

- 4 Three fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.

(a) Find the probability that the score is 4 on a single throw of the three dice.

[3]

A	B	C	
1	1	2	}
1	2	1	
2	1	1	

$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$$3 \times \frac{1}{216} = \underline{\underline{\frac{1}{72}}}$$

- (b) Find the probability that a score of 18 is obtained for the first time on the 5th throw of the three dice. [3]

A B C

$$6 \quad 6 \quad 6 \quad \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$$X \sim \text{Geo} \left(\frac{1}{216} \right)$$

$$P(X=5) = \left(\frac{1}{216} \right) \left(\frac{215}{216} \right)^4$$

$$= \underline{0.00454}$$

- 1 Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time.

(a) Find the probability that two tails are obtained for the first time on the 7th throw. [2]

$$P(TT) = \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

$$X \sim \text{Geo}(0.25)$$

$$P(X=7) = 0.25 \times 0.75^6$$

$$= \underline{\underline{0.0445}}$$

(b) Find the probability that it takes more than 9 throws to obtain two tails for the first time. [2]

$$P(X > 9) = 9^9$$

← probability of 9 failures

$$= 0.75^9$$

$$= \underline{\underline{0.0751}}$$

- 1 Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X .

(a) State the value of $E(X)$.

[1]

$$P(H \cap H) = \frac{1}{2} \times \frac{1}{2} \\ = \frac{1}{4}$$

$$X \sim \text{Geo}\left(\frac{1}{4}\right)$$

$$\mu = \frac{1}{\left(\frac{1}{4}\right)} = \underline{\underline{4}}$$

(b) Find the probability that exactly 5 throws are required to obtain a pair of heads.

[1]

$$P(X=5) = \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4$$

$$= \underline{\underline{0.0791}}$$

(c) Find the probability that fewer than 7 throws are required to obtain a pair of heads.

[2]

$$P(X < 7) = P(X \leq 6)$$

$$= 1 - 9^6$$

↑ probability of 6 failures

$$= 1 - \left(\frac{3}{4}\right)^6$$

$$= \underline{\underline{0.822}}$$

- 1 A fair spinner with 5 sides numbered 1, 2, 3, 4, 5 is spun repeatedly. The score on each spin is the number on the side on which the spinner lands.

(a) Find the probability that a score of 3 is obtained for the first time on the 8th spin. [1]

$$X \sim \text{Geo}(0.2)$$

$$\uparrow P(3) = \frac{1}{5}$$

$$P(X=8) = 0.2 \times 0.8^7$$

$$= \underline{\underline{0.0419}}$$

(b) Find the probability that fewer than 6 spins are required to obtain a score of 3 for the first time. [2]

$$P(X < 6) = P(X \leq 5)$$

$$= 1 - 0.8^5$$

$\uparrow$ probability of 5 failures

$$= 1 - 0.8^5$$

$$= \underline{\underline{0.67232}}$$

- 2 An ordinary fair die is thrown until a 6 is obtained.

- (a) Find the probability that obtaining a 6 takes more than 8 throws.

[2]

$$D \sim \text{Geo}\left(\frac{1}{6}\right)$$

$$P(D > 8) = 9^8$$

$$= \left(\frac{5}{6}\right)^8$$

↑ probability of 8 failures

$$= \underline{\underline{0.233}}$$

Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X .

- (b) Find the expected value of X .

[1]

$$P(6 \cap 6) = \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{36}$$

$$X \sim \text{Geo}\left(\frac{1}{36}\right)$$

$$\mu = \frac{1}{p} = \frac{1}{\frac{1}{36}}$$

$$= \underline{\underline{36}}$$

- (c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws.

[2]

$$X \sim \text{Geo}\left(\frac{1}{36}\right)$$

$$P(X = 10) + P(X = 11) = \left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^9 + \left(\frac{1}{36}\right)\left(\frac{35}{36}\right)^{10}$$

$$= \underline{\underline{0.0425}}$$

- 1 The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

(a) Show that the probability that the score is 4 is $\frac{1}{12}$.

[1]

$$\begin{aligned}
 P(\text{score of } 4) &= P(1,3) + P(3,1) + P(2,2) \\
 &= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \\
 &= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} \\
 &= \underline{\underline{\frac{1}{12}}}
 \end{aligned}$$

The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X .

(b) Find the mean of X .

[1]

$$\begin{aligned}
 X &\sim \text{Geo}\left(\frac{1}{12}\right) \\
 \mu &= \frac{1}{p} = \frac{1}{\frac{1}{12}} \\
 &= \underline{\underline{12}}
 \end{aligned}$$

(c) Find the probability that a score of 4 is first obtained on the 6th throw.

[1]

$$\begin{aligned}
 P(X=6) &= \frac{1}{12} \times \left(\frac{11}{12}\right)^5 \\
 &= \underline{\underline{0.0539}}
 \end{aligned}$$

(d) Find $P(X < 8)$.

[2]

$$\begin{aligned}
 P(X < 8) &= P(X \leq 7) \\
 &= 1 - \left(\frac{11}{12}\right)^7 \\
 &= 1 - \left(\frac{11}{12}\right)^7 \quad \text{probability of 7 failures} \\
 &= \underline{\underline{0.456}}
 \end{aligned}$$

- 3 Three fair 6-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time repeatedly. The score on each throw is the sum of the numbers on the uppermost faces.

(a) Find the probability that a score of 17 or more is first obtained on the 6th throw. [3]

$$\begin{array}{ccc}
 \text{A} & \text{B} & \text{C} \\
 6 & 6 & 6 \\
 6 & 6 & 5 \\
 6 & 5 & 6 \\
 5 & 6 & 6
 \end{array}
 \left. \vphantom{\begin{array}{ccc} 6 & 6 & 6 \\ 6 & 6 & 5 \\ 6 & 5 & 6 \\ 5 & 6 & 6 \end{array}} \right\} = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$$4 \times \frac{1}{216} = \frac{1}{54}$$

$$X \sim \text{Geo}\left(\frac{1}{54}\right)$$

$$P(X=6) = \left(\frac{1}{54}\right) \times \left(\frac{53}{54}\right)^5$$

$$= \underline{\underline{0.0169}}$$

(b) Find the probability that a score of 17 or more is obtained in fewer than 8 throws. [2]

$$P(X < 8) = P(X \leq 7)$$

$$= 1 - 9^7$$

↑ probability of 7 failures

$$= 1 - \left(\frac{53}{54}\right)^7$$

$$= \underline{\underline{0.123}}$$

- 4 Three fair 4-sided spinners each have sides labelled 1, 2, 3, 4. The spinners are spun at the same time and the number on the side on which each spinner lands is recorded. The random variable X denotes the highest number recorded.

(a) Show that $P(X = 2) = \frac{7}{64}$.

[3]

A	B	C	
2	2	2	$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$ $= \frac{1}{64}$
2	2	1	
2	1	2	
1	2	2	
2	1	1	
1	2	1	
1	1	2	

$7 \times \frac{1}{64} = \frac{7}{64}$ QED

(b) Complete the probability distribution table for X .

[2]

x	1	2	3	4
$P(X = x)$	$\frac{1}{64}$	$\frac{7}{64}$	$\frac{19}{64}$	$\frac{37}{64}$

$$P(X=1) = P(1,1,1)$$

$$= \frac{1}{64}$$

$$P(X=4) = 1 - \left(\frac{1}{64} + \frac{7}{64} + \frac{19}{64} \right)$$

$$= 1 - \frac{27}{64}$$

$$= \frac{37}{64}$$

On another occasion, one of the fair 4-sided spinners is spun repeatedly until a 3 is obtained. The random variable Y is the number of spins required to obtain a 3.

(c) Find $P(Y = 6)$.

[1]

$$Y \sim \text{Geo}(0.25)$$

↑ probability of getting a 3

$$P(Y=6) = 0.25 \times 0.75^5$$

$$= \underline{0.0593}$$

(d) Find $P(Y > 4)$.

[2]

$$P(Y > 4) = 9^4$$

↑ probability of 4 failures

$$= 0.75^4$$

$$= \underline{\frac{81}{256}}$$

- 1 An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X .

(a) Write down the mean of X .

[1]

$$P(5) = \frac{1}{6} \quad \mu = \frac{1}{p}$$

$$= \frac{1}{\frac{1}{6}} = \underline{6}$$

(b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw.

[2]

$$P(3 < X < 8) = P(4 \leq X \leq 7)$$

$$= P(X \leq 7) - P(X \leq 3)$$

$$= (1 - q^7) - (1 - q^3)$$

7 failures $\rightarrow$ $\leftarrow$ 3 failures

$$= \left(1 - \left(\frac{5}{6}\right)^7\right) - \left(1 - \left(\frac{5}{6}\right)^3\right)$$

$$= \underline{0.300}$$

(c) Find the probability that a 5 is first obtained in fewer than 10 throws.

[2]

$$P(X < 10) = P(X \leq 9)$$

$$= 1 - q^9$$

$\leftarrow$ probability of 9 failures

$$= 1 - \left(\frac{5}{6}\right)^9$$

$$= \underline{0.806}$$