

- 1 The function f is defined by $f(x) = \frac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither.

[3]

$$f(x) = (3x+2)^{-1} + x^2$$

$$f'(x) = -3(3x+2)^{-2} + 2x$$

$$= \frac{-3}{(3x+2)^2} + 2x$$

negative

$$\frac{-3}{(3x+2)^2} + 2x$$

always positive

negative because $x < -1$

$$\text{so } f'(x) < 0$$

therefore decreasing function

11 (c) The function g is defined by $g(x) = -(3x+1)^{-1} + 3x$ for $x \geq 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

$$g(x) = -(3x+1)^{-1} + 3x$$

$$g'(x) = 3(3x+1)^{-2} + 3$$

positive

$$= \frac{3}{(3x+1)^2} + 3$$

positive

always positive

$$\text{so } g'(x) > 0$$

therefore increasing function

- 9 A function f is such that $f'(x) = 6(2x-3)^2 - 6x$ for $x \in \mathbb{R}$.

(a) Determine the set of values of x for which $f(x)$ is decreasing.

[4]

decreasing when $f'(x) < 0$:

$$6(2x-3)^2 - 6x < 0$$

$$6(4x^2 - 12x + 9) - 6x < 0$$

$$24x^2 - 72x + 54 - 6x < 0$$

$$24x^2 - 78x + 54 < 0$$

 $\div 6$ $\div 6$ $\div 6$ $\div 6$

$$4x^2 - 13x + 9 < 0$$

find critical values:

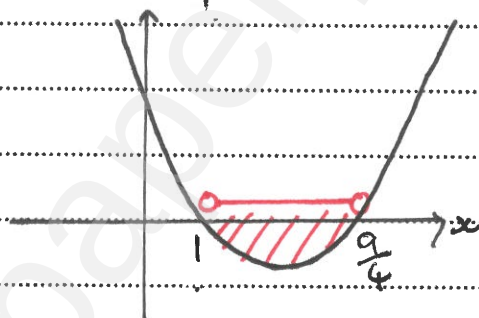
factorise: $ac: 36 \rightarrow -9, -4$

$$4x^2 - 4x - 9x + 9 = 0$$

$$4x(x-1) - 9(x-1) = 0$$

$$(4x-9)(x-1) = 0$$

$$x = \frac{9}{4} \text{ or } x = 1$$



$$1 < x < \frac{9}{4}$$

- 8 The function f is defined by $f(x) = 2 - \frac{3}{4x-p}$ for $x > \frac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

$$f(x) = 2 - 3(4x-p)^{-1}$$

$$f'(x) = 3 \times 4(4x-p)^{-2}$$

$$= 12(4x-p)^{-2}$$

$$= \frac{12}{(4x-p)^2}$$

positive
always positive

$$\text{so } f'(x) > 0$$

therefore increasing function

- 3 (a) Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants.

[2]

$$5[y^2 - 6y] + 50$$

$$5[(y-3)^2 - 9] + 50$$

$$5(y-3)^2 - 45 + 50$$

$$\underline{5(y-3)^2 + 5}$$

- (b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither.

[3]

$$f'(x) = 5x^4 - 30x^2 + 50$$

using part (a), with $y = x^2$:

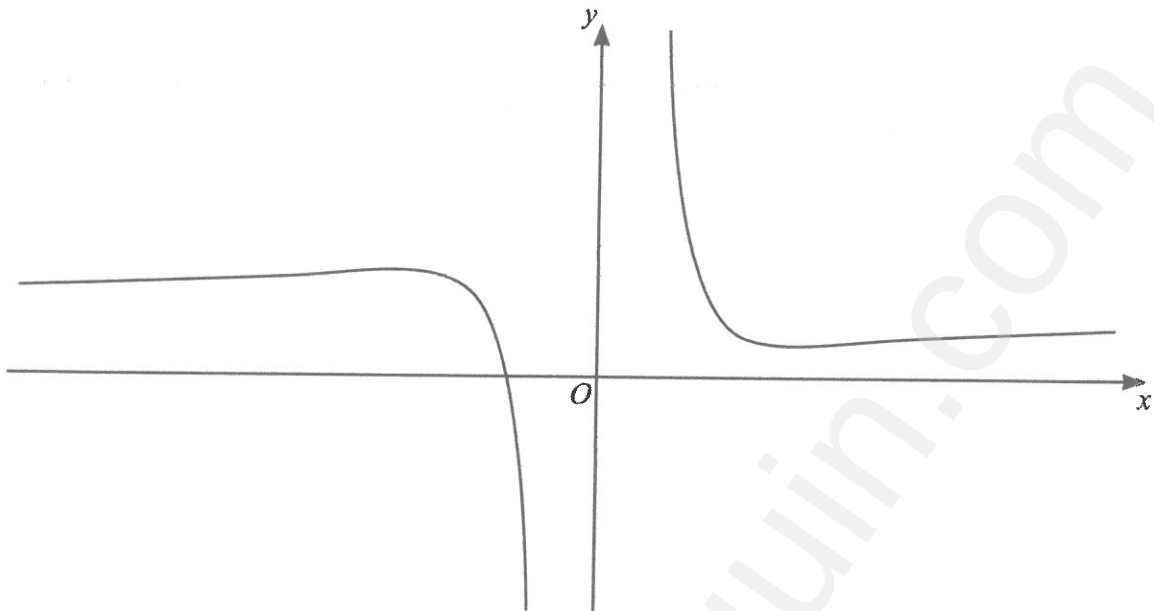
$$f'(x) = 5(x^2 - 3)^2 + 5$$

always
positive

positive

$$\text{so } f'(x) > 0$$

therefore increasing function



A function is defined by $f(x) = \frac{4}{x^3} - \frac{3}{x} + 2$ for $x \neq 0$. The graph of $y = f(x)$ is shown in the diagram.

- (a) Find the set of values of x for which $f(x)$ is decreasing.

[5]

$$y = 4x^{-3} - 3x^{-1} + 2$$

$$\frac{dy}{dx} = -12x^{-4} + 3x^{-2}$$

decreasing, so $\frac{dy}{dx} < 0$:

$$\frac{-12}{x^4} + \frac{3}{x^2} < 0$$

$\times x^4$ $\times x^4$ $\times x^4$

$$-12 + 3x^2 < 0$$

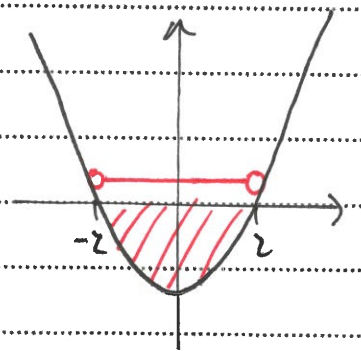
$$3x^2 - 12 < 0$$

$$x^2 - 4 < 0$$

Critical values: $x^2 - 4 = 0$

$$(x - 2)(x + 2) = 0$$

$$x = 2 \text{ or } x = -2$$



so $-2 < x < 2$

but wait! $x \neq 0$ so:

$$\underline{\underline{-2 < x < 2, x \neq 0}}$$



- 9 The equation of a curve is $y = 4 + 5x + 6x^2 - 3x^3$.

(a) Find the set of values of x for which y decreases as x increases.

[4]

ie. when is it a decreasing function

$$\frac{dy}{dx} = 5 + 12x - 9x^2$$

decreasing function so < 0 :

$$-9x^2 + 12x + 5 < 0$$

Find critical values:

$$-9x^2 + 12x + 5 = 0$$

$$x-1 \quad x-1 \quad x-1 \quad x-1$$

$$9x^2 - 12x - 5 = 0$$

9c: -45. Two numbers with product -45 and sum -12: -15 and 3

$$9x^2 + 3x - 15x - 5 = 0$$

$$3x(3x + 1) - 5(3x + 1) = 0$$

$$(3x - 5)(3x + 1) = 0$$

$$3x - 5 = 0$$

$$3x + 1 = 0$$

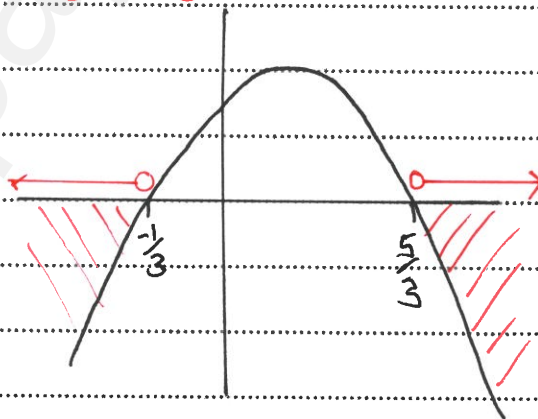
$$3x = 5$$

$$3x = -1$$

$$x = \frac{5}{3}$$

$$x = -\frac{1}{3}$$

Sketch: $-9x^2 + 12x + 5$:



$$-9x^2 + 12x + 5 < 0$$

so we want bits
below x -axis.

$$x < -\frac{1}{3}, \quad x > \frac{5}{3}$$

- 2 The function f is defined by $f(x) = \frac{1}{3}(2x-1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a .

[4]

$$f(x) = \frac{1}{3}(2x-1)^{\frac{3}{2}} - 2x$$

$$f'(x) = \frac{1}{3} \times \frac{3}{2} \times 2(2x-1)^{\frac{1}{2}} - 2$$

$$= (2x-1)^{\frac{1}{2}} - 2$$

decreasing, so $f'(x) < 0$:

$$(2x-1)^{\frac{1}{2}} - 2 < 0$$

$$\sqrt{2x-1} < 2$$

$$2x-1 < 4$$

$$2x < 5$$

$$x < \frac{5}{2}$$

$$\text{so } a = \frac{5}{2}$$