

- 1 Find the exact value of $\int_3^{\infty} \frac{2}{x^2} dx$.

[3]

$$\int_3^{\infty} 2x^{-2} dx = \left[-2x^{-1} \right]_3^{\infty}$$

$$= \left[\frac{-2}{x} \right]_3^{\infty}$$

$$= \left[\frac{-2}{\infty} \right] - \left[\frac{-2}{3} \right]$$

$$= \left[0 \right] - \left[\frac{-2}{3} \right]$$

$$= \underline{\underline{\frac{2}{3}}}$$

- 2 The function f is defined by $f(x) = \frac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\int_1^{\infty} f(x) dx$.

[3]

$$\begin{aligned} & \int_1^{\infty} 2(x+2)^{-2} dx \\ &= \left[-2(x+2)^{-1} \right]_1^{\infty} \\ &= \left[-2(\infty+2)^{-1} \right] - \left[-2(1+2)^{-1} \right] \\ &= \left[-2(\infty)^{-1} \right] - \left[-2(3)^{-1} \right] \\ & \quad \quad \quad \uparrow = 0 \\ &= \left[0 \right] - \left[-\frac{2}{3} \right] \\ &= \underline{\underline{\frac{2}{3}}} \end{aligned}$$

10 The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^{\infty} f(x) dx$.

[4]

$$\begin{aligned}
 & \int_1^{\infty} (4x + 2)^{-2} dx \\
 &= \left[-\frac{1}{4} (4x + 2)^{-1} \right]_1^{\infty} \\
 &= \left[-\frac{1}{4} (4(\infty) + 2)^{-1} \right] - \left[-\frac{1}{4} (4(1) + 2)^{-1} \right] \\
 &= \left[-\frac{1}{4} (\infty)^{-1} \right] - \left[-\frac{1}{4} (6)^{-1} \right] \\
 &= \left[0 \right] - \left[-\frac{1}{24} \right] \\
 &= \frac{1}{24}
 \end{aligned}$$

10 (a) Find $\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}} dx$.

[4]

$$\int_1^{\infty} (3x-2)^{-\frac{3}{2}} dx$$

$$= \left[-2 \times \frac{1}{3} (3x-2)^{-\frac{1}{2}} \right]_1^{\infty}$$

$$= \left[-\frac{2}{3} (3x-2)^{-\frac{1}{2}} \right]_1^{\infty}$$

$$= \left[-\frac{2}{3} (3(\infty)-2)^{-\frac{1}{2}} \right] - \left[-\frac{2}{3} (3(1)-2)^{-\frac{1}{2}} \right]$$

$$= \left[-\frac{2}{3} (\infty)^{-\frac{1}{2}} \right] - \left[-\frac{2}{3} (1)^{-\frac{1}{2}} \right]$$

$$= \left[0 \right] - \left[-\frac{2}{3} \right]$$

$$= \underline{\underline{\frac{2}{3}}}$$