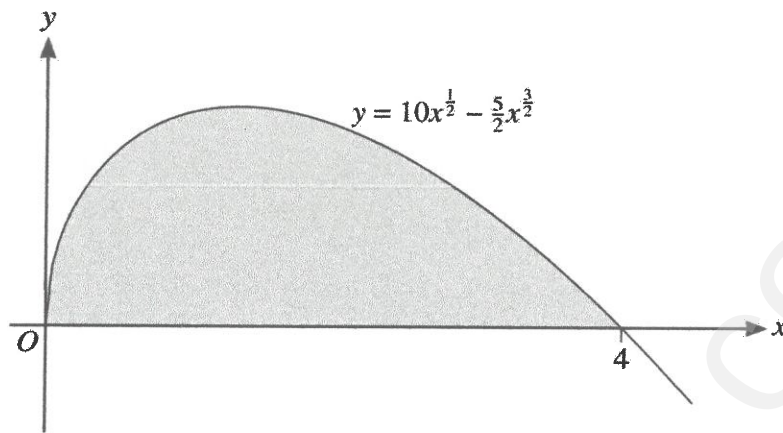


5



The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x -axis at the points $(0, 0)$ and $(4, 0)$.

Find the area of the shaded region.

[4]

$$\int_0^4 \left(10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} \right) dx$$

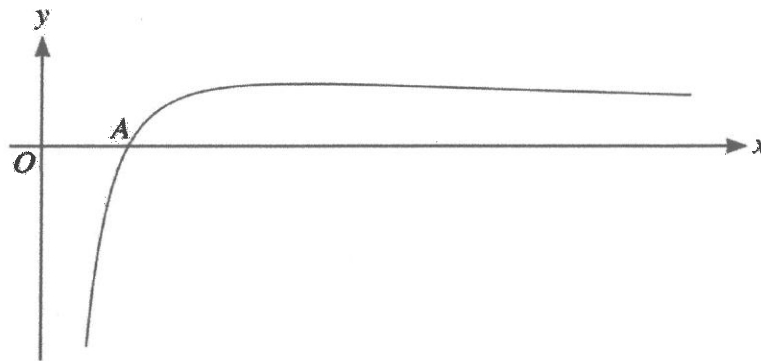
$$= \left[\frac{2}{3} \times 10x^{\frac{3}{2}} - \frac{2}{5} \times \frac{5}{2}x^{\frac{5}{2}} \right]_0^4$$

$$= \left[\frac{20}{3}x^{\frac{3}{2}} - x^{\frac{5}{2}} \right]_0^4$$

$$= \left[\frac{20}{3}(4)^{\frac{3}{2}} - (4)^{\frac{5}{2}} \right] - [0]$$

$$= \frac{20}{3} \times 8 - 32$$

$$= \frac{64}{3}$$



from part (a) $A = (4, 0)$

The diagram shows the curve with equation $y = 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})$. The curve crosses the x -axis at the point A .

(d) Find the area of the region bounded by the curve, the x -axis and the line $x = 9$.

[4]

$$\begin{aligned}
 & \int_4^9 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}) dx \\
 & 9 \int_4^9 (x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}) dx \\
 & 9 \left[2x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} \right]_4^9 \\
 & = 9 \left[2(9)^{\frac{1}{2}} + 8(9)^{-\frac{1}{2}} - (2(4)^{\frac{1}{2}} + 8(4)^{-\frac{1}{2}}) \right] \\
 & = 9 \left[2 \times 3 + 8 \times \frac{1}{3} - (2 \times 2 + 8 \times \frac{1}{2}) \right] \\
 & = 9 \left[\left(6 + \frac{8}{3} \right) - (4 + 4) \right] \\
 & = 9 \left[6 + \frac{8}{3} - 8 \right] \\
 & = 9 \left[\frac{2}{3} \right] \\
 & = \underline{\underline{6}}
 \end{aligned}$$

11 The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(d) Find the exact area of the region bounded by the curve, the x -axis and the lines $x = 0$ and $x = 4$.
[4]

$$\int_0^4 [2(3x+4)^{\frac{1}{2}} - x] dx$$

$$\left[\frac{2}{3} \times 2 \times \frac{3}{2} (3x+4)^{\frac{3}{2}} - \frac{1}{2} x^2 \right]_0^4$$

$$= \left[\frac{4}{9} (3x+4)^{\frac{3}{2}} - \frac{1}{2} x^2 \right]_0^4$$

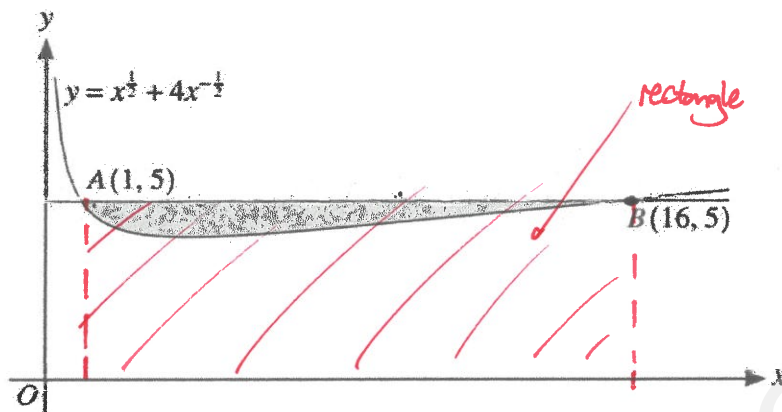
$$= \left[\frac{4}{9} (3(4)+4)^{\frac{3}{2}} - \frac{1}{2} (4)^2 \right] - \left[\frac{4}{9} (3(0)+4)^{\frac{3}{2}} - 0 \right]$$

$$= \left[\frac{4}{9} (16)^{\frac{3}{2}} - \frac{1}{2} \times 16 \right] - \left[\frac{4}{9} (4)^{\frac{3}{2}} \right]$$

$$= \left[\frac{4}{9} \times 64 - 8 \right] - \left[\frac{4}{9} \times 8 \right]$$

$$= \frac{256}{9} - 8 - \frac{32}{9}$$

$$= \frac{152}{9}$$



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A(1, 5)$ and $B(16, 5)$.

(b) Calculate the area of the shaded region.

[4]

Area under curve: $\int_1^{16} (x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}) dx$

$$= \left[\frac{2}{3} x^{\frac{3}{2}} + 8x^{\frac{1}{2}} \right]_1^{16}$$

$$= \left[\frac{2}{3} (16)^{\frac{3}{2}} + 8(16)^{\frac{1}{2}} \right] - \left[\frac{2}{3} (1)^{\frac{3}{2}} + 8(1)^{\frac{1}{2}} \right]$$

$$= \left[\frac{2}{3} \times 64 + 8 \times 4 \right] - \left[\frac{2}{3} \times 1 + 8 \times 1 \right]$$

$$= \left[\frac{128}{3} + 32 \right] - \left[\frac{2}{3} + 8 \right]$$

$$= \left[\frac{224}{3} \right] - \left[\frac{26}{3} \right]$$

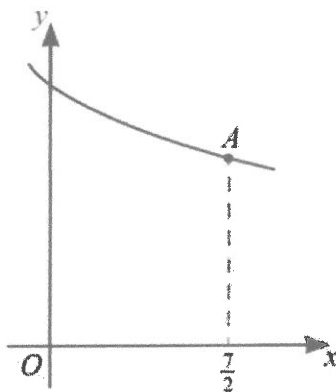
$$= 66$$

Area of rectangle: Area = base $\times$ height

$$= 15 \times 5$$

$$= 75$$

Shaded Area: $75 - 66 = \underline{\underline{9}}$

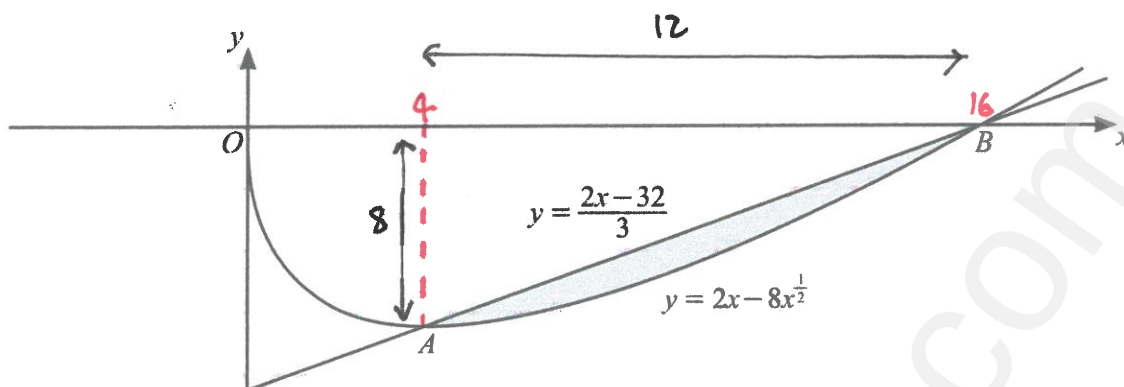


The diagram shows part of the curve with equation $y = \frac{12}{\sqrt[3]{2x+1}}$. The point A on the curve has coordinates $(\frac{7}{2}, 6)$.

- (b) Find the area of the region bounded by the curve and the lines $x = 0$, $x = \frac{7}{2}$ and $y = 0$. [4]

$$\begin{aligned}
 & \int_0^{\frac{7}{2}} 12(2x+1)^{-\frac{1}{3}} dx \\
 &= \left[\frac{3}{2} \times \frac{1}{2} \times 12(2x+1)^{\frac{2}{3}} \right]_0^{\frac{7}{2}} \\
 &= \left[9(2x+1)^{\frac{2}{3}} \right]_0^{\frac{7}{2}} \\
 &= \left[9\left(2\left(\frac{7}{2}\right)+1\right)^{\frac{2}{3}} \right] - \left[9(2(0)+1)^{\frac{2}{3}} \right] \\
 &= \left[9(8)^{\frac{2}{3}} \right] - \left[9(1)^{\frac{2}{3}} \right] \\
 &= 36 - 9 \\
 &= \underline{\underline{27}}
 \end{aligned}$$

6. (b)



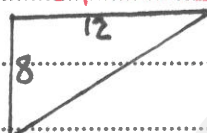
The diagram shows the curve with equation $y = 2x - 8x^{\frac{1}{2}}$ and the line AB . It is given that the equation of AB is $y = \frac{2x - 32}{3}$.

Find the area of the shaded region between the curve and the line.

[5]

from part (a): $A = (4, -8)$ and $B = (16, 0)$

Shaded Area = Area enclosed by $y = 2x - 8x^{\frac{1}{2}}$ - Area of triangle

Area of Triangle:  $\frac{b \times h}{2} = \frac{8 \times 12}{2} = \underline{48}$

Area enclosed by curve:

$$\int_4^{16} (2x - 8x^{\frac{1}{2}}) dx = \left[x^2 - \frac{16}{3} x^{\frac{3}{2}} \right]_4^{16}$$

$$= \left[16^2 - \frac{16}{3} (16)^{\frac{3}{2}} \right] - \left[4^2 - \frac{16}{3} (4)^{\frac{3}{2}} \right]$$

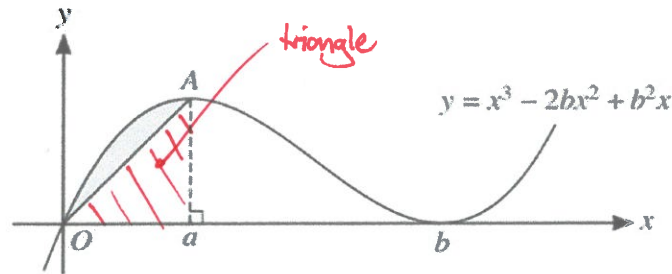
$$= \left[256 - \frac{1024}{3} \right] - \left[16 - \frac{128}{3} \right]$$

$$= 256 - \frac{1024}{3} - 16 + \frac{128}{3}$$

$$= -\frac{176}{3} \leftarrow \text{negative because below } x\text{-axis. Area} = \frac{176}{3}$$

$$\text{Shaded Area} = \frac{176}{3} - 48$$

$$= \frac{32}{3}$$



from (a): $b = 3a$

The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA , where A is the maximum point on the curve. The x -coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

- (b) Show that the area of the shaded region between the line and the curve is ka^4 , where k is a fraction to be found. [7]

$$y = x^3 - 6ax^2 + 9a^2x$$

Area under curve:

$$\int_0^a (x^3 - 6ax^2 + 9a^2x) dx$$

$$= \left[\frac{1}{4}x^4 - 2ax^3 + \frac{9}{2}a^2x^2 \right]_0^a$$

$$= \left[\frac{1}{4}a^4 - 2a^4 + \frac{9}{2}a^4 \right] - [0]$$

$$= \frac{11}{4}a^4$$

Area of triangle:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

For height, need y -coordinate of A :

Sub. $x = a$ into y :

$$\begin{aligned} y &= a^3 - 6a \times a^2 + 9a^2 \times a \\ &= a^3 - 6a^3 + 9a^3 \\ &= 4a^3 \end{aligned}$$

$$\begin{aligned} \rightarrow \text{Area} &= \frac{1}{2} \times a \times 4a^3 \\ &= 2a^4 \end{aligned}$$

$$\text{Shaded Area: } \frac{11}{4}a^4 - 2a^4 = \frac{3}{4}a^4$$

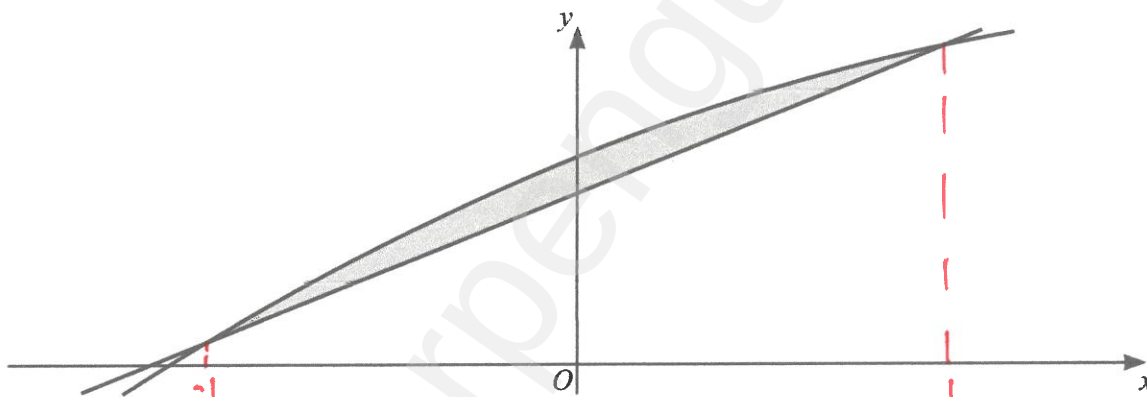


- 7 (a) By expressing $-2x^2 + 8x + 11$ in the form $-a(x-b)^2 + c$, where a , b and c are positive integers, find the coordinates of the vertex of the graph with equation $y = -2x^2 + 8x + 11$. [3]

$$\begin{aligned}
 & -2[x^2 - 4x] + 11 \quad \text{or: } -a(x-b)^2 + c \\
 & = -2[(x-2)^2 - 4] + 11 \quad -a(x^2 - 2bx + b^2) + c \\
 & = -2(x-2)^2 + 8 + 11 \quad -ax^2 + 2abx - ab^2 + c \\
 & = -2(x-2)^2 + 19 \quad \text{compare coeffs:} \\
 & \quad \quad \quad \begin{array}{lcl} x^2: & -2 = -a & x: \quad 8 = 2ab \\ & a = 2 & 8 = 4b \\ & & b = 2 \\ & & 11 = -8 + c \\ & & c = 19 \end{array}
 \end{aligned}$$

Vertex at (2, 19)

(b)



The diagram shows part of the curve with equation $y = -2x^2 + 8x + 11$ and the line with equation $y = 8x + 9$.

Find the area of the shaded region. [5]

Find points of intersection:

$$-2x^2 + 8x + 11 = 8x + 9$$

$$-2x^2 + 2 = 0$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = \pm 1$$

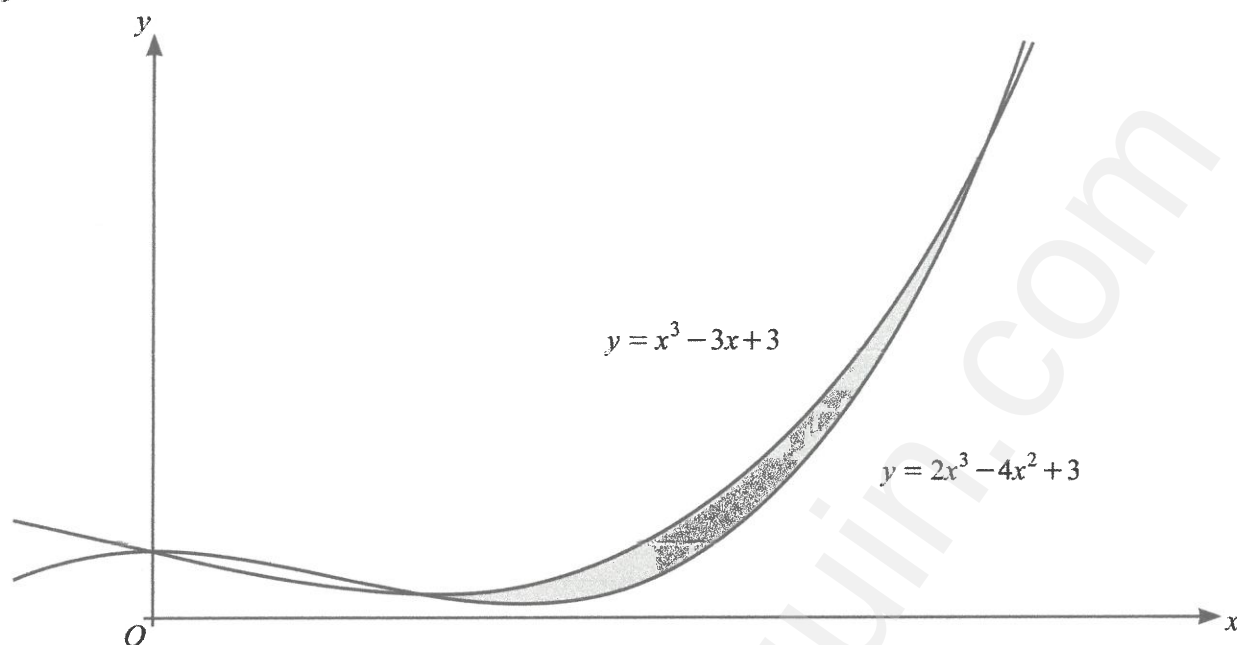




area ¹¹ under
curve

area under
straight line

$$\begin{aligned}
 \text{Area} &= \int_{-1}^1 (-2x^2 + 8x + 11) dx - \int_{-1}^1 (8x + 9) dx \\
 &= \left[-\frac{2x^3}{3} + 4x^2 + 11x \right]_{-1}^1 - \left[4x^2 + 9x \right]_{-1}^1 \\
 &= \left[\left(-\frac{2}{3} + 4 + 11 \right) - \left(\frac{2}{3} + 4 - 11 \right) \right] - \left[(4 + 9) - (4 - 9) \right] \\
 &= \left[\left(\frac{43}{3} \right) - \left(-\frac{19}{3} \right) \right] - \left[(13) - (-5) \right] \\
 &= \frac{62}{3} - 18 \\
 &= \underline{\underline{\frac{8}{3}}}
 \end{aligned}$$



The diagram shows the curves with equations $y = x^3 - 3x + 3$ and $y = 2x^3 - 4x^2 + 3$.

- (a) Find the x -coordinates of the points of intersection of the curves.

[3]

$$2x^3 - 4x^2 + 3 = x^3 - 3x + 3$$

$$x^3 - 4x^2 + 3x = 0$$

$$x[x^2 - 4x + 3] = 0$$

$$x(x - 3)(x - 1) = 0$$

$$\underline{x=0}, \underline{x=3}, \underline{x=1}$$

(b) Find the area of the shaded region.

[4]

Area is between $x=1$ and $x=3$ ($x=0$ is intersection of y-axis)

$$\int_1^3 [(x^3 - 3x + 3) - (2x^3 - 4x^2 + 3)] dx$$

$$= \int_1^3 (-x^3 + 4x^2 - 3x) dx$$

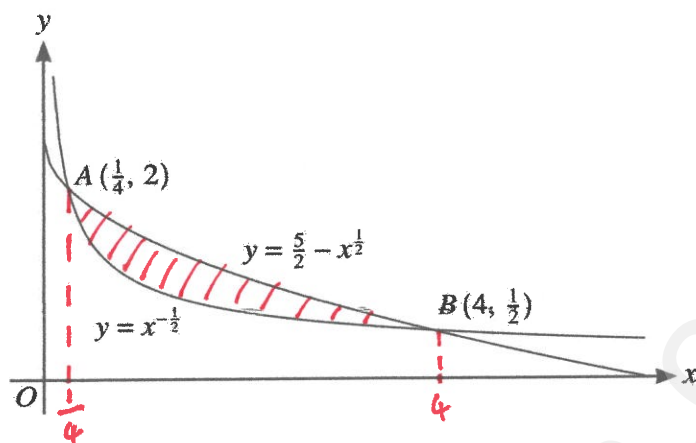
$$= \left[-\frac{x^4}{4} + \frac{4}{3}x^3 - \frac{3}{2}x^2 \right]_1^3$$

$$= \left[-\frac{3^4}{4} + \frac{4}{3} \times 3^3 - \frac{3}{2} \times 3^2 \right] - \left[-\frac{1^4}{4} + \frac{4}{3} \times 1^3 - \frac{3}{2} \times 1^2 \right]$$

$$= \left[-\frac{81}{4} + 36 - \frac{27}{2} \right] - \left[-\frac{1}{4} + \frac{4}{3} - \frac{3}{2} \right]$$

$$= \frac{9}{4} - -\frac{5}{12}$$

$$= \underline{\underline{\frac{8}{3}}}$$

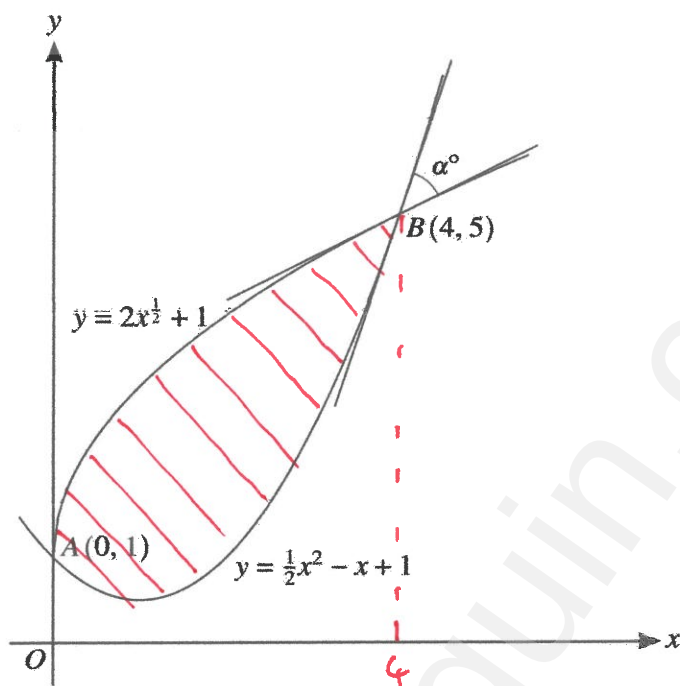


The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A(\frac{1}{4}, 2)$ and $B(4, \frac{1}{2})$.

(a) Find the area of the region between the two curves.

[6]

$$\begin{aligned}
 & \int_{\frac{1}{4}}^4 \left[\left(\frac{5}{2} - x^{\frac{1}{2}} \right) - x^{-\frac{1}{2}} \right] dx \\
 &= \int_{\frac{1}{4}}^4 \left(\frac{5}{2} - x^{\frac{1}{2}} - x^{-\frac{1}{2}} \right) dx \\
 &= \left[\frac{5}{2}x - \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} \right]_{\frac{1}{4}}^4 \\
 &= \left[\frac{5}{2}(4) - \frac{2}{3}(4)^{\frac{3}{2}} - 2(4)^{\frac{1}{2}} \right] - \left[\frac{5}{2}\left(\frac{1}{4}\right) - \frac{2}{3}\left(\frac{1}{4}\right)^{\frac{3}{2}} - 2\left(\frac{1}{4}\right)^{\frac{1}{2}} \right] \\
 &= \left[10 - \frac{2}{3} \times 8 - 2 \times 2 \right] - \left[\frac{5}{8} - \frac{2}{3} \times \frac{1}{8} - 2 \times \frac{1}{2} \right] \\
 &= \left[10 - \frac{16}{3} - 4 \right] - \left[\frac{5}{8} - \frac{1}{12} - 1 \right] \\
 &= \left[\frac{2}{3} \right] - \left[-\frac{11}{24} \right] \\
 &= \frac{9}{8}
 \end{aligned}$$



Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A(0, 1)$ and $B(4, 5)$, as shown in the diagram.

- (a) Find the area of the region between the two curves.

[5]

$$\int_0^4 \left[(2x^{\frac{1}{2}} + 1) - \left(\frac{1}{2}x^2 - x + 1 \right) \right] dx$$

$$\int_0^4 \left(2x^{\frac{1}{2}} - \frac{1}{2}x^2 + x \right) dx$$

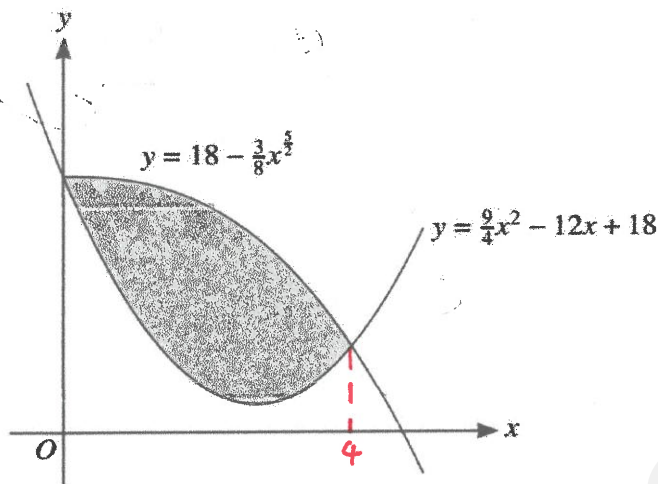
$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{6}x^3 + \frac{1}{2}x^2 \right]_0^4$$

$$= \left[\frac{4}{3}(4)^{\frac{3}{2}} - \frac{1}{6}(4)^3 + \frac{1}{2}(4)^2 \right] - [0]$$

$$= \frac{4}{3} \times 8 - \frac{1}{6} \times 64 + \frac{1}{2} \times 16$$

$$= \frac{32}{3} - \frac{32}{3} + 8$$

$$= \underline{\underline{8}}$$

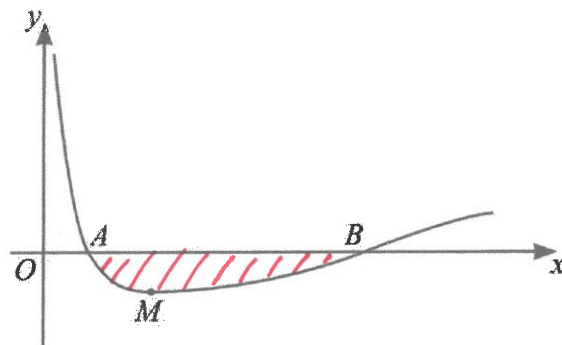


The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

(b) Find the area of the shaded region.

[5]

$$\begin{aligned}
 & \int_0^4 \left[\left(18 - \frac{3}{8}x^{\frac{5}{2}} \right) - \left(\frac{9}{4}x^2 - 12x + 18 \right) \right] dx \\
 &= \int_0^4 \left(-\frac{3}{8}x^{\frac{5}{2}} - \frac{9}{4}x^2 + 12x \right) dx \\
 &= \left[\frac{2}{7} \times -\frac{3}{8}x^{\frac{7}{2}} - \frac{1}{3} \times \frac{9}{4}x^3 + 6x^2 \right]_0^4 \\
 &= \left[-\frac{3}{28}x^{\frac{7}{2}} - \frac{3}{4}x^3 + 6x^2 \right]_0^4 \\
 &= \left[-\frac{3}{28}(4)^{\frac{7}{2}} - \frac{3}{4}(4)^3 + 6(4)^2 \right] - [0] \\
 &= -\frac{3}{28} \times 128 - \frac{3}{4} \times 64 + 6 \times 16 \\
 &= -\frac{96}{7} - 48 + 96 \\
 &= \frac{240}{7}
 \end{aligned}$$



The diagram shows the curve with equation $y = 2x^{-\frac{2}{3}} - 3x^{-\frac{1}{3}} + 1$ for $x > 0$. The curve crosses the x -axis at points A and B and has a minimum point M .

- (b) Find the area of the region bounded by the curve and the line segment AB .

[7]

Find A and B: $0 = 2x^{-\frac{2}{3}} - 3x^{-\frac{1}{3}} + 1$

Let $Y = x^{-\frac{1}{3}}$: $2Y^2 - 3Y + 1 = 0$

$$(2Y - 1)(Y - 1) = 0$$

$$Y = \frac{1}{2} \quad \text{or} \quad Y = 1$$

$$x^{-\frac{1}{3}} = \frac{1}{2} \quad x^{-\frac{1}{3}} = 1$$

$$x = 8$$

$$x = 1$$

$$\int_1^8 (2x^{-\frac{2}{3}} - 3x^{-\frac{1}{3}} + 1) dx$$

$$= \left[3 \times 2x^{\frac{1}{3}} - \frac{3}{2} \times 3x^{\frac{2}{3}} + x \right]_1^8$$

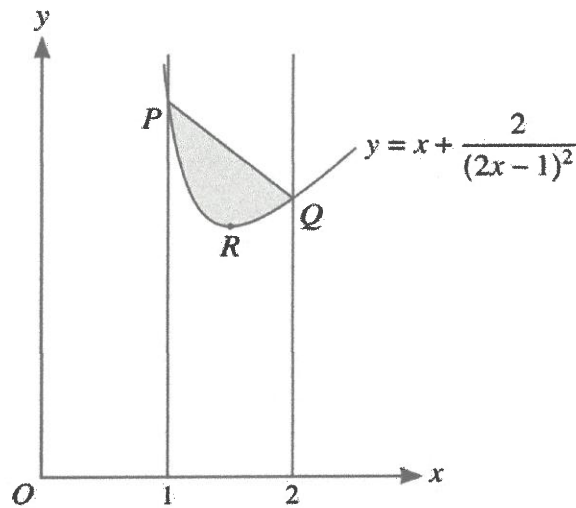
$$= \left[6x^{\frac{1}{3}} - \frac{9}{2}x^{\frac{2}{3}} + x \right]_1^8$$

$$= \left[6(8)^{\frac{1}{3}} - \frac{9}{2}(8)^{\frac{2}{3}} + 8 \right] - \left[6(1)^{\frac{1}{3}} - \frac{9}{2}(1)^{\frac{2}{3}} + 1 \right]$$

$$= \left[6 \times 2 - \frac{9}{2} \times 4 + 8 \right] - \left[6 - \frac{9}{2} + 1 \right]$$

$$= [2] - \left[\frac{5}{2} \right] = -0.5$$

Area can't be negative so = 0.5



The diagram shows part of the curve with equation $y = x + \frac{2}{(2x-1)^2}$. The lines $x = 1$ and $x = 2$ intersect the curve at P and Q respectively and R is the stationary point on the curve.

- (b) Find the exact value of the area of the shaded region.

[6]

Find coords of P and Q:

$$x=1: y = 1 + \frac{2}{(2-1)^2}$$

$$x=2: y = 2 + \frac{2}{(4-1)^2}$$

$$= 1 + \frac{2}{1}$$

$$= 2 + \frac{2}{9}$$

$$= 3$$

$$= \frac{20}{9}$$

Shaded Area = Area of Trapezium - Area under Curve

$$\text{Area of Trapezium} = \frac{1}{2} \left(3 + \frac{20}{9} \right) \times 1$$

$$= \frac{47}{18}$$

Area under Curve:

$$\int_1^2 \left(x + \frac{2}{(2x-1)^2} \right) dx$$

$$= \left[\frac{1}{2} x^2 - 1 \times \frac{1}{2} \times 2(2x-1)^{-1} \right]_1^2$$

$$= \left[\frac{1}{2}x^2 - (2x-1)^{-1} \right]_1^2$$

$$= \left[\frac{1}{2}(2)^2 - (4-1)^{-1} \right] - \left[\frac{1}{2}(1)^2 - (2-1)^{-1} \right]$$

$$= \left[\frac{1}{2} \times 4 - 3^{-1} \right] - \left[\frac{1}{2} \times 1 - 1^{-1} \right]$$

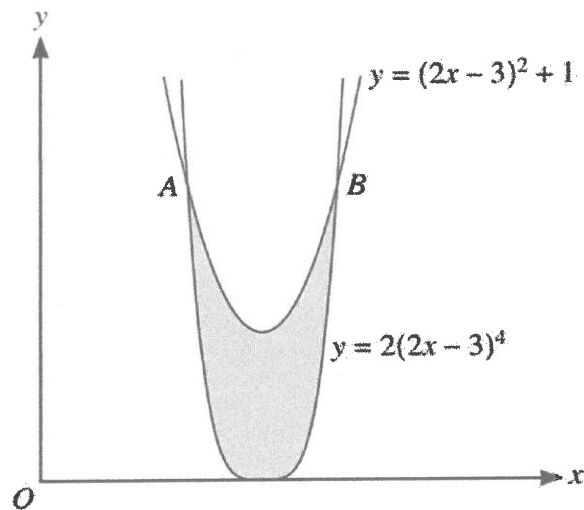
$$= \left[2 - \frac{1}{3} \right] - \left[\frac{1}{2} - 1 \right]$$

$$= \frac{5}{3} - - \frac{1}{2}$$

$$= \frac{13}{6}$$

$$\text{Shaded Area} = \frac{47}{18} - \frac{13}{6}$$

$$= \frac{4}{9}$$



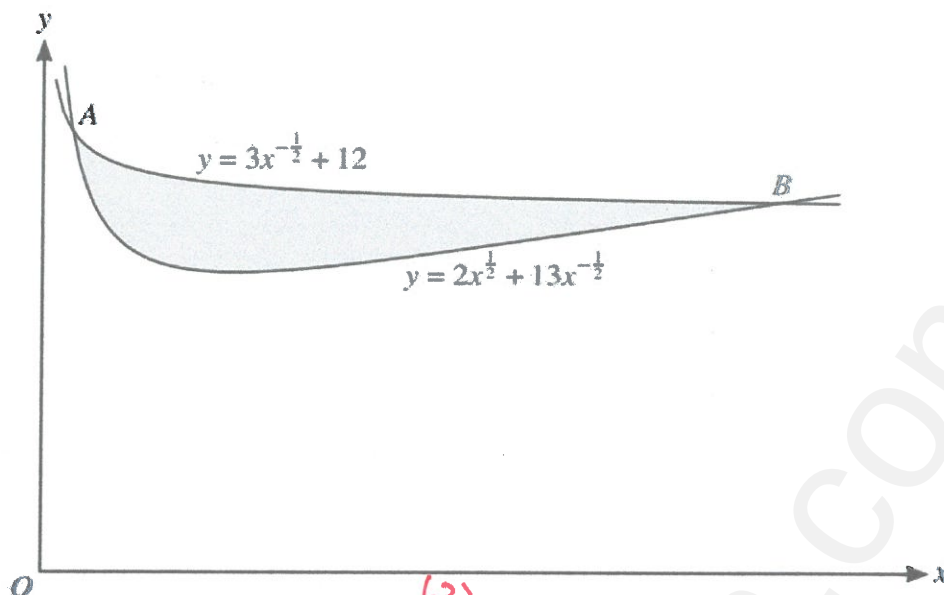
from (a) A: (1, 2) B: (2, 2)

The diagram shows the curves with equations $y = 2(2x - 3)^4$ and $y = (2x - 3)^2 + 1$ meeting at points A and B.

(b) Find the exact area of the shaded region.

[5]

$$\begin{aligned}
 \text{Area} &= \int_1^2 ((2x-3)^2 + 1 - 2(2x-3)^4) dx \\
 &= \left[\frac{1}{3} \times \frac{1}{2} (2x-3)^3 + x - 2 \times \frac{1}{5} \times \frac{1}{2} (2x-3)^5 \right]_1^2 \\
 &= \left[\frac{1}{6} (2x-3)^3 + x - \frac{1}{5} (2x-3)^5 \right]_1^2 \\
 &= \left[\frac{1}{6} (2(2)-3)^3 + 2 - \frac{1}{5} (2(2)-3)^5 \right] - \left[\frac{1}{6} (2(1)-3)^3 + 1 - \frac{1}{5} (2(1)-3)^5 \right] \\
 &= \left[\frac{1}{6} \times 1^3 + 2 - \frac{1}{5} \times 1^5 \right] - \left[\frac{1}{6} \times (-1)^3 + 1 - \frac{1}{5} (-1)^5 \right] \\
 &= \left[\frac{1}{6} + 2 - \frac{1}{5} \right] - \left[-\frac{1}{6} + 1 + \frac{1}{5} \right] \\
 &= \frac{59}{30} - \frac{31}{30} \\
 &= \frac{28}{30} \\
 &= \frac{14}{15}
 \end{aligned}$$



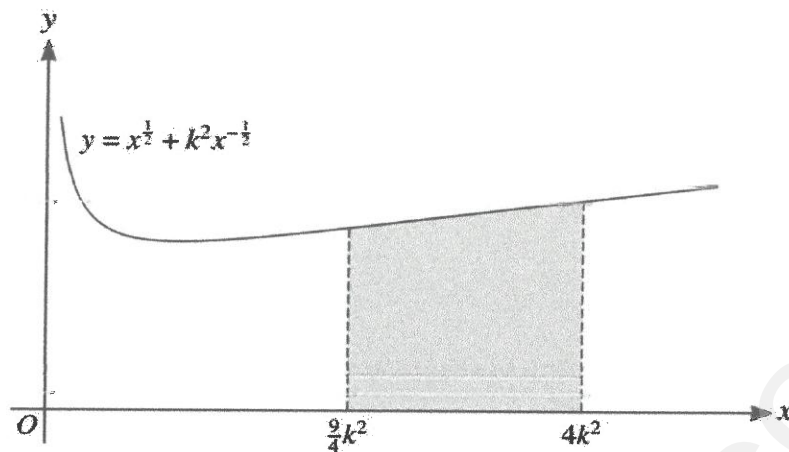
from (a): A: (1, 15) B: (25, $\frac{63}{5}$)

The diagram shows curves with equations $y = 2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}}$ and $y = 3x^{-\frac{1}{2}} + 12$. The curves intersect at points A and B.

(b) Hence find the area of the shaded region.

[5]

$$\begin{aligned}
 & \int_1^{25} (3x^{-\frac{1}{2}} + 12) - (2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}}) dx \\
 &= \int_1^{25} (-10x^{-\frac{1}{2}} + 12 - 2x^{\frac{1}{2}}) dx \\
 &= \left[-20x^{\frac{1}{2}} + 12x - \frac{4}{3}x^{\frac{3}{2}} \right]_1^{25} \\
 &= \left[-20(25)^{\frac{1}{2}} + 12(25) - \frac{4}{3}(25)^{\frac{3}{2}} \right] - \left[-20(1)^{\frac{1}{2}} + 12(1) - \frac{4}{3}(1)^{\frac{3}{2}} \right] \\
 &= \left[-20 \times 5 + 300 - \frac{4}{3} \times 125 \right] - \left[-20 + 12 - \frac{4}{3} \right] \\
 &= \left[-100 + 300 - \frac{500}{3} \right] - \left[-\frac{28}{3} \right] \\
 &= \frac{100}{3} + \frac{28}{3} \\
 &= \frac{128}{3}
 \end{aligned}$$



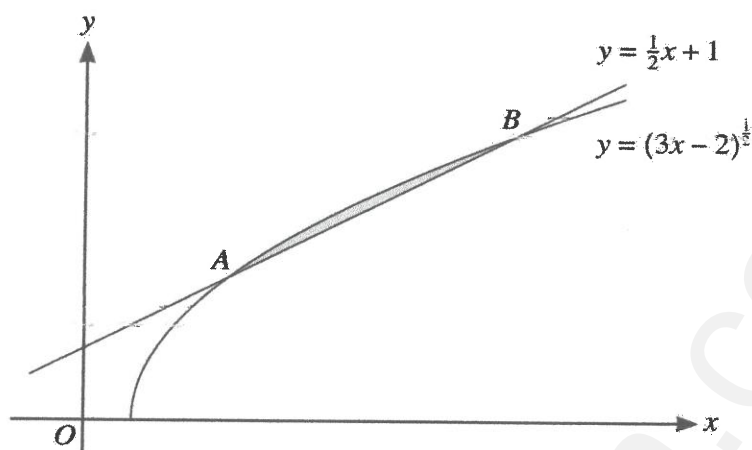
The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2 x^{-\frac{1}{2}}$, where k is a positive constant.

The shaded region is bounded by the curve, the x -axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k .

[3]

$$\begin{aligned}
 & \int_{\frac{9}{4}k^2}^{4k^2} (x^{\frac{1}{2}} + k^2 x^{-\frac{1}{2}}) dx \\
 &= \left[\frac{2}{3} x^{\frac{3}{2}} + 2k^2 x^{\frac{1}{2}} \right]_{\frac{9}{4}k^2}^{4k^2} \\
 &= \left[\frac{2}{3} (4k^2)^{\frac{3}{2}} + 2k^2 (4k^2)^{\frac{1}{2}} \right] - \left[\frac{2}{3} \left(\frac{9}{4}k^2 \right)^{\frac{3}{2}} + 2k^2 \left(\frac{9}{4}k^2 \right)^{\frac{1}{2}} \right] \\
 &= \left[\frac{2}{3} (2k)^3 + 2k^2 (2k) \right] - \left[\frac{2}{3} \left(\frac{3}{2}k \right)^3 + 2k^2 \left(\frac{3}{2}k \right) \right] \\
 &= \left[\frac{2}{3} \times 8k^3 + 4k^3 \right] - \left[\frac{2}{3} \times \frac{27}{8}k^3 + 3k^3 \right] \\
 &= \frac{16}{3}k^3 + 4k^3 - \frac{9}{4}k^3 - 3k^3 \\
 &= \frac{49}{12}k^3
 \end{aligned}$$



The diagram shows the curve with equation $y = (3x - 2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B.

[4]

Solve simultaneously:

$$(3x - 2)^{\frac{1}{2}} = \frac{1}{2}x + 1$$

$$\sqrt{3x - 2} = \frac{x}{2} + 1$$

$$3x - 2 = \left(\frac{x}{2} + 1\right)^2$$

$$3x - 2 = \frac{x^2}{4} + x + 1$$

$$\frac{x^2}{4} - 2x + 3 = 0$$

$$x^2 - 8x + 12 = 0$$

$$(x - 6)(x - 2) = 0$$

$$x = 6 \text{ or } x = 2$$

Sub. $x = 2$ into y :

$$\begin{aligned} y &= (3 \times 2 - 2)^{\frac{1}{2}} \\ &= (4)^{\frac{1}{2}} \\ &= 2 \end{aligned}$$

$$\rightarrow \underline{(2, 2)}_A$$

Sub. $x = 6$ into y :

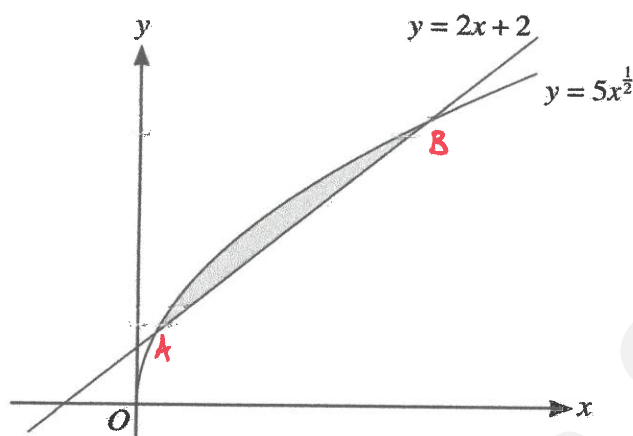
$$\begin{aligned} y &= (3 \times 6 - 2)^{\frac{1}{2}} \\ &= (16)^{\frac{1}{2}} \\ &= 4 \end{aligned}$$

$$\rightarrow \underline{(6, 4)}_B$$

(b) Hence find the area of the region enclosed between the curve and the line.

[5]

$$\begin{aligned}
 & \int_2^6 \left[(3x-2)^{\frac{1}{2}} - \left(\frac{1}{2}x + 1 \right) \right] dx \\
 & \quad \quad \quad \uparrow \text{curve} \quad \quad \quad \uparrow \text{line} \\
 & = \left[\frac{2}{3} \times \frac{1}{3} (3x-2)^{\frac{3}{2}} - \left(\frac{1}{4}x^2 + x \right) \right]_2^6 \\
 & = \left[\frac{2}{9} (3x-2)^{\frac{3}{2}} - \frac{1}{4}x^2 - x \right]_2^6 \\
 & = \left[\frac{2}{9} (3(6)-2)^{\frac{3}{2}} - \frac{1}{4}(6)^2 - 6 \right] - \left[\frac{2}{9} (3(2)-2)^{\frac{3}{2}} - \frac{1}{4}(2)^2 - 2 \right] \\
 & = \left[\frac{2}{9} (16)^{\frac{3}{2}} - \frac{1}{4} \times 36 - 6 \right] - \left[\frac{2}{9} (4)^{\frac{3}{2}} - \frac{1}{4} \times 4 - 2 \right] \\
 & = \left[\frac{2}{9} \times 64 - 9 - 6 \right] - \left[\frac{2}{9} \times 8 - 1 - 2 \right] \\
 & = \left[\frac{-7}{9} \right] - \left[\frac{-11}{9} \right] \\
 & = \frac{4}{9}
 \end{aligned}$$



The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve.

[5]

Find points of intersection:

$$5x^{\frac{1}{2}} = 2x + 2$$

$$x^{\frac{1}{2}} = \frac{2x + 2}{5}$$

Square both sides...

$$x = \frac{4x^2 + 8x + 4}{25}$$

$$4x^2 + 8x + 4 = 25x$$

$$4x^2 - 17x + 4 = 0$$

$$(4x - 1)(x - 4) = 0$$

$$x = \frac{1}{4} \text{ or } x = 4$$

$$\rightarrow \int_{\frac{1}{4}}^4 [5x^{\frac{1}{2}} - (2x + 2)] dx$$

$\uparrow$ curve $\uparrow$ line

$$\left[\frac{2}{3} \times 5x^{\frac{3}{2}} - x^2 - 2x \right]_{\frac{1}{4}}^4$$

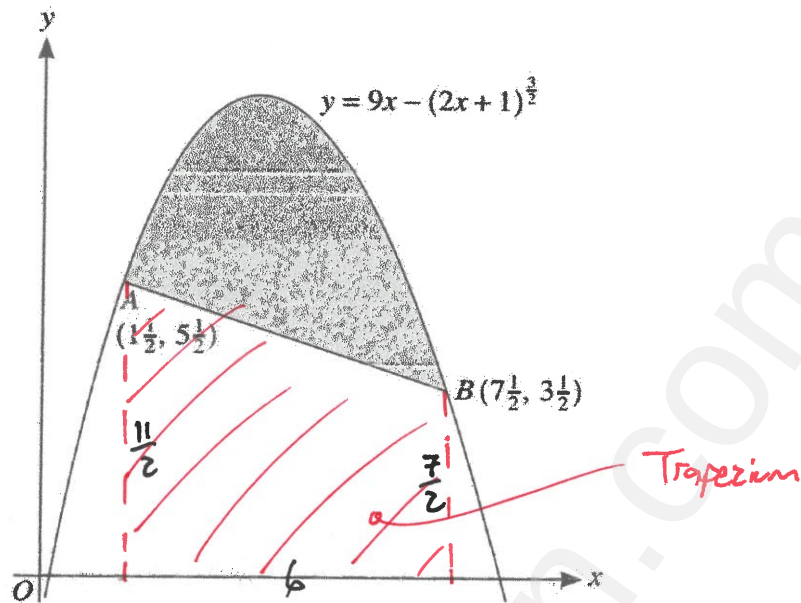
$$\left[\frac{10}{3}x^{\frac{3}{2}} - x^2 - 2x \right]_{\frac{1}{4}}^4$$

$$\left[\frac{10}{3}(4)^{\frac{3}{2}} - (4)^2 - 2(4) \right] - \left[\frac{10}{3}\left(\frac{1}{4}\right)^{\frac{3}{2}} - \left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right) \right]$$

$$= \left[\frac{10}{3} \times 8 - 16 - 8 \right] - \left[\frac{10}{3} \times \frac{1}{8} - \frac{1}{16} - \frac{1}{2} \right]$$

$$= \left[\frac{8}{3} \right] - \left[-\frac{7}{48} \right]$$

$$= \frac{45}{16}$$



The diagram shows the points $A(1\frac{1}{2}, 5\frac{1}{2})$ and $B(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x + 1)^{\frac{3}{2}}$.

(c) Find the area of the shaded region.

[5]

Area under curve between A & B:

$$\int_{\frac{3}{2}}^{\frac{15}{2}} (9x - (2x + 1)^{\frac{3}{2}}) dx$$

$$= \left[\frac{9}{2}x^2 - \frac{2}{5} \times \frac{1}{2} (2x + 1)^{\frac{5}{2}} \right]_{\frac{3}{2}}^{\frac{15}{2}}$$

$$= \left[\frac{9}{2} \left(\frac{15}{2} \right)^2 - \frac{1}{5} (2 \left(\frac{15}{2} \right) + 1)^{\frac{5}{2}} \right] - \left[\frac{9}{2} \left(\frac{3}{2} \right)^2 - \frac{1}{5} (2 \left(\frac{3}{2} \right) + 1)^{\frac{5}{2}} \right]$$

$$= \left[\frac{9}{2} \times \frac{225}{4} - \frac{1}{5} (16)^{\frac{5}{2}} \right] - \left[\frac{9}{2} \times \frac{9}{4} - \frac{1}{5} (4)^{\frac{5}{2}} \right]$$

$$= \left[\frac{2025}{8} - \frac{1024}{5} \right] - \left[\frac{81}{8} - \frac{32}{5} \right]$$

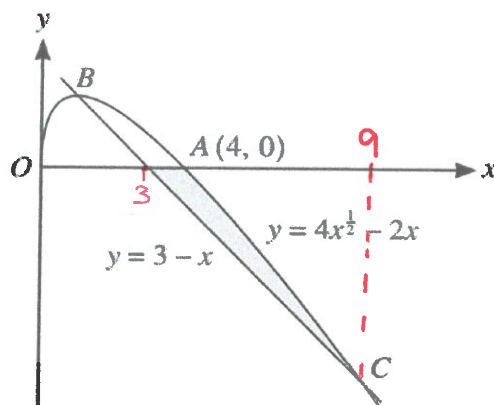
$$= \left[\frac{1933}{40} \right] - \left[\frac{149}{40} \right] = \frac{223}{5}$$

Area of trapezium:

$$\begin{aligned} \text{Area} &= \frac{1}{2}(a+b) \times h \\ &= \frac{1}{2} \left(\frac{11}{2} + \frac{7}{2} \right) \times 6 \\ &= \frac{1}{2}(9) \times 6 \\ &= 27 \end{aligned}$$

→ Shaded Area:

$$\frac{223}{5} - 27 = \underline{\underline{17.6}}$$



Shaded Area =

Area between $y = 3 - x$ and x -axis

– Area between $y = 4x^{1/2} - 2x$ and x -axis

from (a): x-coordinate of C = 9

The diagram shows a curve with equation $y = 4x^{1/2} - 2x$ for $x \geq 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x -axis at A (4, 0) and crosses the straight line at B and C.

(c) Find the area of the shaded region.

[6]

Area of triangle made by $y = 3 - x$:

x -coord of x -intercept: $0 = 3 - x \rightarrow x = 3$

y -coord of point C: $y = 3 - 9 \rightarrow y = -6$

$$\begin{aligned} \rightarrow \text{Area of triangle} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 6 \times 6 \\ &= 18 \end{aligned}$$

Area between curve and x -axis: (limits are $x = 4$ and $x = 9$)

$$\int_4^9 (4x^{1/2} - 2x) dx$$

$$= \left[\frac{2}{3} \times 4x^{3/2} - x^2 \right]_4^9$$

$$= \left[\frac{2}{3} \times 4(9)^{3/2} - (9)^2 \right] - \left[\frac{2}{3} \times 4(4)^{3/2} - (4)^2 \right]$$

$$= \left[\frac{8}{3} \times 27 - 81 \right] - \left[\frac{8}{3} \times 8 - 16 \right]$$

$$= [-9] - \left[\frac{16}{3} \right]$$

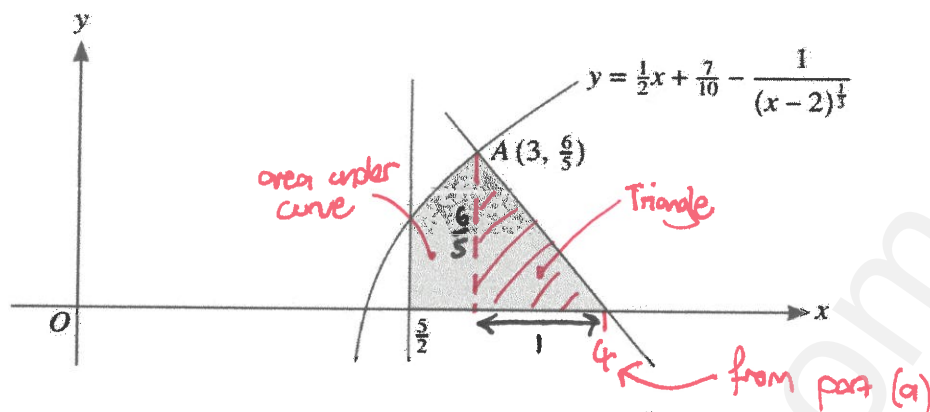
$$= -\frac{43}{3}$$

negative because below axis, so area = $\frac{43}{3}$

→ Shaded area =

$$18 - \frac{43}{3}$$

$$= \frac{11}{3}$$



The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A(3, \frac{6}{5})$.

- (b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

Area of triangle: $\text{Area} = \frac{1}{2} \times b \times h$
 $= \frac{1}{2} \times 1 \times \frac{6}{5}$
 $= 0.6$

Area under curve: $\int_{\frac{5}{2}}^3 \left(\frac{1}{2}x + \frac{7}{10} - (x-2)^{-\frac{1}{3}} \right) dx$
 $= \left[\frac{1}{4}x^2 + \frac{7}{10}x - \frac{3}{2}(x-2)^{\frac{2}{3}} \right]_{\frac{5}{2}}^3$
 $= \left[\frac{1}{4}(3)^2 + \frac{7}{10}(3) - \frac{3}{2}(1)^{\frac{2}{3}} \right] - \left[\frac{1}{4}\left(\frac{5}{2}\right)^2 + \frac{7}{10}\left(\frac{5}{2}\right) - \frac{3}{2}\left(\frac{1}{2}\right)^{\frac{2}{3}} \right]$
 $= \left[\frac{9}{4} + \frac{21}{10} - \frac{3}{2} \right] - \left[\frac{1}{4} \times \frac{25}{4} + \frac{35}{20} - \frac{3}{2} \times 0.5^{\frac{2}{3}} \right]$
 $= \left[\frac{57}{20} \right] - \left[\frac{53}{16} - \frac{3}{2}(0.5)^{\frac{2}{3}} \right]$
 $= -\frac{37}{80} + \frac{3}{2}(0.5)^{\frac{2}{3}}$
 $= 0.482$ *sto*

Shaded Area: $0.6 + 0.482 = \underline{\underline{1.08}}$