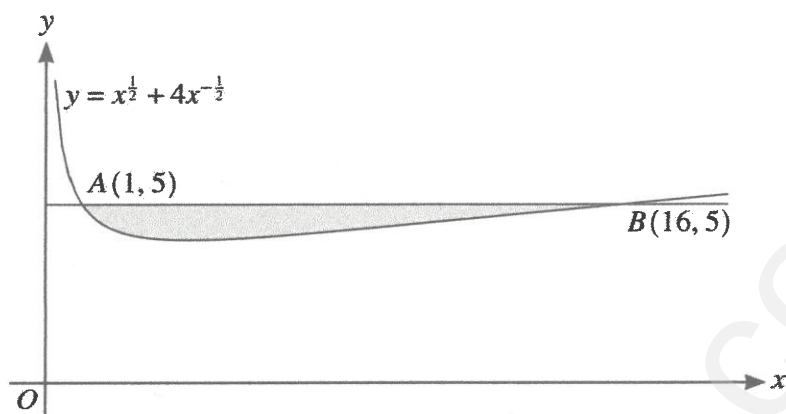




The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A(1, 5)$ and $B(16, 5)$.

- (a) Find the equation of the tangent to the curve at the point A.

[4]

gradient at (1,5): $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$
 $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}$

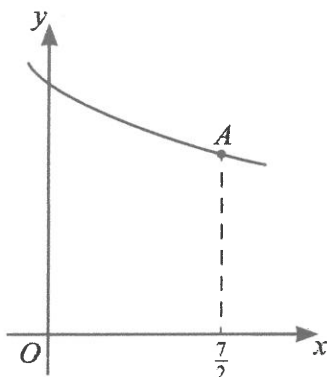
Sub $x=1$: $\frac{1}{2}(1)^{-\frac{1}{2}} - 2(1)^{-\frac{3}{2}}$
 $= \frac{1}{2} - 2$
 $= -\frac{3}{2}$

eqⁿ of tangent: $y - y_1 = m(x - x_1)$
 $y - 5 = -\frac{3}{2}(x - 1)$

$$y - 5 = -\frac{3}{2}x + \frac{3}{2}$$

$$y = -\frac{3}{2}x + \frac{13}{2}$$

7



The diagram shows part of the curve with equation $y = \frac{12}{\sqrt[3]{2x+1}}$. The point A on the curve has coordinates $(\frac{7}{2}, 6)$.

- (a) Find the equation of the tangent to the curve at A . Give your answer in the form $y = mx + c$. [4]

$$y = 12(2x+1)^{-\frac{1}{3}}$$

$$\frac{dy}{dx} = -\frac{1}{3} \times 2 \times 12(2x+1)^{-\frac{4}{3}}$$

$$= -8(2x+1)^{-\frac{4}{3}}$$

Sub. $x = \frac{7}{2}$:

$$\text{gradient} = -8(2(\frac{7}{2})+1)^{-\frac{4}{3}}$$

$$= -8(8)^{-\frac{4}{3}}$$

$$= -8 \times \frac{1}{16}$$

$$= -\frac{1}{2}$$

Eqn of line:

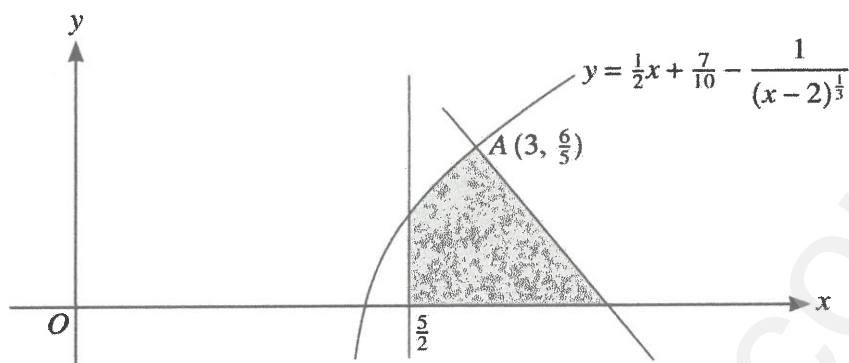
$$y - y_1 = m(x - x_1)$$

$$y - 6 = -\frac{1}{2}(x - \frac{7}{2})$$

$$y - 6 = -\frac{1}{2}x + \frac{7}{4}$$

$$\underline{y = -\frac{1}{2}x + \frac{31}{4}}$$

11



The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A(3, \frac{6}{5})$.

- (a) Find the x -coordinate of the point where the normal to the curve meets the x -axis. [5]

$$y = \frac{1}{2}x + \frac{7}{10} - (x-2)^{-\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{1}{2} + \frac{1}{3}(x-2)^{-\frac{4}{3}}$$

gradient at $x=3$:

$$\frac{1}{2} + \frac{1}{3}(3-2)^{-\frac{4}{3}}$$

$$= \frac{1}{2} + \frac{1}{3}(1)^{-\frac{4}{3}}$$

$$= \frac{1}{2} + \frac{1}{3}$$

$$= \frac{5}{6}$$

gradient of normal:

$$m = -\frac{6}{5}$$

eqn of normal:

$$y - \frac{6}{5} = -\frac{6}{5}(x-3)$$

$$y - \frac{6}{5} = -\frac{6}{5}x + \frac{18}{5}$$

$$y = -\frac{6}{5}x + \frac{24}{5}$$

$y=0$:

$$0 = -\frac{6}{5}x + \frac{24}{5}$$

$$\frac{6}{5}x = \frac{24}{5}$$

$$6x = 24$$

$$\underline{x = 4}$$

11 The equation of a curve is $y = 2\sqrt{3x+4} - x$.

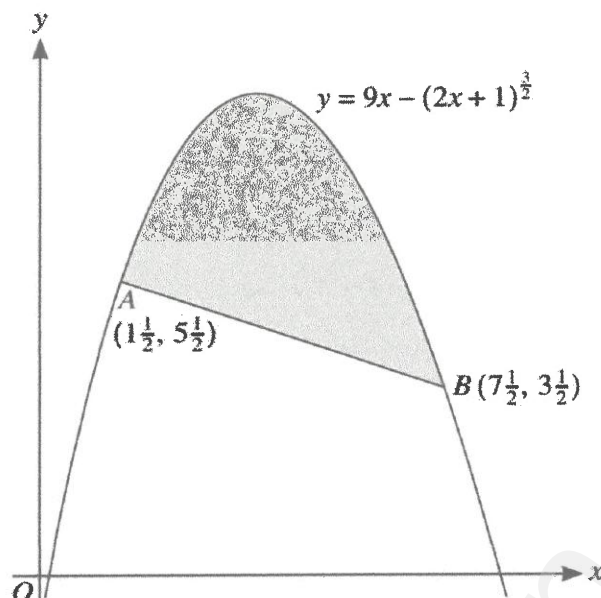
- (a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

gradient at (4, 4): $y = 2(3x+4)^{\frac{1}{2}} - x$
 $\frac{dy}{dx} = 1 \times 3(3x+4)^{-\frac{1}{2}} - 1$
 $= 3(3x+4)^{-\frac{1}{2}} - 1$

sub. $x=4$: $3(3(4)+4)^{-\frac{1}{2}} - 1$
 $3(16)^{-\frac{1}{2}} - 1$
 $3 \times \frac{1}{4} - 1$
 $= -\frac{1}{4}$

→ Gradient of Normal = 4

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= 4(x - 4) \\ y - 4 &= 4x - 16 \\ y &= 4x - 12 \end{aligned}$$



The diagram shows the points $A(1\frac{1}{2}, 5\frac{1}{2})$ and $B(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x + 1)^{\frac{3}{2}}$.

(b) Verify that the line AB is the normal to the curve at A .

[3]

$$y = 9x - (2x + 1)^{\frac{3}{2}}$$

$$\frac{dy}{dx} = 9 - \frac{3}{2} \times 2(2x + 1)^{\frac{1}{2}}$$

$$= 9 - 3(2x + 1)^{\frac{1}{2}}$$

Sub. $x = \frac{3}{2}$:

$$9 - 3(2(\frac{3}{2}) + 1)^{\frac{1}{2}}$$

$$= 9 - 3(4)^{\frac{1}{2}}$$

$$= 9 - 3 \times 2$$

gradient = 3

→ gradient of normal at A:

$$\text{gradient} = -\frac{1}{3}$$

Now find gradient of AB and check that it = $-\frac{1}{3}$:

$$m = \frac{\frac{7}{2} - \frac{11}{2}}{\frac{15}{2} - \frac{3}{2}}$$

$$= \frac{-2}{6}$$

$= -\frac{1}{3}$ therefore AB is the normal to the curve.

- 5 A curve has the equation $y = \frac{3}{2x^2 - 5}$.

Find the equation of the normal to the curve at the point (2, 1), giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. [6]

$$y = 3(2x^2 - 5)^{-1}$$

Chain rule:

$$y = 3t^{-1}$$

$$t = 2x^2 - 5$$

$$\frac{dy}{dt} = -3t^{-2}$$

$$\frac{dt}{dx} = 4x$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= -3t^{-2} \times 4x$$

$$= -3(2x^2 - 5)^{-2} \times 4x$$

gradient at (2, 1):

$$m = -3(2(2)^2 - 5)^{-2} \times 4(2)$$

$$= -3(3)^{-2} \times 8$$

$$= \frac{-3}{9} \times 8$$

$$= -\frac{8}{3}$$

gradient of normal:

$$m_{\text{norm}} = \frac{3}{8}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{3}{8}(x - 2) \quad \times 8$$

$$8y - 8 = 3(x - 2)$$

$$8y - 8 = 3x - 6$$

$$\underline{3x - 8y + 2 = 0}$$

10 The equation of a curve is $y = (5-2x)^{\frac{3}{2}} + 5$ for $x < \frac{5}{2}$.

- (b) Point A on the curve has y -coordinate 32. Point B on the curve is such that the gradient of the curve at B is -3 .

Find the equation of the perpendicular bisector of AB . Give your answer in the form $ax+by+c=0$, where a , b and c are integers. [6]

A: Find x when $y=32$:

$$32 = (5-2x)^{\frac{3}{2}} + 5$$

$$27 = (5-2x)^{\frac{3}{2}}$$

$$3 = (5-2x)^{\frac{1}{2}}$$

$$9 = 5-2x$$

$$4 = -2x$$

$$x = -2 \quad A: (-2, 32)$$

Mid-point of AB :

$$\left(\frac{-2+2}{2}, \frac{32+6}{2} \right)$$

$$(0, 19)$$

Gradient of perpendicular:

$$m_{\text{perp}} = \frac{2}{13}$$

B: $\frac{dy}{dx} = \frac{3}{2}x - 2 \times (5-2x)^{\frac{1}{2}}$

$$= -3(5-2x)^{\frac{1}{2}}$$

gradient $= -3$:

$$-3(5-2x)^{\frac{1}{2}} = -3$$

$$(5-2x)^{\frac{1}{2}} = 1$$

$$5-2x = 1$$

$$-2x = -4$$

$$x = 2$$

Equation of perp. bisector:

$$y-19 = \frac{2}{13}(x-0)$$

$$y-19 = \frac{2}{13}x$$

$$13y - 247 = 2x$$

y-coord: $y = (5-2(2))^{\frac{3}{2}} + 5$

$$= 1^{\frac{3}{2}} + 5$$

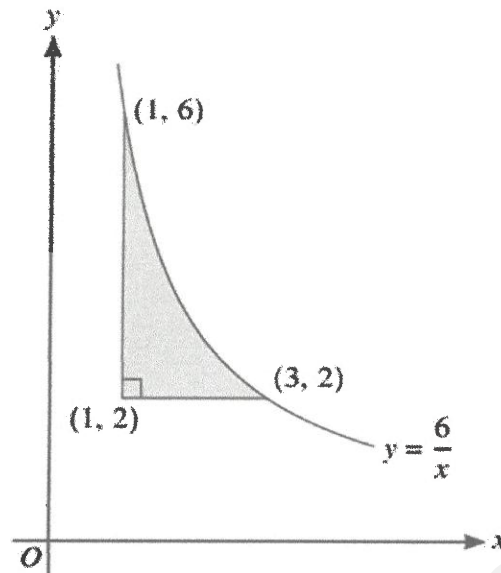
$$= 6$$

$$B: (2, 6)$$

gradient AB : $m = \frac{6-32}{2-(-2)} = \frac{-26}{4}$

$$= \frac{-13}{2}$$

$$2x - 13y + 247 = 0$$



The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

- (b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

$$\hookrightarrow y = -2x$$

Tangent at X is parallel to $y = -2x$, so gradient = -2

$$y = 6x^{-1}$$

$$\frac{dy}{dx} = -6x^{-2}$$

$$m = -2: -2 = -6x^{-2}$$

$$2 = \frac{6}{x^2}$$

$$2x^2 = 6$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

It's not clear whether we should pick $+\sqrt{3}$ or $-\sqrt{3}$ (these papers weren't used because of the covid pandemic so maybe weren't checked properly). Let's go with $x = \sqrt{3}$.

Sub. $x = \sqrt{3}$ into $y = \frac{6}{x}$:

$$y = \frac{6}{\sqrt{3}}$$

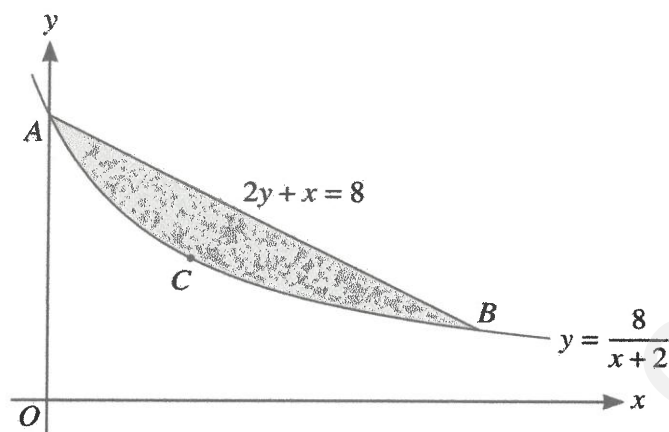
$$y = 2\sqrt{3} \rightarrow (\sqrt{3}, 2\sqrt{3})$$

→ Show this lies on $y = 2x$:

$$y = 2 \times (\sqrt{3})$$

$$= 2\sqrt{3}$$

so it lies on $y = 2x$



The diagram shows part of the curve $y = \frac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

- (a) Find, by calculation, the coordinates of A, B and C.

[6]

Rearrange ②: $2y + x = 8$

$$2y = 8 - x$$

$$y = \frac{8-x}{2}$$

Solve simultaneously:

$$\frac{8}{x+2} = \frac{8-x}{2}$$

$$16 = (x+2)(8-x)$$

$$16 = -x^2 + 6x + 16$$

$$x^2 - 6x = 0$$

$$x(x-6) = 0$$

$$x=0 \text{ or } x=6$$

Sub. $x=0$ into ①:

$$y = \frac{8}{2}$$

$$y = 4 \rightarrow (0, 4) \text{ (A)}$$

Sub. $x=6$ into ①:

$$y = \frac{8}{8}$$

$$y = 1 \rightarrow (6, 1) \text{ (B)}$$

At C, gradient is the same as

AB: $2y = -x + 8$

$$y = -\frac{1}{2}x + 4$$

→ Gradient = $-\frac{1}{2}$

Differentiate ①:

$$y = 8(x+2)^{-1}$$

$$\frac{dy}{dx} = -8(x+2)^{-2}$$

gradient = $-\frac{1}{2}$:

$$-\frac{1}{2} = -8(x+2)^{-2}$$

$$\frac{1}{2} = \frac{8}{(x+2)^2}$$

$$(x+2)^2 = 16$$

$$x+2 = \pm\sqrt{16}$$

$$x+2 = \pm 4$$

$$x = -2 \pm 4$$

$$x = -6 \text{ or } x = 2$$

→ Sub. into ①:

$$y = \frac{8}{4}$$

$$= 2$$

$$\rightarrow (2, 2)$$

11 The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

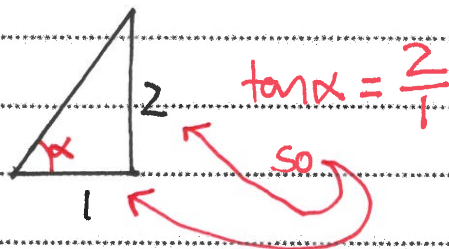
- (c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x -axis. [4]

$$y = 10.5(4x+1)^{\frac{1}{2}} - x + 5$$

$$\frac{dy}{dx} = \frac{21}{4} \times 4(4x+1)^{-\frac{1}{2}} - 1$$

$$= 21(4x+1)^{-\frac{1}{2}} - 1$$

$\alpha = \tan^{-1}(2)$:



$$\text{gradient} = \frac{2}{1} = 2$$

$$\rightarrow 21(4x+1)^{-\frac{1}{2}} - 1 = 2$$

$$21(4x+1)^{-\frac{1}{2}} = 3$$

$$\frac{1}{\sqrt{4x+1}} = \frac{1}{7}$$

$$7 = \sqrt{4x+1}$$

$$49 = 4x+1$$

$$48 = 4x$$

$$x = 12$$

find y -coord: $y = 10.5\sqrt{4(12)+1} - 12 + 5$
 $= 10.5\sqrt{49} - 7$
 $= \frac{133}{2}$

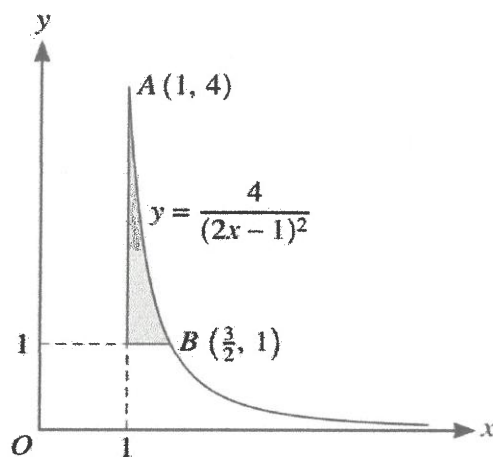
eqn of normal:

$$\text{gradient} = -\frac{1}{2}$$

$$y - \frac{133}{2} = -\frac{1}{2}(x - 12)$$

$$y - \frac{133}{2} = -\frac{1}{2}x + 6$$

$$y = -\frac{1}{2}x + \frac{145}{2}$$



The diagram shows part of the curve with equation $y = \frac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A(1, 4)$ and $B(\frac{3}{2}, 1)$.

- (b) A triangle is formed from the tangent to the curve at B , the normal to the curve at B and the x -axis.

Find the area of this triangle.

[6]

gradient at $(\frac{3}{2}, 1)$: $y = 4(2x-1)^{-2}$

$$\frac{dy}{dx} = -8 \times 2(2x-1)^{-3}$$

$$= -16(2x-1)^{-3}$$

sub. $x = \frac{3}{2}$: $-16(2(\frac{3}{2})-1)^{-3}$

$$= -16(2)^{-3}$$

$$= -2$$

eqⁿ of tangent:

$$y - 1 = -2(x - \frac{3}{2})$$

$$y - 1 = -2x + 3$$

$$y = -2x + 4$$

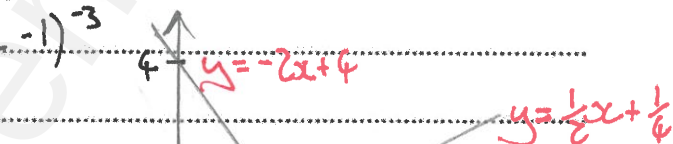
eqⁿ of normal:

$$\text{gradient} = \frac{1}{2} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x - \frac{3}{2})$$

$$y - 1 = \frac{1}{2}x - \frac{3}{4}$$

$$y = \frac{1}{2}x + \frac{1}{4}$$



$y = -2x + 4$ meets x -axis at B :

$$0 = -2x + 4$$

$$2x = 4$$

$$x = 2$$

$y = \frac{1}{2}x + \frac{1}{4}$ meets x -axis at A :

$$0 = \frac{1}{2}x + \frac{1}{4}$$

$$\frac{1}{2}x = -\frac{1}{4}$$

$$x = -\frac{1}{2}$$

Area: $\text{Area} = \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 2.5 \times 1 = \frac{5}{4}$$