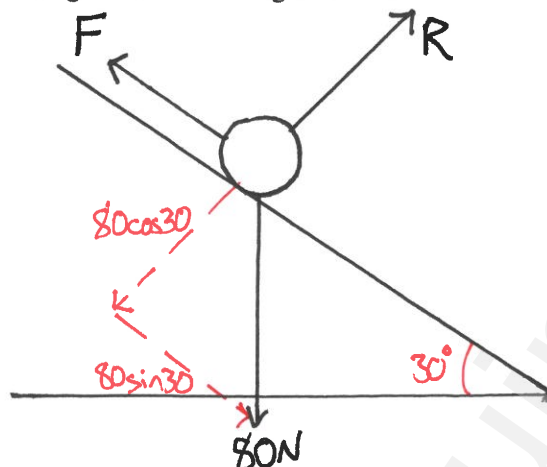


- 3 A block of mass 8 kg slides down a rough plane inclined at 30° to the horizontal, starting from rest. The coefficient of friction between the block and the plane is μ . The block accelerates uniformly down the plane at 2.4 m s^{-2} .

(a) Draw a diagram showing the forces acting on the block.

[1]



(b) Find the value of μ .

[4]

$$R(\uparrow): R - 80 \cos 30 = 0$$

$$R = 80 \cos 30$$

$$= 40\sqrt{3} \text{ N}$$

$$R(\downarrow): F = ma$$

$$80 \sin 30 - F = 8 \times 2.4$$

$$40 - \mu R = 19.2$$

$$\mu R = 20.8$$

$$\mu \times 40\sqrt{3} = 20.8$$

$$\underline{\underline{\mu = 0.300}}$$

(c) Find the speed of the block after it has moved 3 m down the plane.

[1]

$$s = 3 \quad v^2 = u^2 + 2as$$

$$u = 0 \quad v^2 = 0^2 + 2 \times 2.4 \times 3$$

$$v = \quad v^2 = 14.4$$

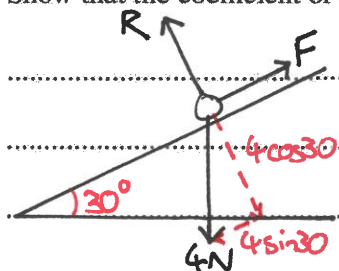
$$a = 2.4 \quad v = \underline{\underline{3.79 \text{ m s}^{-1}}}$$

$$t =$$

- 2 A particle P of mass 0.4 kg is in limiting equilibrium on a plane inclined at 30° to the horizontal.

(a) Show that the coefficient of friction between the particle and the plane is $\frac{1}{3}\sqrt{3}$.

[3]



$$R(\perp): R - 4 \cos 30 = 0$$

$$R = 2\sqrt{3} \text{ N}$$

$$R(\parallel): 4 \sin 30 - F = 0$$

$$2 - \mu R = 0$$

$$2 - 2\sqrt{3} \mu = 0$$

$$2\sqrt{3} \mu = 2$$

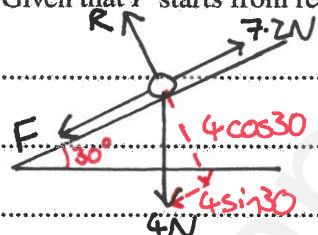
$$\mu = \frac{2}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3} \quad \text{QED}$$

A force of magnitude 7.2 N is now applied to P directly up a line of greatest slope of the plane.

(b) Given that P starts from rest, find the time that it takes for P to move 1 m up the plane.

[4]



$$R(\perp): 7.2 - 4 \sin 30 - \mu R = ma$$

$$7.2 - 2 - \frac{\sqrt{3}}{3} \times 2\sqrt{3} = 0.4a$$

$$5.2 - 2 = 0.4a$$

$$3.2 = 0.4a$$

$$a = 8 \text{ m s}^{-2}$$

$$s = 1 \quad s = ut + \frac{1}{2}at^2$$

$$u = 0 \quad 1 = 0 + \frac{1}{2}(8)t^2$$

$$v = \quad 1 = 4t^2$$

$$a = 8 \quad t^2 = \frac{1}{4}$$

$$t = \quad t = 0.5 \text{ s}$$

- 5 A straight slope of length 60 m is inclined at an angle of 12° to the horizontal. A bobsled starts at the top of the slope with a speed of 5 ms^{-1} . The bobsled slides directly down the slope.

(a) It is given that there is no resistance to the bobsled's motion.

Find its speed when it reaches the bottom of the slope.

[3]



$$R(\downarrow): F = ma$$

$$10m \sin 12 = ma$$

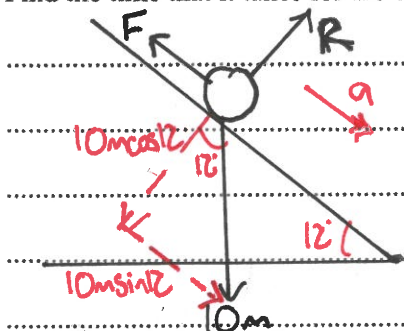
$$10 \sin 12 = a$$

$s = 60$	$v^2 = u^2 + 2as$
$u = 5$	$= 5^2 + 2 \times 10 \sin 12 \times 60$
$v =$	$v^2 = 274.494$
$a = 10 \sin 12$	$v = \underline{\underline{16.6 \text{ ms}^{-1}}}$
$t =$	

- (b) It is given instead that the coefficient of friction between the bobsled and the slope is 0.03 .

Find the time that it takes for the bobsled to reach the bottom of the slope.

[5]



$$R(\uparrow): R - 10m \cos 12 = 0$$

$$R = 10m \cos 12$$

$$R(\downarrow): F = ma$$

$$10m \sin 12 - \mu R = ma$$

$$10m \sin 12 - 0.03 \times 10m \cos 12 = ma$$

$$10m \sin 12 - 0.3m \cos 12 = ma$$

$$10 \sin 12 - 0.3 \cos 12 = a$$

$$a = 1.785 \text{ ms}^{-2} \text{ STO}$$

↓

$$S = 60$$

$$u = 5$$

$$v =$$

$$a = 1.785 \dots$$

$$t =$$

$$S = ut + \frac{1}{2}at^2$$

$$60 = 5t + \frac{1}{2} \times 1.785 \cdot t^2$$

$$0.8928t^2 + 5t - 60 = 0$$

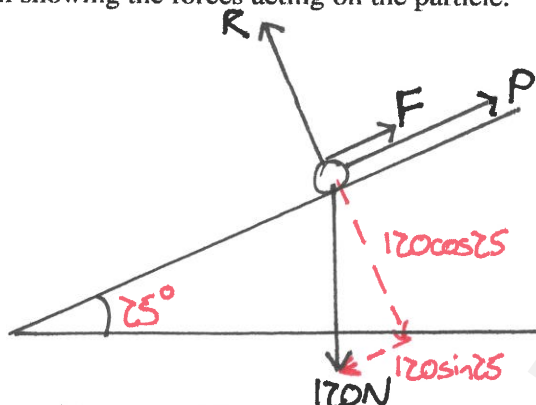
← STO

$$t = \frac{-5 \pm \sqrt{5^2 - 4(0.8928)(-60)}}{2(0.8928)}$$

$$t = \underline{5.86s} \text{ or } t = -11.5s \times$$

- 4 A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A force of magnitude P N acting parallel to a line of greatest slope of the plane is used to prevent the particle sliding down the plane. The coefficient of friction between the particle and the plane is 0.35.

(a) Draw a sketch showing the forces acting on the particle. [1]



(b) Find the least possible value of P . [5]

Least possible value of P is when particle is in limiting equilibrium, about to slide down the plane, so friction is up the plane.

$$R(\perp): R - 120 \cos 25 = 0$$

$$R = 120 \cos 25 \text{ N}$$

$$R(\parallel): 120 \sin 25 - F - P = 0$$

$$120 \sin 25 - \mu R - P = 0$$

$$120 \sin 25 - 0.35 \times 120 \cos 25 - P = 0$$

$$120 \sin 25 - 42 \cos 25 - P = 0$$

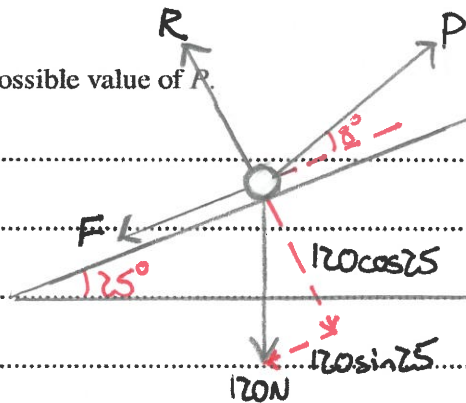
$$P = 120 \sin 25 - 42 \cos 25$$

$$= \underline{\underline{12.6 \text{ N}}}$$

- 4 A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A pulling force of magnitude P N acts at an angle of 8° above a line of greatest slope of the plane. This force is used to keep the particle in equilibrium. The coefficient of friction between the particle and the plane is 0.3.

Find the greatest possible value of P .

[6]



Greatest value of P is when particle is in limiting equilibrium, about to move up the plane, so friction is acting down the plane.

$$R(\uparrow): R + P \sin 8 - 120 \cos 25$$

$$R = 120 \cos 25 - P \sin 8$$

$$R(\rightarrow): P \cos 8 - 120 \sin 25 - F = 0$$

$$P \cos 8 - 120 \sin 25 - \mu R = 0$$

$$P \cos 8 - 120 \sin 25 - 0.3(120 \cos 25 - P \sin 8) = 0$$

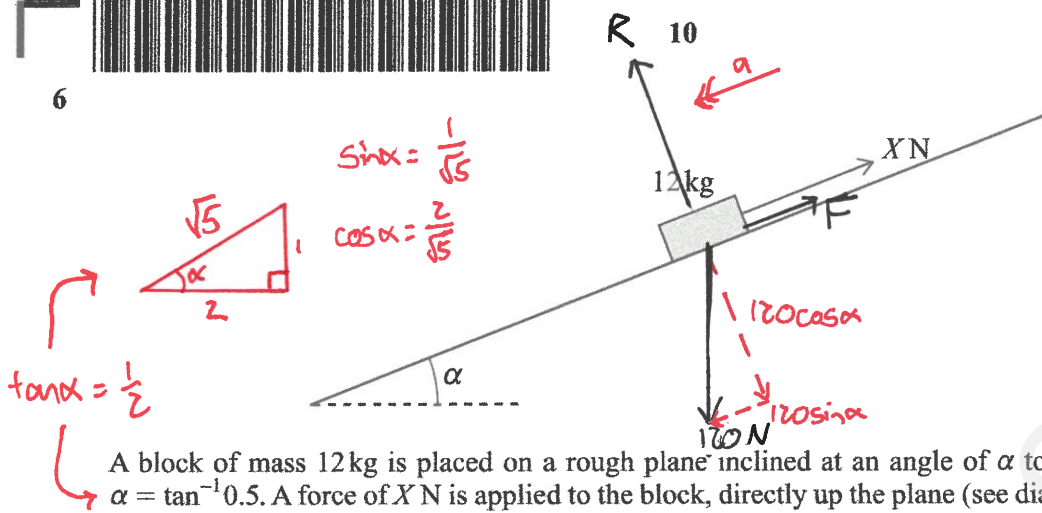
$$P \cos 8 - 120 \sin 25 - 36 \cos 25 + 0.3P \sin 8 = 0$$

$$P \cos 8 + 0.3P \sin 8 = 120 \sin 25 + 36 \cos 25$$

$$P(\cos 8 + 0.3 \sin 8) = 120 \sin 25 + 36 \cos 25$$

$$P = \frac{120 \sin 25 + 36 \cos 25}{\cos 8 + 0.3 \sin 8}$$

$$P = \underline{\underline{80.8 \text{ N}}}$$



A block of mass 12 kg is placed on a rough plane inclined at an angle of α to the horizontal, where $\alpha = \tan^{-1} 0.5$. A force of X N is applied to the block, directly up the plane (see diagram). The coefficient of friction between the block and the plane is μ .

- (a) It is given that $\mu = 0.15$ and $X = 20$.

Find the time that it takes for the block to move 2 m down the plane from rest.

[6]

Since it's moving down the plane, friction must be acting up the plane.

$$R(\uparrow): R - 120 \cos \alpha = 0$$

$$R = 120 \times \frac{2}{\sqrt{5}}$$

$$= 48\sqrt{5} \text{ N}$$

$$R(\downarrow): F = ma$$

$$120 \sin \alpha - X - F = 12a$$

$$120 \times \frac{1}{\sqrt{5}} - 20 - \mu R = 12a$$

$$24\sqrt{5} - 20 - 0.15 \times 48\sqrt{5} = 12a$$

$$17.565... = 12a$$

$$a = 1.46... \text{ ms}^{-2} \text{ STO}$$

$s = 2$	$s = ut + \frac{1}{2}at^2$
$u = 0$	$2 = 0 + \frac{1}{2} \times 1.46 \times t^2$
$v =$	$2 = 0.7319... t^2$
$a = 1.46...$	$t^2 = 2.7325...$
$t =$	$t = 1.65 \text{ s}$



- (b) It is given instead that $\mu \neq 0.15$ and that when $X = 10$, the block is on the point of moving **down** the plane.

Find the value of μ and the value of X for which the block is on the point of moving **up** the plane. [4]

On the point of moving down (F acting up):

$$R(\downarrow): \quad F = ma$$

$$120 \sin \alpha - X - F = 0 \quad \leftarrow \text{not moving}$$

$$24\sqrt{5} - 10 - \mu \times 120 \cos \alpha = 0$$

$$24\sqrt{5} - 10 - 48\sqrt{5} \mu = 0$$

$$48\sqrt{5} \mu = -10 + 24\sqrt{5}$$

$$\mu = \frac{-10 + 24\sqrt{5}}{48\sqrt{5}}$$

$$\mu = 0.407 \quad \text{STO}$$

On the point of moving up (F acting down):

$$R(\uparrow): \quad F = ma$$

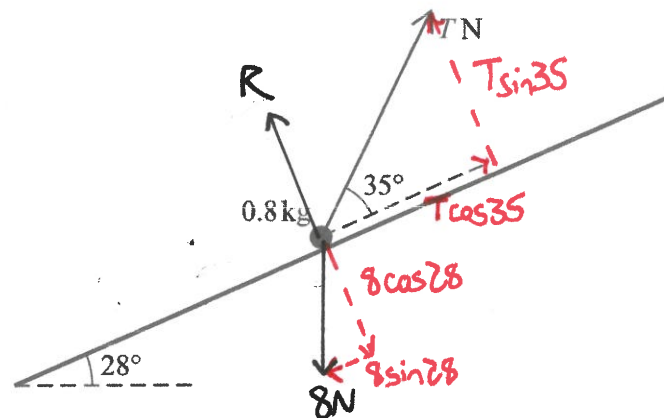
$$X - 120 \sin \alpha - F = 0$$

$$X - 24\sqrt{5} - \mu R = 0$$

$$X - 24\sqrt{5} - 0.407 \times 48\sqrt{5} = 0$$

$$X = 24\sqrt{5} + 0.407 \times 48\sqrt{5}$$

$$= 97.3 \text{ N}$$



A particle of mass 0.8 kg lies on a rough plane which is inclined at an angle of 28° to the horizontal. The particle is kept in equilibrium by a force of magnitude T N. This force acts at an angle of 35° above a line of greatest slope of the plane (see diagram). The coefficient of friction between the particle and the plane is 0.2.

Find the least and greatest possible values of T .

[8]

For greatest value of T , particle is about to move up the plane and friction is acting down the plane:

$$R(\uparrow): R + T \sin 35 - 8 \cos 28 = 0$$

$$R = 8 \cos 28 - T \sin 35$$

$$R(\nearrow): T \cos 35 - 8 \sin 28 - F = 0$$

$$T \cos 35 - 8 \sin 28 - \mu R = 0$$

$$T \cos 35 - 8 \sin 28 - 0.2(8 \cos 28 - T \sin 35) = 0$$

$$T \cos 35 - 8 \sin 28 - 1.6 \cos 28 + 0.2 T \sin 35 = 0$$

$$T \cos 35 + 0.2 T \sin 35 = 8 \sin 28 + 1.6 \cos 28$$

$$T(\cos 35 + 0.2 \sin 35) = 8 \sin 28 + 1.6 \cos 28$$

$$T = \frac{8 \sin 28 + 1.6 \cos 28}{\cos 35 + 0.2 \sin 35}$$

$$T = \underline{\underline{5.53 \text{ N}}}$$

continued...

For least value, particle is about to move down the plane and friction is acting up the plane.

$$R(\nearrow): T \cos 35 + F - 8 \sin 28 = 0$$

$$T \cos 35 + \mu R - 8 \sin 28 = 0$$

$$T \cos 35 + 0.2(8 \cos 28 - T \sin 35) - 8 \sin 28 = 0$$

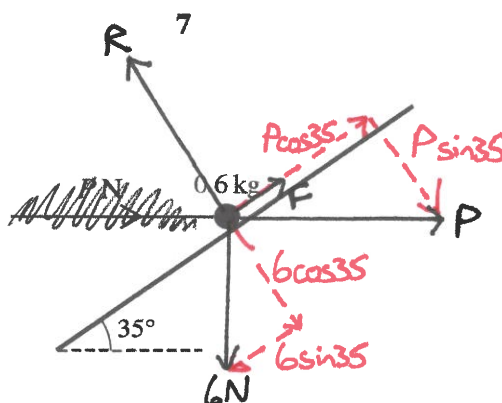
$$T \cos 35 + 1.6 \cos 28 - 0.2T \sin 35 - 8 \sin 28 = 0$$

$$T \cos 35 - 0.2T \sin 35 = 8 \sin 28 - 1.6 \cos 28$$

$$T(\cos 35 - 0.2 \sin 35) = 8 \sin 28 - 1.6 \cos 28$$

$$T = \frac{8 \sin 28 - 1.6 \cos 28}{\cos 35 - 0.2 \sin 35}$$

$$T = \underline{\underline{3.33 \text{ N}}}$$



A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of P .

[6]

For the least possible value of P , the particle is on the point of moving down the plane and friction is acting up the plane.

$$R(\perp): R - 6\cos 35 - P\sin 35 = 0$$

$$R = 6\cos 35 + P\sin 35$$

$$R(\parallel): 6\sin 35 - P\cos 35 - F = 0$$

$$6\sin 35 - P\cos 35 - \mu R = 0$$

$$6\sin 35 - P\cos 35 - 0.4(6\cos 35 + P\sin 35) = 0$$

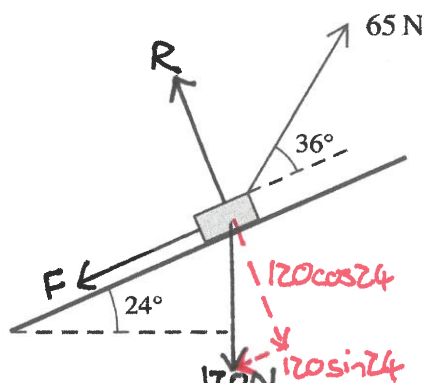
$$6\sin 35 - P\cos 35 - 2.4\cos 35 - 0.4P\sin 35 = 0$$

$$6\sin 35 - 2.4\cos 35 = P\cos 35 + 0.4P\sin 35$$

$$6\sin 35 - 2.4\cos 35 = P(\cos 35 + 0.4\sin 35)$$

$$P = \frac{6\sin 35 - 2.4\cos 35}{\cos 35 + 0.4\sin 35}$$

$$P = \underline{\underline{1.4 \text{ N}}}$$



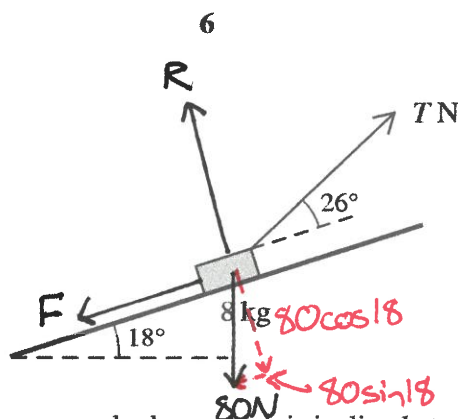
A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in the string is 65 N (see diagram). The coefficient of friction between the block and plane is μ . The block is in limiting equilibrium and is on the point of sliding up the plane.

Find μ .

[6]

$$\begin{aligned}
 R(\uparrow): R + 65 \sin 36 - 120 \cos 24 &= 0 \\
 R &= 120 \cos 24 - 65 \sin 36 \\
 &= 71.419 \text{ N} \quad \text{STO}
 \end{aligned}$$

$$\begin{aligned}
 R(\uparrow): 65 \cos 36 - 120 \sin 24 - F &= 0 \\
 65 \cos 36 - 120 \sin 24 - \mu R &= 0 \\
 65 \cos 36 - 120 \sin 24 - 71.419 \mu &= 0 \\
 71.419 \mu &= 65 \cos 36 - 120 \sin 24 \\
 \mu &= \frac{65 \cos 36 - 120 \sin 24}{71.419} \\
 \mu &= 0.0529
 \end{aligned}$$



A block of mass 8 kg is placed on a rough plane which is inclined at an angle of 18° to the horizontal. The block is pulled up the plane by a light string that makes an angle of 26° above a line of greatest slope. The tension in the string is T N (see diagram). The coefficient of friction between the block and plane is 0.65.

- (a) The acceleration of the block is 0.2 m s^{-2} .

Find T .

[7]

$$R(\perp): R + T \sin 26 - 80 \cos 18 = 0$$

$$R = 80 \cos 18 - T \sin 26$$

$$R(\parallel): T \cos 26 - 80 \sin 18 - F = ma$$

$$T \cos 26 - 80 \sin 18 - \mu R = 8 \times 0.2$$

$$T \cos 26 - 80 \sin 18 - 0.65(80 \cos 18 - T \sin 26) = 1.6$$

$$T \cos 26 - 80 \sin 18 - 52 \cos 18 + 0.65 T \sin 26 = 1.6$$

$$T \cos 26 + 0.65 T \sin 26 = 1.6 + 80 \sin 18 + 52 \cos 18$$

$$T(\cos 26 + 0.65 \sin 26) = 1.6 + 80 \sin 18 + 52 \cos 18$$

$$T = \frac{1.6 + 80 \sin 18 + 52 \cos 18}{\cos 26 + 0.65 \sin 26}$$

$$T = \underline{\underline{64.0 \text{ N}}}$$

- (b) The block is initially at rest.

Find the distance travelled by the block during the fourth second of motion.

[2]

$t=3:$

$$\begin{array}{l} S = \\ u = 0 \\ v = \\ a = 0.2 \\ t = 3 \end{array} \quad \begin{array}{l} S = ut + \frac{1}{2}at^2 \\ = 0 + \frac{1}{2}(0.2) \times 3^2 \\ S = \underline{0.9 \text{ m}} \end{array}$$

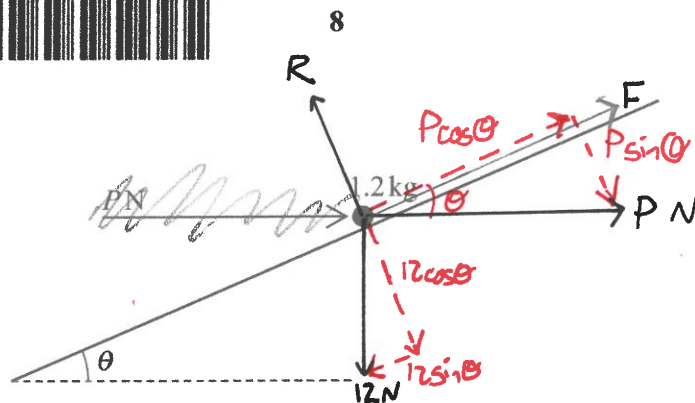
$t=4:$

$$\begin{array}{l} S = \\ u = 0 \\ v = \\ a = 0.2 \\ t = 4 \end{array} \quad \begin{array}{l} S = ut + \frac{1}{2}at^2 \\ = 0 + \frac{1}{2}(0.2) \times 4^2 \\ S = \underline{1.6 \text{ m}} \end{array}$$

so distance travelled in fourth second = $1.6 - 0.9$
 $= \underline{0.7 \text{ m}}$



6



A particle of mass 1.2 kg is placed on a rough plane which is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{7}{25}$. The particle is kept in equilibrium by a horizontal force of magnitude $P \text{ N}$ acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and the plane is 0.15 .

Find the least possible value of P .

$$\sin \theta = \frac{7}{25} \rightarrow \begin{array}{c} 25 \\ 24 \end{array} \quad \cos \theta = \frac{24}{25} \quad \tan \theta = \frac{7}{24} \quad [6]$$

P is at its least when the particle is on the point of sliding down the plane, so friction is acting up the plane.

$$R(\uparrow): R - 12 \cos \theta - P \sin \theta = 0$$

$$R - 12 \times \frac{24}{25} - P \times \frac{7}{25} = 0$$

$$R - 11.52 - \frac{7}{25}P = 0 \quad \times 25$$

$$25R - 288 - 7P = 0$$

$$25R - 7P = 288 \quad (1)$$

$$R(\downarrow): 12 \sin \theta - P \cos \theta - F = 0$$

$$12 \times \frac{7}{25} - P \times \frac{24}{25} - \mu R = 0$$

$$3.36 - \frac{24}{25}P - 0.15R = 0 \quad \times 25$$

$$84 - 24P - 3.75R = 0$$

$$3.75R + 24P = 84 \quad (2)$$

Cont...





$$\textcircled{1} \times 24: 600R - 168P = 6912 \quad +$$

$$\textcircled{2} \times 7: 26.25R + 168P = 588$$

$$\text{add: } 626.25R = 7500$$

$$R = 11.97...$$

$$\rightarrow \textcircled{2}: 3.75(11.97...) + 24P = 84$$

$$44.91... + 24P = 84$$

$$24P = 39.089...$$

$$P = \underline{\underline{1.63 \text{ N}}}$$



- 6 A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ .

(a)

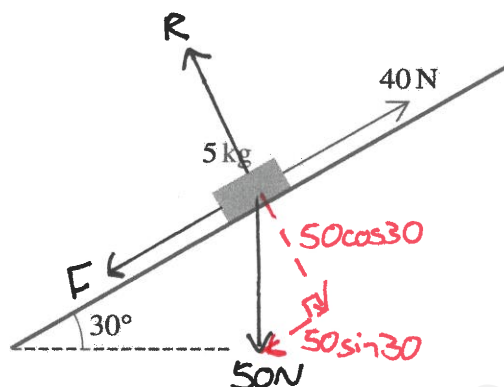


Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{3}$.

[4]

$$R(\uparrow): R - 50 \cos 30 = 0$$

$$R = 50 \cos 30$$

$$R = 25\sqrt{3} \text{ N}$$

$$R(\nearrow): 40 - 50 \sin 30 - F > 0$$

$$40 - 25 - \mu R > 0 \quad \text{because particle is accelerating up the plane}$$

$$15 - \mu \times 25\sqrt{3} > 0$$

$$15 > 25\sqrt{3}\mu$$

$$25\sqrt{3}\mu < 15$$

$$\mu < \frac{15}{25\sqrt{3}}$$

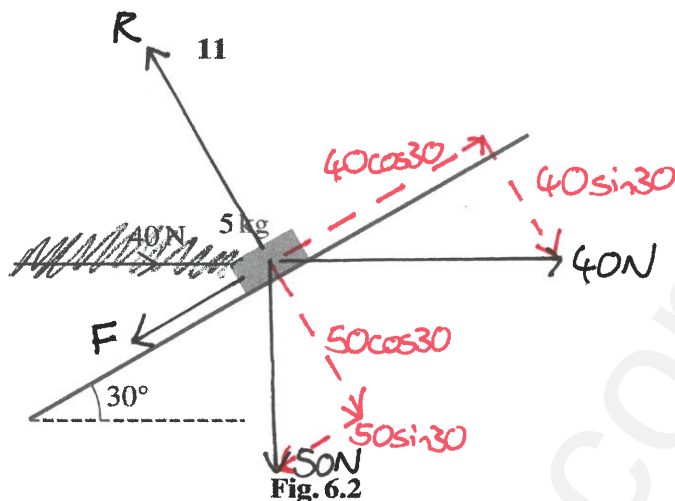
$$\mu < \frac{3}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\mu < \frac{3\sqrt{3}}{15}$$

$$\mu < \frac{\sqrt{3}}{5}$$

QED

(b)



When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152.

[4]

$40 \cos 30 > 50 \sin 30$, so without friction it would move up the plane, so friction is down the plane.

Least possible value of μ is when block is in limiting equilibrium ($F = \mu R$)

$$R(\uparrow): R - 40 \sin 30 - 50 \cos 30 = 0$$

$$R = 40 \sin 30 + 50 \cos 30$$

$$R = 20 + 25\sqrt{3}$$

$$R(\downarrow): 40 \cos 30 - 50 \sin 30 - F = 0$$

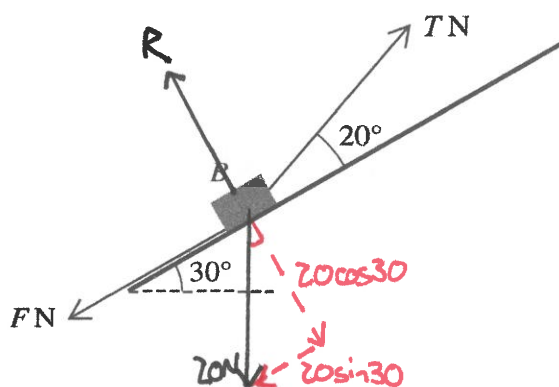
$$20\sqrt{3} - 25 - \mu R = 0$$

$$20\sqrt{3} - 25 - \mu(20 + 25\sqrt{3}) = 0$$

$$\mu(20 + 25\sqrt{3}) = 20\sqrt{3} - 25$$

$$\mu = \frac{20\sqrt{3} - 25}{20 + 25\sqrt{3}}$$

$$\mu = \underline{\underline{0.152}} \text{ QED}$$



A block B , of mass 2 kg , lies on a rough inclined plane sloping at 30° to the horizontal. A light rope, inclined at an angle of 20° above a line of greatest slope, is attached to B . The tension in the rope is $T\text{ N}$. There is a friction force of $F\text{ N}$ acting on B (see diagram). The coefficient of friction between B and the plane is μ .

(a) It is given that $F = 5$ and that the acceleration of B up the plane is 1.2 ms^{-2} .

(i) Find the value of T .

[3]

$$\begin{aligned}
 R(\uparrow): T \cos 20 - 20 \sin 30 - F &= ma \\
 T \cos 20 - 10 - 5 &= 2 \times 1.2 \\
 T \cos 20 - 15 &= 2.4 \\
 T \cos 20 &= 17.4 \\
 T &= \frac{17.4}{\cos 20} \\
 T &= \underline{18.5\text{ N}} \quad \text{sto}
 \end{aligned}$$

(ii) Find the value of μ .

[3]

$$\begin{aligned}
 R(\uparrow): R + T \sin 20 - 20 \cos 30 &= 0 \\
 R + 18.5 \sin 20 - 20 \cos 30 &= 0 \\
 R &= 20 \cos 30 - 18.5 \sin 20 \\
 R &= \underline{10.987\text{ N}} \quad \text{sto} \\
 \text{Block is in motion so } F &= \mu R: \\
 5 &= \mu \times 10.987 \\
 \mu &= \underline{0.455}
 \end{aligned}$$

- (b) It is given instead that $\mu = 0.8$ and $T = 15$.

Determine whether B will move up the plane.

[3]

$$R(\uparrow): R + 15\sin 20 - 20\cos 30 = 0$$

$$R = 20\cos 30 - 15\sin 20$$

$$R = 12.19 \text{ N}_{\text{sto}}$$

Max possible value of friction, $F = \mu R$:

$$F = 0.8 \times 12.19$$

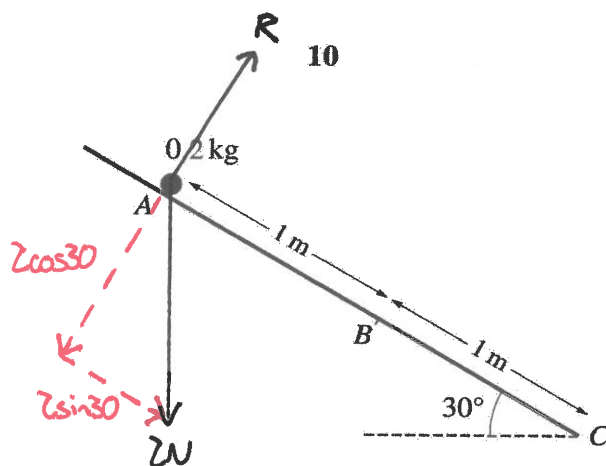
$$= 9.75 \text{ N}_{\text{sto}}$$

Resolve up the plane (without friction)

$$R(\uparrow): 15\cos 20 - 20\sin 30$$

$$= 4.095 \text{ N}$$

→ Resultant force up the plane (without friction) is 4.095 N , but the maximum possible value of friction is greater than this (9.75 N) so the block won't move.



Three points A, B and C lie on a line of greatest slope of a plane inclined at an angle of 30° to the horizontal, with $AB = 1$ m and $BC = 1$ m, as shown in the diagram. A particle of mass 0.2 kg is released from rest at A and slides down the plane. The part of the plane from A to B is smooth. The part of the plane from B to C is rough, with coefficient of friction μ between the plane and the particle.

- (a) Given that $\mu = \frac{1}{2}\sqrt{3}$, find the speed of the particle at C.

[8]

$$R(\uparrow): R - 2\cos 30 = 0$$

$$R = 2\cos 30$$

$$R = \sqrt{3} \text{ N}$$

Smooth section:

$$R(\downarrow): 2\sin 30 = ma$$

$$1 = 0.2a$$

$$a = 5 \text{ ms}^{-2}$$

$s = 1$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2 \times 5 \times 1$
$v =$	$v^2 = 10$
$a = 5$	$v = \sqrt{10} \text{ ms}^{-1}$
$t =$	

Rough section:

$$R(\downarrow): 2\sin 30 - \mu R = ma$$

$$1 - \frac{1}{2}\sqrt{3} \times \sqrt{3} = 0.2a$$

$$1 - 1.5 = 0.2a$$

$$-0.5 = 0.2a$$

$$a = -2.5 \text{ ms}^{-2}$$

$$s = 1$$

$$v^2 = u^2 + 2as$$

$$u = \sqrt{10}$$

$$= (\sqrt{10})^2 + 2(-2.5) \times 1$$

$$v =$$

$$= 10 - 5$$

$$a = -2.5$$

$$v^2 = 5$$

$$t =$$

$$v = \sqrt{5}$$

$$\rightarrow V = 2.24 \text{ ms}^{-1}$$

- (b) Given instead that the particle comes to rest at C, find the exact value of μ .

[4]

Find required acceleration for $v=0$:

$$s = 1$$

$$v^2 = u^2 + 2as$$

$$u = \sqrt{10}$$

$$0^2 = (\sqrt{10})^2 + 2 \times a \times 1$$

$$v = 0$$

$$0 = 10 + 2a$$

$$a =$$

$$2a = -10$$

$$t =$$

$$a = -5 \text{ ms}^{-2}$$

Now resolve forces for rough section again:

$$R(\downarrow): 2 \sin 30 - \mu R = ma$$

$$1 - \mu \times \sqrt{3} = 0.2 \times -5$$

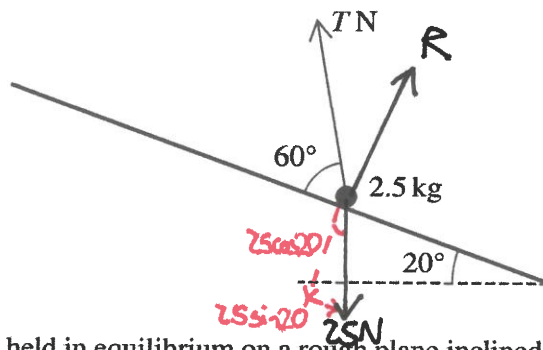
$$1 - \sqrt{3} \mu = -1$$

$$- \sqrt{3} \mu = -2$$

$$\sqrt{3} \mu = 2$$

$$\mu = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\mu = \frac{2\sqrt{3}}{3}$$



A particle of mass 2.5 kg is held in equilibrium on a rough plane inclined at 20° to the horizontal by a force of magnitude T N making an angle of 60° with a line of greatest slope of the plane (see diagram). The coefficient of friction between the particle and the plane is 0.3.

Find the greatest and least possible values of T .

[8]

For greatest possible value of T , the particle is about to move up the plane and friction is acting down the plane:

$$R(\uparrow): R + T \sin 60 - 25 \cos 20 = 0$$

$$R = 25 \cos 20 - T \sin 60$$

$$R(\rightarrow): T \cos 60 - 25 \sin 20 - F = 0$$

$$T \cos 60 - 25 \sin 20 - \mu R = 0$$

$$T \cos 60 - 25 \sin 20 - 0.3(25 \cos 20 - T \sin 60) = 0$$

$$T \cos 60 - 25 \sin 20 - 7.5 \cos 20 + 0.3 T \sin 60 = 0$$

$$T \cos 60 + 0.3 T \sin 60 = 7.5 \cos 20 + 25 \sin 20$$

$$T(\cos 60 + 0.3 \sin 60) = 7.5 \cos 20 + 25 \sin 20$$

$$T = \frac{7.5 \cos 20 + 25 \sin 20}{\cos 60 + 0.3 \sin 60}$$

$$T = \underline{20.5 \text{ N}}$$

Continued $\rightarrow$

For least possible value of T , the particle is about to move down the plane and friction is acting up the plane.

$$R \text{ is unchanged: } R = 25 \cos 20^\circ - T \sin 60^\circ$$

$$R(\downarrow): 25 \sin 20^\circ - T \cos 60^\circ - F = 0$$

$$25 \sin 20^\circ - T \cos 60^\circ - \mu R = 0$$

$$25 \sin 20^\circ - T \cos 60^\circ - 0.3(25 \cos 20^\circ - T \sin 60^\circ) = 0$$

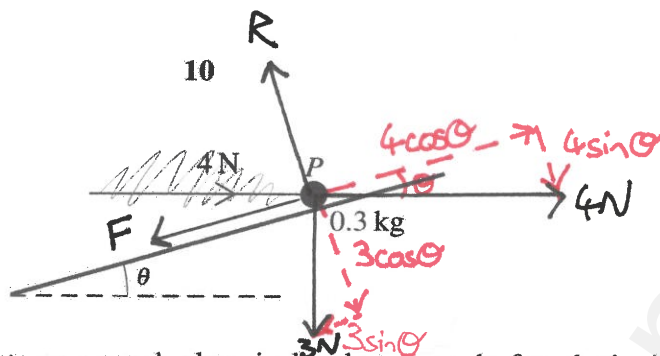
$$25 \sin 20^\circ - T \cos 60^\circ - 7.5 \cos 20^\circ + 0.3T \sin 60^\circ = 0$$

$$0.3T \sin 60^\circ - T \cos 60^\circ = 7.5 \cos 20^\circ - 25 \sin 20^\circ$$

$$T(0.3 \sin 60^\circ - \cos 60^\circ) = 7.5 \cos 20^\circ - 25 \sin 20^\circ$$

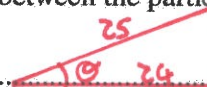
$$T = \frac{7.5 \cos 20^\circ - 25 \sin 20^\circ}{0.3 \sin 60^\circ - \cos 60^\circ}$$

$$T = \underline{\underline{6.26 \text{ N}}}$$



A particle P of mass 0.3 kg rests on a rough plane inclined at an angle θ to the horizontal, where $\sin \theta = \frac{7}{25}$. A horizontal force of magnitude 4 N , acting in the vertical plane containing a line of greatest slope of the plane, is applied to P (see diagram). The particle is on the point of sliding up the plane.

- (a) Show that the coefficient of friction between the particle and the plane is $\frac{3}{4}$. [4]

$\sin \theta = \frac{7}{25} \rightarrow$  $\cos \theta = \frac{24}{25}$ $\tan \theta = \frac{7}{24}$

$$R(\uparrow): R - 3 \cos \theta - 4 \sin \theta = 0$$

$$R - 3 \times \frac{24}{25} - 4 \times \frac{7}{25} = 0$$

$$R - 4 = 0$$

$$R = 4 \text{ N}$$

$$R(\rightarrow): 4 \cos \theta - 3 \sin \theta - F = 0$$

$$4 \times \frac{24}{25} - 3 \times \frac{7}{25} - \mu R = 0$$

$$3 - \mu R = 0$$

$$3 - 4\mu = 0$$

$$4\mu = 3$$

$$\mu = \frac{3}{4} \quad \text{QED}$$

The force acting horizontally is replaced by a force of magnitude 4 N acting up the plane parallel to a line of greatest slope.

- (b) Find the acceleration of P . [3]

$$R(\uparrow): R - 3 \cos \theta = 0$$

$$R = 3 \times \frac{24}{25}$$

$$R = 2.88 \text{ N}$$

$$R(\rightarrow): 4 - 3 \sin \theta - F = ma$$

$$4 - 3 \times \frac{7}{25} - \mu R = 0.3a$$

$$4 - 0.84 - \frac{3}{4} \times 2.88 = 0.3a$$

$$4 - 0.84 - 2.16 = 0.3a$$

$$1 = 0.3a$$

$$a = \underline{3.33 \text{ ms}^{-2}}$$

STO

- (c) Starting with P at rest, the force of 4 N parallel to the plane acts for 3 seconds and is then removed.

Find the total distance travelled until P comes to instantaneous rest.

[3]

$$S =$$

$$u = 0$$

$$v =$$

$$a = \frac{10}{3} \text{ (3.33.)}$$

$$t = 3$$

$$S = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times \left(\frac{10}{3}\right) \times 3^2$$

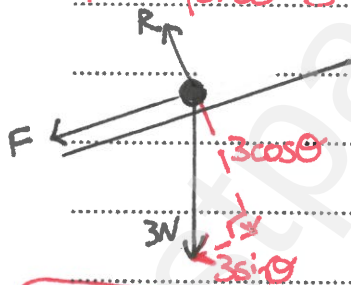
$$S = \underline{15 \text{ m}}$$

$$v = u + at$$

$$= 0 + \frac{10}{3} \times 3$$

$$v = \underline{10 \text{ ms}^{-1}}$$

After 3 s , P has travelled 15 m and is moving at 10 ms^{-1} .
 4 N force is removed. Particle is still moving UP, so friction is acting down.



$$R(\uparrow): -3\sin\theta - F = ma$$

$$-3 \times \frac{7}{25} - \mu R = 0.3a$$

$$-0.84 - \frac{3}{4} \times 2.88 = 0.3a$$

$$-3 = 0.3a$$

$$\underline{a = -10}$$

$$S =$$

$$u = 10$$

$$v = 0$$

$$a = -10$$

$$t =$$

$$v^2 = u^2 + 2as$$

$$0^2 = 10^2 + 2(-10)s$$

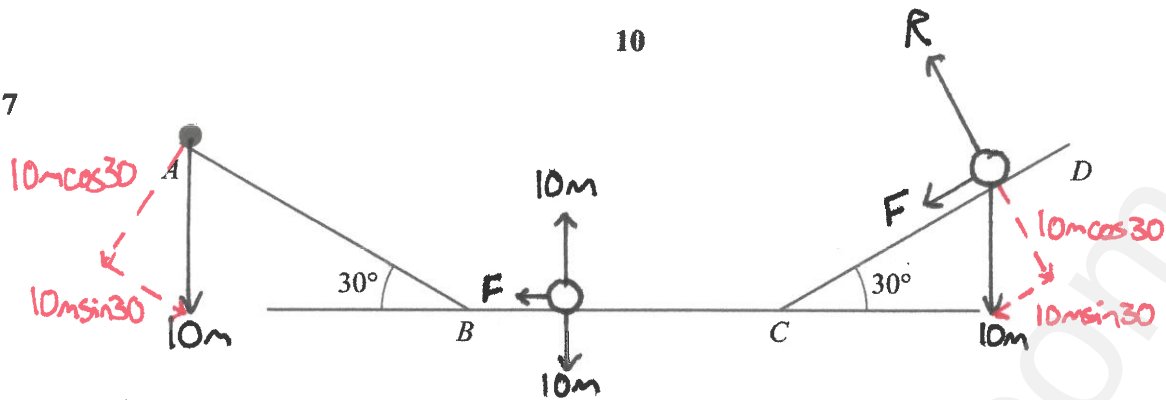
$$0 = 100 - 20s$$

$$20s = 100$$

$$\underline{s = 5 \text{ m}}$$

$\rightarrow P$ moves $15 + 5 = \underline{20 \text{ m}}$

7



The diagram shows a track $ABCD$ which lies in a vertical plane. The section AB is a straight line inclined at an angle of 30° to the horizontal and is smooth. The section BC is a horizontal straight line and is rough. The section CD is a straight line inclined at an angle of 30° to the horizontal and is rough. The lengths AB , BC and CD are each 2 m .

A particle is released from rest at A . The coefficient of friction between the particle and both BC and CD is μ . There is no change in the speed of the particle when it passes through either of the points B or C .

(a) It is given that $\mu = 0.1$.

Find the distance which the particle has moved up the section CD when its speed is 1 m s^{-1} . [5]

AB:
(smooth)

$R(\downarrow): F = ma$	$S = 2$	$V^2 = u^2 + 2as$
$10m \sin 30 = ma$	$u = 0$	$= 0^2 + 2 \times 5 \times 2$
$5m = ma$	$V =$	$V^2 = 20$
$a = 5$	$a = 5$	$V = \sqrt{20}$
	$t =$	

BC:
(rough)

$R(\rightarrow): F = ma$	$S = 2$	$V^2 = u^2 + 2as$
$-\mu R = ma$	$u = \sqrt{20}$	$= \sqrt{20}^2 + 2(-1) \times 2$
$-0.1 \times 10m = ma$	$V =$	$= 20 - 4$
$-1m = ma$	$a = -1$	$V^2 = 16$
$a = -1$	$t =$	$V = 4$

CD:
(rough)

$R(\rightarrow): F = ma$	$S =$	$V^2 = u^2 + 2as$
$-10m \sin 30 - \mu R = ma$	$u = 4$	$1^2 = 4^2 + 2 \times -5.866 \times s$
$-5m - 0.1 \times 10m \cos 30 = ma$	$V = 1$	$1 = 16 - 11.73s$
$-5m - \frac{\sqrt{3}}{2}m = ma$	$a = -5.866$	$11.73s = 15$
$-5 - \frac{\sqrt{3}}{2} = a$	$t =$	$s = 1.28\text{ m}$
$a = -5.866$		

- (b) It is given instead that with a different value of μ the particle travels 1 m up the track from C before it comes instantaneously to rest.

Find the value of μ and the speed of the particle at the instant that it passes C for the second time.

[4]

No change to AB section.

BC: (rough)	$R(\rightarrow): F = ma$	$S = 2$	$V^2 = u^2 + 2as$
	$-\mu R = ma$	$u = \sqrt{20}$	$= \sqrt{20}^2 + 2 \times -10\mu \times 2$
	$-\mu \times 10m = ma$	$v =$	$= 20 - 40\mu$
	$a = -10\mu$	$a = -10\mu$	$v = \sqrt{20 - 40\mu}$
		$t =$	

1m up
CD:
(rough)

$R(\nearrow): F = ma$

$-10m \sin 30 - \mu R = ma$

$-5m - \mu \times 10m \cos 30 = ma$

$-5m - 5\sqrt{3}\mu m = ma$

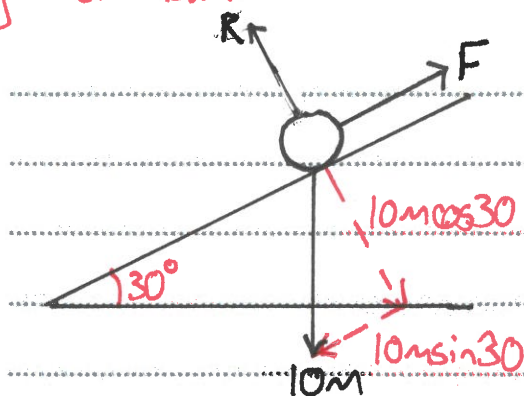
$a = -5 - 5\sqrt{3}\mu$

+1

$S = 1$	$V^2 = u^2 + 2as$
$u = \sqrt{20 - 40\mu}$	$0^2 = \sqrt{20 - 40\mu}^2 + 2(-5 - 5\sqrt{3}\mu) \times 1$
$v = 0$	$0 = 20 - 40\mu - 10 - 10\sqrt{3}\mu$
$a = -5 - 5\sqrt{3}\mu$	$0 = 10 - 40\mu - 10\sqrt{3}\mu$
$t =$	$40\mu + 10\sqrt{3}\mu = 10$
	$\mu(40 + 10\sqrt{3}) = 10$
	$\mu = \frac{10}{40 + 10\sqrt{3}}$
	$\mu = 0.174$ STO

CONTINUED

On way back down:



$$R(\leftarrow): F = ma$$

$$10m \sin 30 - \mu R = ma$$

$$5m - 0.174 \times 10m \cos 30 = ma$$

$$5m - 1.511m = ma$$

$$3.489m = ma$$

$$a = 3.489 \dots \text{STO}$$

$$s = 1$$

$$u = 0$$

$$v =$$

$$a = 3.489$$

$$t =$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 3.489 \times 1$$

$$v^2 = 6.9783$$

$$v = \underline{\underline{2.64 \text{ ms}^{-1}}}$$