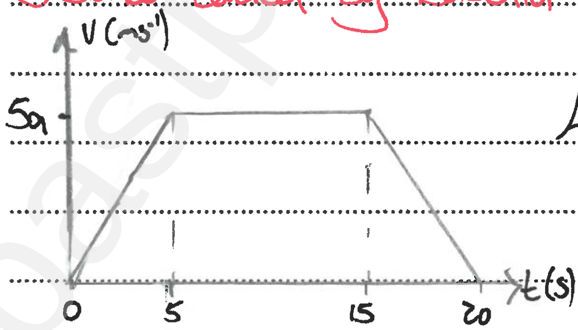
$= \frac{1}{2} \times 30 \times 5.5$ $= 27.5 + 50 + 12.5$

$= 82.5\,\mathrm{m}$ $= 90\,\mathrm{m}$

So Maria travels further.

(b) Find the value of a for which the two cyclists travel the same distance. [2]

Distance covered by Isabella $= 90$:



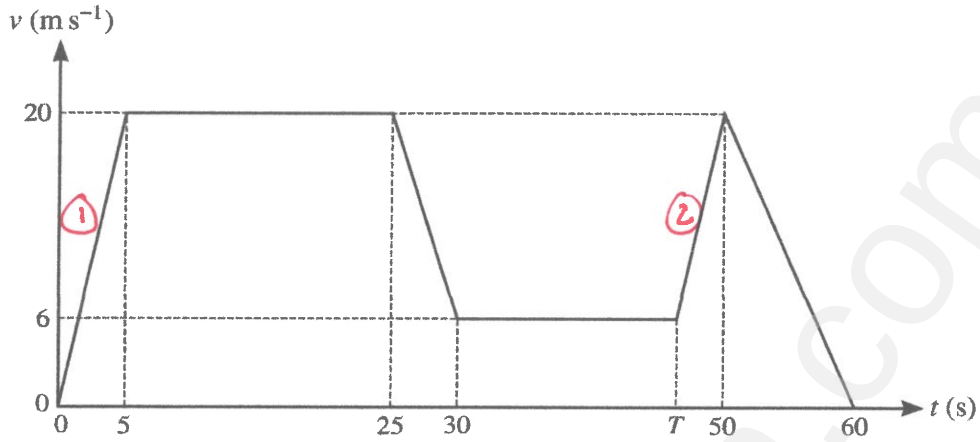
Area $= \frac{1}{2}(10+20) \times 5a$

$= 75a$

$75a = 90$

$a = 1.2\,\mathrm{ms^{-2}}$

1



The diagram shows a velocity-time graph which models the motion of a car. The graph consists of six straight line segments. The car accelerates from rest to a speed of $20\,\mathrm{m\,s^{-1}}$ over a period of 5 s, and then travels at this speed for a further 20 s. The car then decelerates to a speed of $6\,\mathrm{m\,s^{-1}}$ over a period of 5 s. This speed is maintained for a further $(T - 30)$ s. The car then accelerates again to a speed of $20\,\mathrm{m\,s^{-1}}$ over a period of $(50 - T)$ s, before decelerating to rest over a period of 10 s.

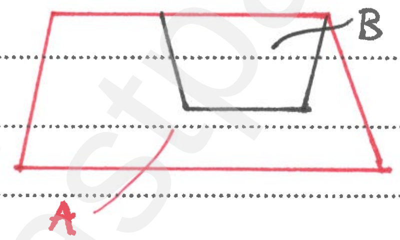
(a) Given that during the two stages of the motion when the car is accelerating, the acceleration are equal, find the value of T. [2]

1: $s =$	$v = u + at$	2: $s =$	$v = u + at$
$u = 0$	$20 = 0 + 5a$	$u = 6$	$20 = 6 + 4t$
$v = 20$	$5a = 20$	$v = 20$	$14 = 4t$
$a =$	$a = 4$	$a = 4$	$t = 3.5$
$t = 5$		$t =$	

$T = 50 - 3.5$

$= 46.5\,\mathrm{s}$

(b) Find the total distance travelled by the car during the motion. [2]



Area = Trapezium A − Trapezium B

Area A : $A = \frac{1}{2}(60 + 45) \times 20$

$= \frac{1}{2} \times 105 \times 20$

$= 1050$

Area B: $B = \frac{1}{2}(25 + 16.5) \times 14$

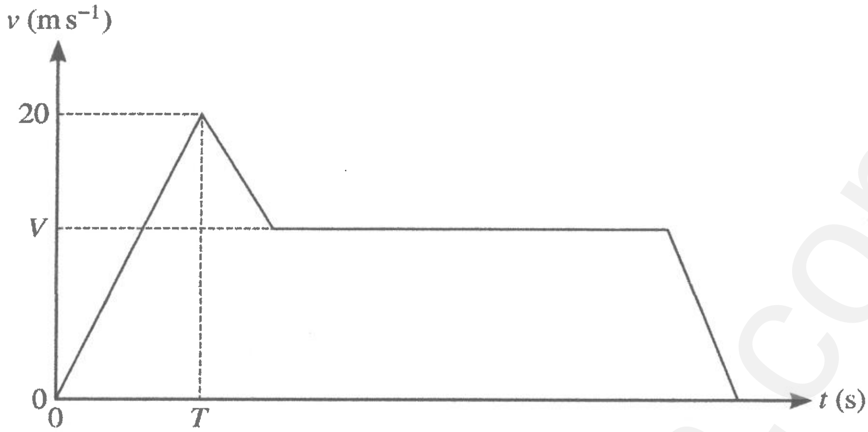
$= \frac{1}{2} \times 41.5 \times 14$

$= 290.5$

$A - B = 1050 - 290.5$

$= 759.5\,\mathrm{m}$

4

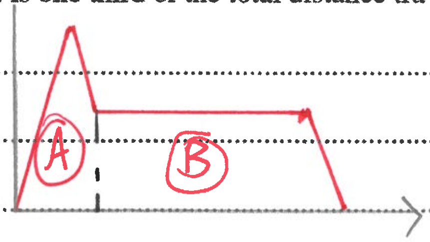


The diagram shows a velocity-time graph which models the motion of a car. The graph consists of four straight line segments. The car accelerates at a constant rate of $2\,\text{m}\,\text{s}^{-2}$ from rest to a speed of $20\,\text{m}\,\text{s}^{-1}$ over a period of T s. It then decelerates at a constant rate for 5 seconds before travelling at a constant speed of $V\,\text{m}\,\text{s}^{-1}$ for 27.5 s. The car then decelerates to rest at a constant rate over a period of 5 s.

(a) Find T. [1]

$s =$	$v = u + at$
$u = 0$	$20 = 0 + 2t$
$v = 20$	$20 = 2t$
$a = 2$	$t = 10$
$t =$	$\rightarrow$ $\underline{T = 10\text{ s}}$

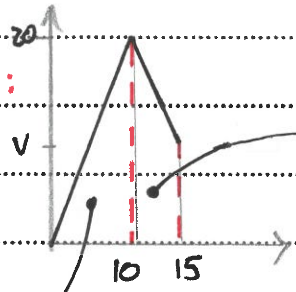
(b) Given that the distance travelled up to the point at which the car begins to move with constant speed is one third of the total distance travelled, find V. [4]



If (A) $= \frac{1}{3}$ Total and (B) $= \frac{2}{3}$ Total

Then (B) $= 2 \times$ (A)

Area (A):



area $= \frac{1}{2}(a+b) \times h$

$= \frac{1}{2}(20+V) \times 5$

$= 2.5(20+V)$

$= 50 + 2.5V$

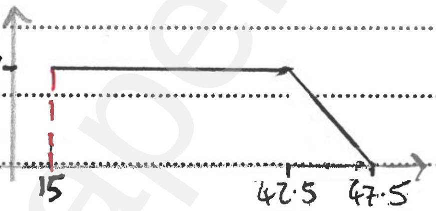
area $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 10 \times 20$

$= 100$ m

→ (A) $= 150 + 2.5V$

Area (B):



Area $= \frac{1}{2}(a+b) \times h$

$= \frac{1}{2}(27.5 + 32.5) \times V$

$= \frac{1}{2} \times 60 \times V$

$= 30V$

(B) $= 2 \times$ (A) :

$30V = 2(150 + 2.5V)$

$30V = 300 + 5V$

$25V = 300$

$V = 12 \text{ ms}^{-1}$