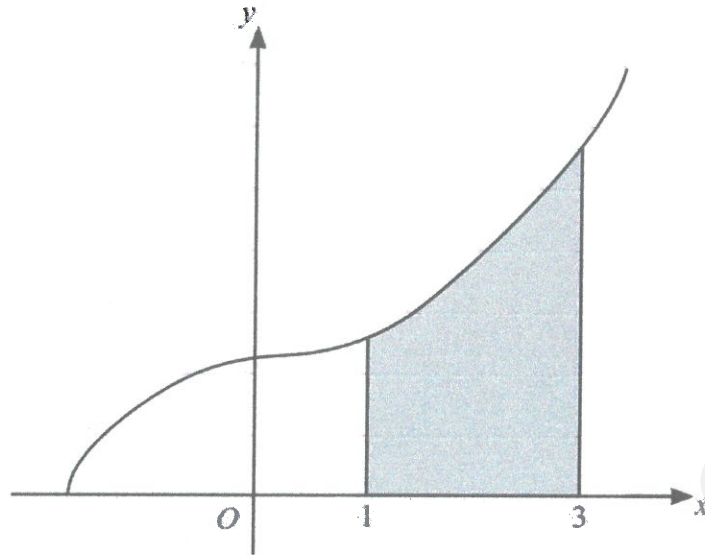




The diagram shows the curve with equation $y = \sqrt{2x^3 + 10}$.

- (b) The region shaded in the diagram is enclosed by the curve and the straight lines $x = 1$, $x = 3$ and $y = 0$.

Find the volume of the solid obtained when the shaded region is rotated through 360° about the x -axis. [3]

$$V = \pi \int y^2 dx$$

$$= \pi \int_1^3 (2x^3 + 10) dx$$

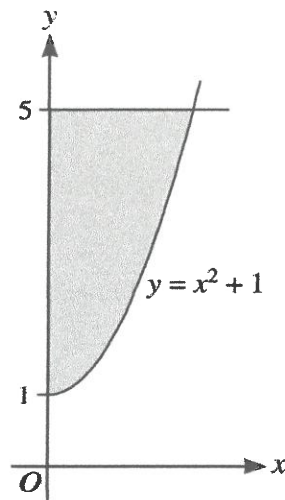
$$= \pi \left[\frac{1}{2}x^4 + 10x \right]_1^3$$

$$= \pi \left[\frac{1}{2} \times 3^4 + 10(3) \right] - \pi \left[\frac{1}{2} \times 1^4 + 10(1) \right]$$

$$= \pi \left[\frac{81}{2} + 30 \right] - \pi \left[\frac{1}{2} + 10 \right]$$

$$= 70.5\pi - 10.5\pi$$

$$= \underline{\underline{60\pi}}$$



$$\rightarrow x^2 = y - 1$$

The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y -axis and the line $y = 5$ is rotated through 360° about the y -axis.

Find the volume obtained.

[4]

$$\text{Volume} = \pi \int x^2 dy$$

$$= \pi \int_1^5 (y-1) dy$$

$$= \pi \left[\frac{1}{2} y^2 - y \right]_1^5$$

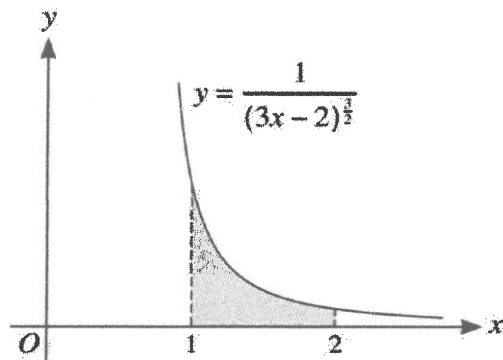
$$= \pi \left[\frac{1}{2} (5)^2 - 5 \right] - \pi \left[\frac{1}{2} (1)^2 - 1 \right]$$

$$= \pi \left[\frac{15}{2} \right] - \pi \left[-\frac{1}{2} \right]$$

$$= \frac{15\pi}{2} + \frac{\pi}{2}$$

$$= \underline{\underline{8\pi}}$$

10.



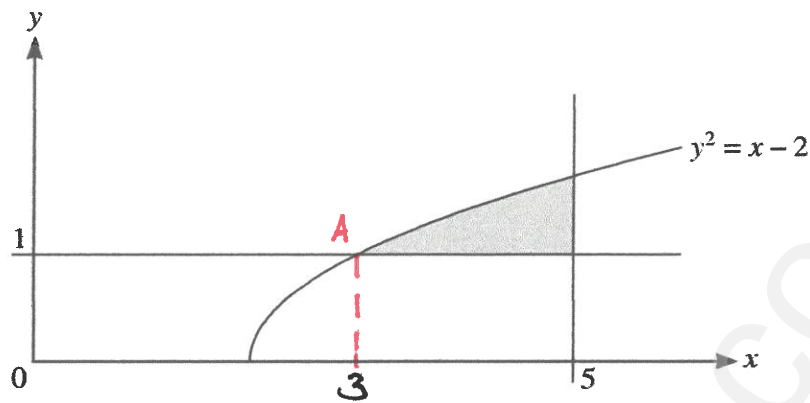
The diagram shows the curve with equation $y = \frac{1}{(3x-2)^{3/2}}$. The shaded region is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x -axis.

(b) Find the volume of revolution.

[4]

$$\begin{aligned}
 \text{Volume} &= \pi \int_1^2 y^2 dx \\
 &= \pi \int_1^2 \left(\frac{1}{(3x-2)^{3/2}} \right)^2 dx \\
 &= \pi \int_1^2 \frac{1}{(3x-2)^3} dx = \pi \int_1^2 (3x-2)^{-3} dx \\
 &= \pi \left[-\frac{1}{2} \times \frac{1}{3} (3x-2)^{-2} \right]_1^2 \\
 &= \pi \left[-\frac{1}{6} (3x-2)^{-2} \right]_1^2 \\
 &= \pi \left[-\frac{1}{6} (3(2)-2)^{-2} \right] - \pi \left[-\frac{1}{6} (3(1)-2)^{-2} \right] \\
 &= \pi \left[-\frac{1}{6} \times 4^{-2} \right] - \pi \left[-\frac{1}{6} \times 1^{-2} \right] \\
 &= -\frac{\pi}{96} - -\frac{\pi}{6} = \underline{\underline{\frac{5\pi}{32}}}
 \end{aligned}$$

9



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x -axis.

Find the volume obtained.

[6]

find point A: sub. $y=1$: $1^2 = x - 2 \rightarrow x = 3$

Volume of shaded region = Volume bounded by $y^2 = x - 2$ - Volume of cylinder bounded by line $y = 1$

Volume bounded by $y^2 = x - 2$:

$$\text{Volume} = \pi \int y^2 dx$$

$$= \pi \int_3^5 (x - 2) dx$$

$$= \pi \left[\frac{1}{2} x^2 - 2x \right]_3^5$$

$$= \pi \left[\frac{1}{2} (5)^2 - 2(5) \right] - \pi \left[\frac{1}{2} (3)^2 - 2(3) \right]$$

$$= \pi \left[\frac{25}{2} - 10 \right] - \pi \left[\frac{9}{2} - 6 \right]$$

$$= \frac{5\pi}{2} - -\frac{3\pi}{2} = 4\pi$$

Volume of cylinder:

$$V = \pi r^2 h$$

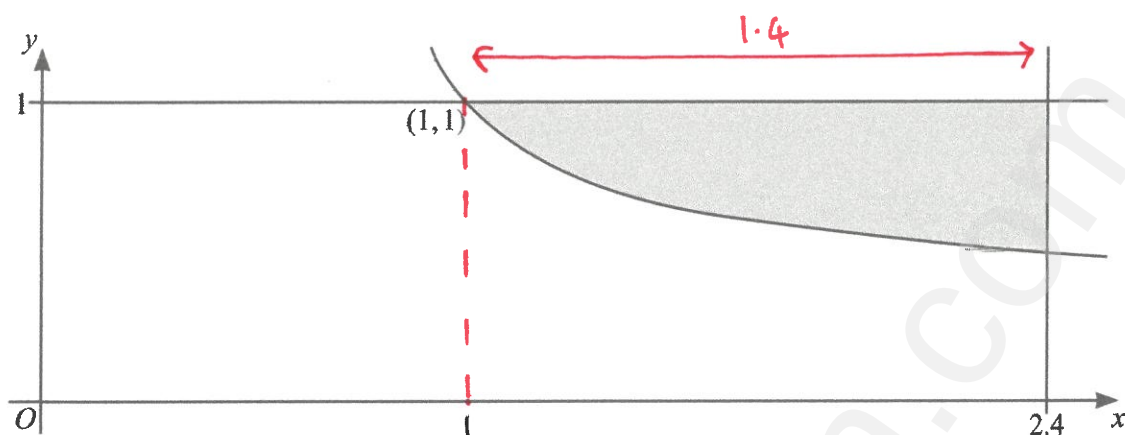
$$= \pi \times 1^2 \times 2$$

$$= 2\pi$$

Volume of shaded region:

$$4\pi - 2\pi = 2\pi$$

9



The diagram shows part of the curve with equation $y = \frac{1}{(5x-4)^{\frac{1}{3}}}$ and the lines $x = 2.4$ and $y = 1$. The curve intersects the line $y = 1$ at the point $(1, 1)$.

Find the exact volume of the solid generated when the shaded region is rotated through 360° about the x -axis. [6]

Volume of shaded region = volume of cylinder - volume bounded by $y = \frac{1}{(5x-4)^{\frac{1}{3}}}$

Volume of cylinder: $V = \pi r^2 h$
 $= \pi \times 1^2 \times 1.4$
 $= 1.4\pi$

Volume bounded by $y = \frac{1}{(5x-4)^{\frac{1}{3}}}$:

$$\begin{aligned} V &= \pi \int_{2.4}^1 y^2 dx \\ &= \pi \int_1^{2.4} \left(\frac{1}{(5x-4)^{\frac{1}{3}}} \right)^2 dx \\ &= \pi \int_1^{2.4} \frac{1}{(5x-4)^{\frac{2}{3}}} dx \\ &= \pi \int_1^{2.4} (5x-4)^{-\frac{2}{3}} dx \end{aligned}$$

$$= \pi \left[3 \times \frac{1}{5} (5x-4)^{\frac{1}{3}} \right]_1^{2.4}$$

$$= \pi \left[\frac{3}{5} (5(2.4)-4)^{\frac{1}{3}} \right] - \pi \left[\frac{3}{5} (5-4)^{\frac{1}{3}} \right]$$

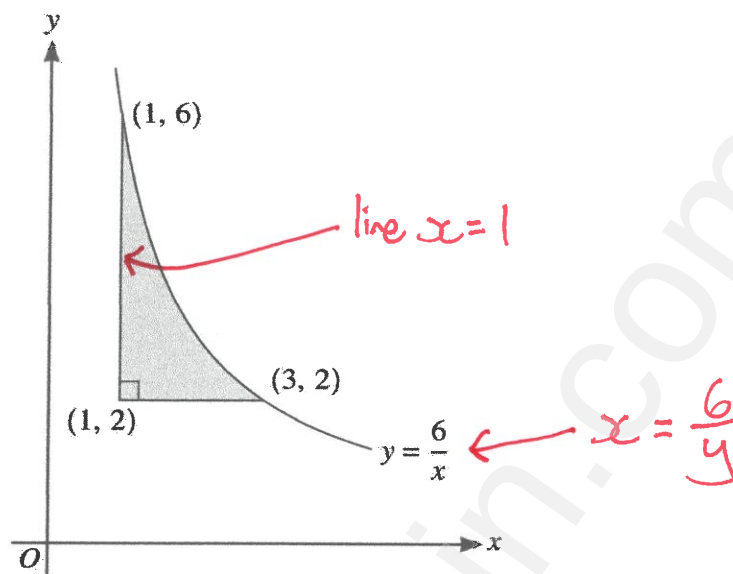
$$= \pi \left[\frac{3}{5} \times 8^{\frac{1}{3}} \right] - \pi \left[\frac{3}{5} \times 1^{\frac{1}{3}} \right]$$

$$= \pi \left[\frac{6}{5} \right] - \pi \left[\frac{3}{5} \right]$$

$$= \frac{3}{5} \pi$$

$$\text{Volume} = 1.4\pi - \frac{3}{5}\pi$$

$$= \frac{4}{5}\pi$$



The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

- (a) Find the volume generated when the shaded region is rotated through 360° about the y-axis. [5]

Volume bounded by shaded region = Volume bounded by $y = \frac{6}{x}$
 - volume of cylinder bounded by line $x = 1$

Volume bounded by $y = \frac{6}{x}$:

$$\text{Volume} = \pi \int x^2 dy$$

$$= \pi \int_2^6 \left(\frac{6}{y}\right)^2 dy$$

$$= \pi \int_2^6 36y^{-2} dy$$

$$= \pi [-36y^{-1}]_2^6$$

$$= \pi [-36(6)^{-1}] - \pi [-36(2)^{-1}]$$

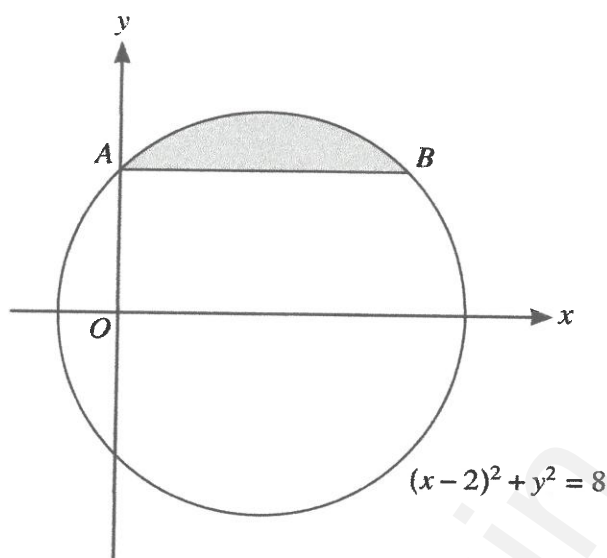
$$= -6\pi - -18\pi = \underline{12\pi}$$

Volume of cylinder bounded by line $x=1$:

$$\begin{aligned}\text{Volume} &= \pi r^2 h \\ &= \pi \times 1^2 \times 4 \\ &= 4\pi\end{aligned}$$

Volume bounded by shaded region:

$$12\pi - 4\pi = \underline{8\pi}$$



The diagram shows the circle with equation $(x-2)^2 + y^2 = 8$. The chord AB of the circle intersects the positive y -axis at A and is parallel to the x -axis.

- (a) Find, by calculation, the coordinates of A and B .

[3]

At point A , $x=0$:

$$(0-2)^2 + y^2 = 8$$

$$(-2)^2 + y^2 = 8$$

$$y^2 = 4$$

$$y = \pm 2$$

↳ $y=2$ (from diagram)

At point B , $y=2$:

$$(x-2)^2 + 2^2 = 8$$

$$x^2 - 4x + 4 + 4 = 8$$

$$x^2 - 4x + 8 = 8$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x=0, \quad x=4$$

→ $(0, 2)^A$

→ $(4, 2)^B$

- (b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

Volume from shaded segment = Volume from circle - cylinder bounded by the line $y=2$

Volume from circle:

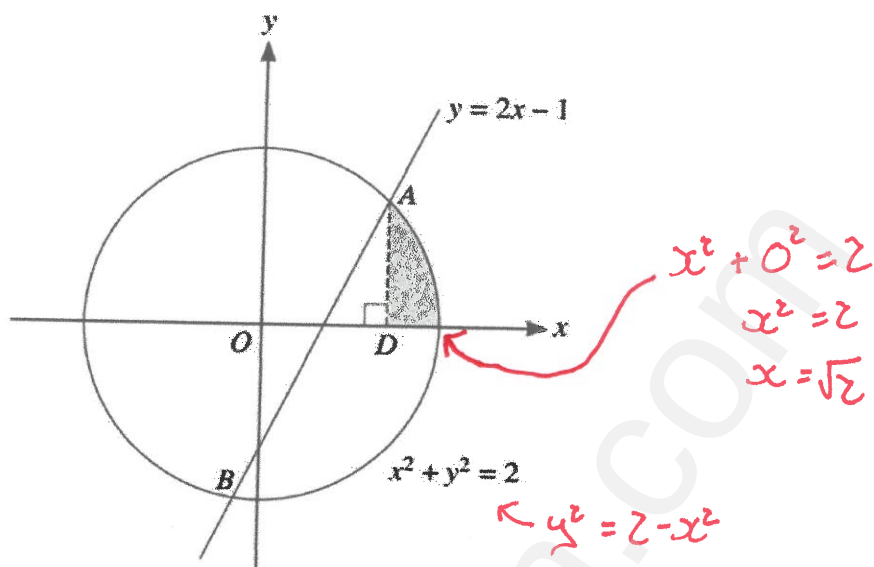
$$\begin{aligned}
 \text{Volume} &= \pi \int y^2 dx \\
 &= \pi \int_0^4 (8 - (x-2)^2) dx \\
 &= \pi \left[8x - \frac{1}{3}(x-2)^3 \right]_0^4 \\
 &= \pi \left[8(4) - \frac{1}{3}(4-2)^3 \right] - \pi \left[0 - \frac{1}{3}(0-2)^3 \right] \\
 &= \pi \left[32 - \frac{1}{3}(2)^3 \right] - \pi \left[-\frac{1}{3}(-2)^3 \right] \\
 &= \pi \left[32 - \frac{8}{3} \right] - \pi \left[\frac{8}{3} \right] \\
 &= \frac{88\pi}{3} - \frac{8\pi}{3} = \frac{80\pi}{3}
 \end{aligned}$$

Volume of cylinder bounded by $y=2$:

$$\begin{aligned}
 \text{Volume} &= \pi r^2 h \\
 &= \pi \times 2^2 \times 4 \\
 &= 16\pi
 \end{aligned}$$

Volume from shaded segment:

$$\frac{80\pi}{3} - 16\pi = \frac{32\pi}{3}$$



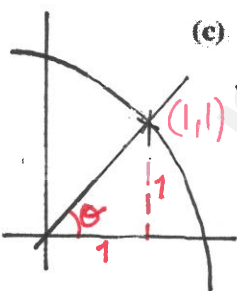
from (a): $A = (1, 1)$

The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

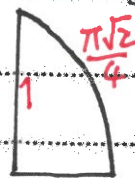
- (b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}(b\sqrt{c} - d)$, where a, b, c and d are integers. [4]

$$\begin{aligned}
 \text{Volume} &= \pi \int_1^{\sqrt{2}} y^2 dx = \pi \int_1^{\sqrt{2}} (2 - x^2) dx \\
 &= \pi \left[2x - \frac{1}{3}x^3 \right]_1^{\sqrt{2}} \\
 &= \pi \left[2\sqrt{2} - \frac{1}{3}(\sqrt{2})^3 \right] - \pi \left[2(1) - \frac{1}{3}(1)^3 \right] \\
 &= \pi \left[2\sqrt{2} - \frac{1}{3} \times 2\sqrt{2} \right] - \pi \left[2 - \frac{1}{3} \right] \\
 &= \pi \left[\frac{4\sqrt{2}}{3} \right] - \pi \left[\frac{5}{3} \right] \\
 &= \pi \left(\frac{4\sqrt{2}}{3} - \frac{5}{3} \right) = \frac{\pi}{3} (4\sqrt{2} - 5)
 \end{aligned}$$

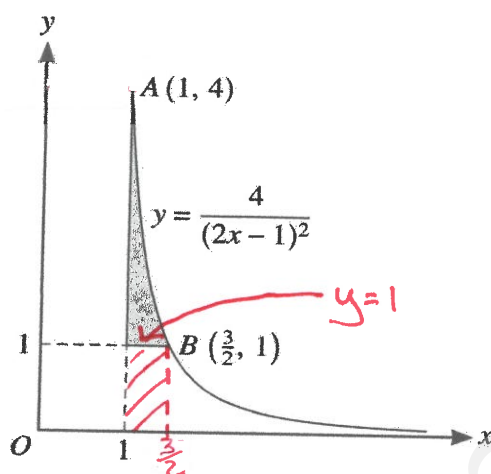
- (c) Find an exact expression for the perimeter of the shaded region. [2]



$$\begin{aligned}
 \tan \theta &= \frac{1}{1} \rightarrow \theta = \frac{\pi}{4} \\
 \text{Arc length} &= r\theta \\
 &= \sqrt{2} \times \frac{\pi}{4}
 \end{aligned}$$



$$\begin{aligned}
 \text{perimeter} &= \frac{\pi\sqrt{2}}{4} + 1 + \sqrt{2} - 1 \\
 &= \sqrt{2} + \frac{\pi\sqrt{2}}{4}
 \end{aligned}$$



The diagram shows part of the curve with equation $y = \frac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A(1, 4)$ and $B(\frac{3}{2}, 1)$.

- (a) Find the exact volume generated when the shaded region is rotated through 360° about the x -axis. [5]

Volume of shaded region = Volume bounded by $y = \frac{4}{(2x-1)^2}$ - Volume of cylinder bounded by line $y = 1$

Volume bounded by $y = \frac{4}{(2x-1)^2}$: $V = \pi \int y^2 dx$

$$V = \pi \int_1^{\frac{3}{2}} \frac{16}{(2x-1)^4} dx = \pi \int_1^{\frac{3}{2}} 16(2x-1)^{-4} dx$$

$$= \pi \left[-\frac{16}{3} \times \frac{1}{2} (2x-1)^{-3} \right]_1^{\frac{3}{2}} = \pi \left[-\frac{8}{3} (2x-1)^{-3} \right]_1^{\frac{3}{2}}$$

$$= \pi \left[-\frac{8}{3} (2(\frac{3}{2})-1)^{-3} \right] - \pi \left[-\frac{8}{3} (2(1)-1)^{-3} \right]$$

$$= \pi \left[-\frac{8}{3} \times 2^{-3} \right] - \pi \left[-\frac{8}{3} \times 1^{-3} \right]$$

$$= -\frac{\pi}{3} - -\frac{8\pi}{3} = \frac{7\pi}{3}$$

Volume of cylinder:

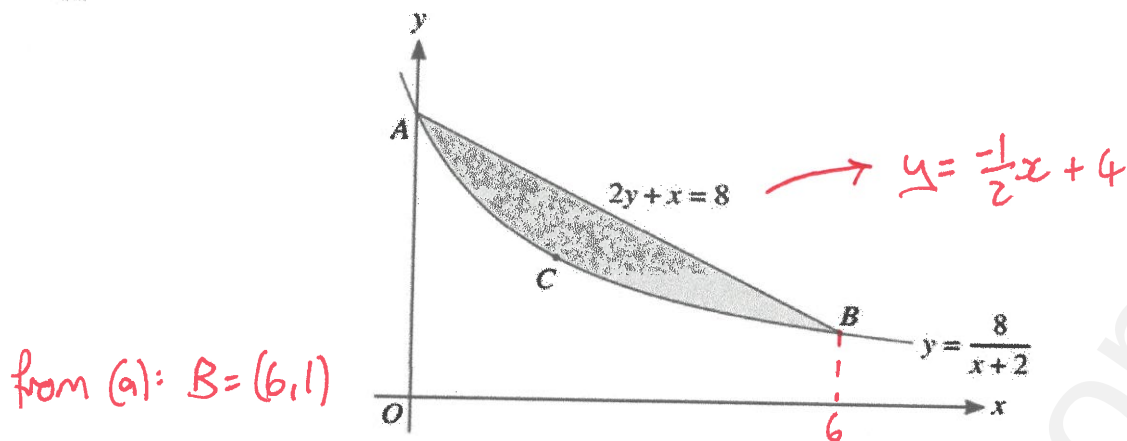
$$V = \pi r^2 h$$

$$= \pi \times 1^2 \times 0.5$$

$$= \frac{\pi}{2}$$

Volume of shaded region:

$$\frac{7\pi}{3} - \frac{\pi}{2} = \frac{11\pi}{6}$$



The diagram shows part of the curve $y = \frac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

- (b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

Volume bounded by shaded region = Volume bounded by $2y + x = 8$
 - Volume bounded by $y = \frac{8}{x+2}$

Volume bounded by $2y + x = 8$:

$$\text{Volume} = \pi \int y^2 dx$$

$$= \pi \int_0^6 \left(-\frac{1}{2}x + 4 \right)^2 dx$$

$$= \pi \int_0^6 \left(\frac{1}{4}x^2 - 4x + 16 \right) dx$$

$$= \pi \left[\frac{1}{12}x^3 - 2x^2 + 16x \right]_0^6$$

$$= \pi \left[\frac{1}{12}(6)^3 - 2(6)^2 + 16(6) \right] - \pi[0]$$

$$= \pi[18 - 72 + 96]$$

$$= 42\pi$$

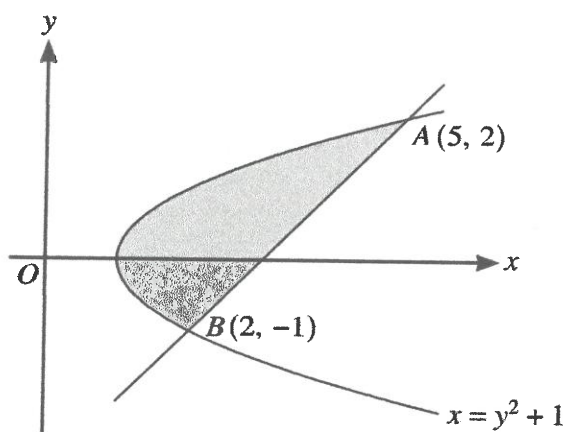
Volume bounded by $y = \frac{8}{x+2}$:

$$\begin{aligned}\text{Volume} &= \pi \int y^2 dx \\&= \pi \int_0^6 \left(\frac{8}{x+2} \right)^2 dx \\&= \pi \int_0^6 \frac{64}{(x+2)^2} dx \\&= \pi \int_0^6 64(x+2)^{-2} dx \\&= \pi \left[-64(x+2)^{-1} \right]_0^6 \\&= \pi \left[-64(6+2)^{-1} \right] - \pi \left[-64(0+2)^{-1} \right] \\&= \pi \left[-64 \times \frac{1}{8} \right] - \pi \left[-64 \times \frac{1}{2} \right] \\&= -8\pi - -32\pi = \underline{24\pi}\end{aligned}$$

Volume bounded by shaded region:

$$42\pi - 24\pi = \underline{18\pi}$$

11



The diagram shows the curve with equation $x = y^2 + 1$. The points $A(5, 2)$ and $B(2, -1)$ lie on the curve.

- (a) Find an equation of the line AB .

[2]

Gradient AB : $m = \frac{2 - (-1)}{5 - 2}$

$= \frac{3}{3} = 1$

$y - 2 = 1(x - 5)$

$y - 2 = x - 5$

$y = x - 3$

- (b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the y -axis.

[9]

Volume from shaded region = Volume bounded by $y = x - 3$ - Volume bounded by $x = y^2 + 1$

Volume bounded by $y = x - 3$:

Volume = $\pi \int x^2 dy$

$= \pi \int_{-1}^2 (y + 3)^2 dy$

$= \pi \int_{-1}^2 (y^2 + 6y + 9) dy$

$$\begin{aligned}
 &= \pi \left[\frac{1}{3}y^3 + 3y^2 + 9y \right]_{-1}^2 \\
 &= \pi \left[\frac{1}{3}(2)^3 + 3(2)^2 + 9(2) \right] - \pi \left[\frac{1}{3}(-1)^3 + 3(-1)^2 + 9(-1) \right] \\
 &= \pi \left[\frac{8}{3} + 12 + 18 \right] - \pi \left[-\frac{1}{3} + 3 - 9 \right] \\
 &= \frac{98\pi}{3} - -\frac{19\pi}{3} = \underline{39\pi}
 \end{aligned}$$

Volume bounded by $x = y^2 + 1$.

$$\begin{aligned}
 \text{Volume} &= \pi \int x^2 dy \\
 &= \pi \int_{-1}^2 (y^2 + 1)^2 dy \\
 &= \pi \int_{-1}^2 (y^4 + 2y^2 + 1) dy \\
 &= \pi \left[\frac{1}{5}y^5 + \frac{2}{3}y^3 + y \right]_{-1}^2 \\
 &= \pi \left[\frac{1}{5}(2)^5 + \frac{2}{3}(2)^3 + 2 \right] - \pi \left[\frac{1}{5}(-1)^5 + \frac{2}{3}(-1)^3 + -1 \right] \\
 &= \pi \left[\frac{32}{5} + \frac{16}{3} + 2 \right] - \pi \left[-\frac{1}{5} - \frac{2}{3} - 1 \right] \\
 &= \frac{206\pi}{15} - -\frac{28\pi}{15} = \underline{\frac{78\pi}{5}}
 \end{aligned}$$

Volume from shaded region: $39\pi - \frac{78\pi}{5} = \underline{\underline{\frac{117\pi}{5}}}$