7 A curve is such that the gradient at a general point with coordinates (x, y) is proportional to $\frac{y}{\sqrt{x+1}}$. The curve passes through the points with coordinates $(0, 1)$ and $(3, e)$.

By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [7]

$$\frac{dy}{dx} \propto \frac{y}{\sqrt{x+1}}$$

$$\frac{dy}{dx} = \frac{ky}{\sqrt{x+1}}$$

$$\int \frac{1}{y}\,dy = \int \frac{k}{\sqrt{x+1}}\,dx$$

use reverse chain rule

$$\int \frac{1}{y}\,dy = \int k(x+1)^{-\frac{1}{2}}\,dx$$

$$\ln|y| = 2k(x+1)^{\frac{1}{2}} + C$$

$(0,1)$:

$$\ln 1 = 2k(1)^{\frac{1}{2}} + C$$

$$0 = 2k + C$$

$$\underline{C = -2k}$$

$$\rightarrow \ln|y| = 2k(x+1)^{\frac{1}{2}} - 2k$$

$(3,e)$:

$$\ln e = 2k(4)^{\frac{1}{2}} - 2k$$

$$1 = 4k - 2k$$

$$1 = 2k$$

$$\underline{k = \tfrac{1}{2}}$$

$$\rightarrow \ln|y| = (x+1)^{\frac{1}{2}} - 1$$

$$\underline{y = e^{(x+1)^{\frac{1}{2}} - 1}}$$

10 A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\frac{dV}{dt} = a - bV$.

(a) Write down the values of the constants a and b. [1]

$$\frac{dV}{dt} = 30 - 0.01V$$

$a = 30 \quad b = 0.01$

(b) Solve the differential equation and find the value of t when $V = 1000$. [6]

$$\int \frac{1}{30 - 0.01V}\, dV = \int 1\, dt$$

$$-100\ln|30 - 0.01V| = t + C$$

* $t = 0,\ V = 0$:

$$-100\ln 30 = 0 + C$$

$$C = -100\ln 30$$

$$\rightarrow t = 100\ln 30 - 100\ln|30 - 0.01V|$$

$V = 1000$:

$$t = 100\ln 30 - 100\ln|30 - 10|$$

$$= 100\ln 30 - 100\ln 20$$

$$= 40.5 \text{ (3sf)}$$

(c) Obtain an expression for V in terms of t and hence state what happens to V as t becomes large. [2]

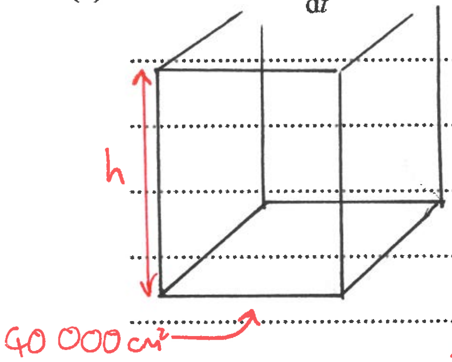
$$t = 100\ln 30 - 100\ln|30 - 0.01V|$$
$$100\ln|30-0.01V| = 100\ln 30 - t$$
$$\ln|30-0.01V| = \ln 30 - 0.01t$$
$$30 - 0.01V = e^{\ln 30 - 0.01t}$$
$$30 - 0.01V = e^{\ln 30} \times e^{-0.01t}$$
$$30 - 0.01V = 30e^{-0.01t}$$
$$0.01V = 30 - 30e^{-0.01t}$$
$$V = 3000 - 3000e^{-0.01t}$$
$$= 3000(1 - e^{-0.01t})$$

As $t \to \infty$, $e^{-0.01t} \to 0$

so this becomes: $V = 3000(1-0)$
$$= \underline{3000}$$

10 A water tank is in the shape of a cuboid with base area $40000\,\text{cm}^2$. At time t minutes the depth of water in the tank is h cm. Water is pumped into the tank at a rate of $50000\,\text{cm}^3$ per minute. Water is leaking out of the tank through a hole in the bottom at a rate of $600h\,\text{cm}^3$ per minute. *

(a) Show that $200\dfrac{dh}{dt} = 250 - 3h$. [3]



Volume $= 40\,000 \times h$

$V = 40\,000h$

$\dfrac{dV}{dh} = 40\,000$

* $\dfrac{dV}{dt} = 50\,000 - 600h$

$\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt}$

$= \dfrac{1}{40\,000} \times (50\,000 - 600h)$

$\dfrac{dh}{dt} = \dfrac{50\,000 - 600h}{40\,000}$

$\times 200 \qquad \times 200$

$200\dfrac{dh}{dt} = \dfrac{50\,000 - 600h}{200}$

$200\dfrac{dh}{dt} = 250 - 3h$

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(b) It is given that when $t = 0$, $h = 50$.

Find the time taken for the depth of water in the tank to reach 80 cm. Give your answer correct to 2 significant figures. [5]

$$200\frac{dh}{dt} = 250 - 3h$$

$$\int \frac{200}{250-3h}\,dh = \int 1\,dt$$

$$\int \frac{1}{250-3h}\,dh = \int \frac{1}{200}\,dt$$

$$-\frac{1}{3}\int \frac{-3}{250-3h}\,dh = \int \frac{1}{200}\,dt$$

$$-\frac{1}{3}\ln|250-3h| = \frac{1}{200}t + C$$

Sub. $t=0$, $h=50$:

$$-\frac{1}{3}\ln|250-150| = 0 + C$$

$$C = -\frac{1}{3}\ln 100$$

$$\rightarrow -\frac{1}{3}\ln|250-3h| = \frac{1}{200}t - \frac{1}{3}\ln 100$$

Sub. $h = 80$:

$$-\frac{1}{3}\ln|250-240| = \frac{1}{200}t - \frac{1}{3}\ln 100$$

$$-\frac{1}{3}\ln 10 = \frac{1}{200}t - \frac{1}{3}\ln 100$$

$$\frac{1}{200}t = \frac{1}{3}\ln 100 - \frac{1}{3}\ln 10$$

$$t = 200\left(\frac{1}{3}\ln 100 - \frac{1}{3}\ln 10\right)$$

$$= 153.5 \quad = \underline{150} \quad (2sf)$$

10 A balloon in the shape of a sphere has volume V and radius r. Air is pumped into the balloon at a constant rate of 40π starting when time $t = 0$ and $r = 0$. At the same time, air begins to flow out of the balloon at a rate of $0.8\pi r$. The balloon remains a sphere at all times.

(a) Show that r and t satisfy the differential equation

$$\frac{dr}{dt} = \frac{50-r}{5r^2}.$$

[3]

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

* $\frac{dV}{dt} = 40\pi - 0.8\pi r$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$= \frac{1}{4\pi r^2} \times (40\pi - 0.8\pi r)$

$= \frac{40\cancel{\pi} - 0.8\cancel{\pi} r}{4\cancel{\pi} r^2}$

$= \frac{40 - 0.8r}{4r^2}$ ÷4 ÷4

$= \frac{10 - 0.2r}{r^2}$ ×5 ×5

$= \frac{50 - r}{5r^2}$ QED

(b) Find the quotient and remainder when $5r^2$ is divided by $50 - r$. $-r + 50$ [3]

$$\begin{array}{r|l} & -5r - 250 \\ -r + 50 & 5r^2 + 0r + 0 \\ & 5r^2 - 250r \\ & \quad 250r + 0 \\ & \quad 250r - 12500 \\ & \quad\quad 12500 \end{array}$$

$\frac{5r^2}{50-r} = \underbrace{-5r - 250}_{\text{quotient}} + \frac{12500}{50-r}$ remainder

(c) Solve the differential equation in part **(a)**, obtaining an expression for t in terms of r. [6]

$$\frac{dr}{dt} = \frac{50 - r}{5r^2}$$

$$\frac{5r^2}{50 - r}\,dr = dt$$

$$\int \left(-5r - 250 + \frac{12500}{50 - r} \right) dr = \int 1\,dt$$

$$-\frac{5}{2}r^2 - 250r - 12500\ln|50 - r| = t + C$$

Sub. $t = 0,\ r = 0$:

$$0 - 0 - 12500\ln 50 = 0 + C$$

$$C = -12500\ln 50$$

$$-\frac{5}{2}r^2 - 250r - 12500\ln|50 - r| = t - 12500\ln 50$$

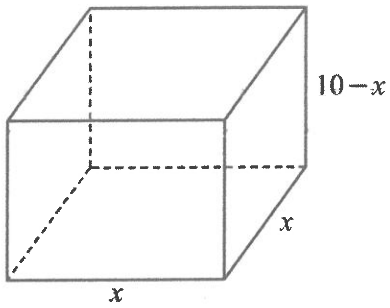
$$t = \frac{-5}{2}r^2 - 250r - 12500\ln(50 - r) + 12500\ln 50$$

(d) Find the value of t when the radius of the balloon is 12. [1]

$$t = \frac{-5}{2}(12)^2 - 250(12) - 12500\ln 38 + 12500\ln 50$$

$$= 70.5$$

9



A container in the shape of a cuboid has a square base of side x and a height of $(10-x)$. It is given that x varies with time, t, where $t > 0$. The container decreases in volume at a rate which is inversely proportional to t. *

When $t = \frac{1}{10}$, $x = \frac{1}{2}$ and the rate of decrease of x is $\frac{20}{37}$.

rate of decrease so $\frac{dx}{dt} = \frac{-20}{37}$

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{-1}{2t(20x-3x^2)}.$$

[5]

$$\text{Volume} = x \times x \times (10-x)$$

$$V = 10x^2 - x^3$$

$$\frac{dV}{dx} = 20x - 3x^2$$

$$*:\ \frac{dV}{dt} \propto \frac{-1}{t}$$

$$\frac{dV}{dt} = \frac{-k}{t}$$

$$\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dx}{dV}$$

$$\frac{dx}{dt} = \frac{-k}{t} \times \frac{1}{20x-3x^2}$$

$\frac{dx}{dt} = \frac{-20}{37}$, $t = \frac{1}{10}$, $x = \frac{1}{2}$:

$$-\frac{20}{37} = -10k \times \frac{1}{10-\frac{3}{4}}$$

$$-\frac{20}{37} = \frac{-10k}{37/4}$$

$$\frac{-20}{37} = \frac{-40k}{37}$$

$$-20 = -40k$$

$$k = \frac{1}{2}$$

$$\frac{dx}{dt} = \frac{-(\frac{1}{2})}{t} \times \frac{1}{20x-3x^2}$$

$$= \frac{-1}{2t} \times \frac{1}{20x-3x^2}$$

$$= \frac{-1}{2t(20x-3x^2)}$$

QED

(b) Solve the differential equation, obtaining an expression for t in terms of x. [6]

$$\int (20x - 3x^2)\,dx = \int \frac{-1}{2t}\,dt$$

$$\int (20x - 3x^2)\,dx = \frac{-1}{2}\int \frac{1}{t}\,dt$$

$$10x^2 - x^3 = \frac{-1}{2}\ln t + C$$

$t = \frac{1}{10}, x = \frac{1}{2}$:

$$10\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^3 = \frac{-1}{2}\ln\left(\frac{1}{10}\right) + C$$

$$\frac{10}{4} - \frac{1}{8} = \frac{1}{2}\ln 10 + C$$

$$\frac{19}{8} - \frac{1}{2}\ln 10 = C$$

$$\rightarrow 10x^2 - x^3 = \frac{-1}{2}\ln t + \frac{19}{8} - \frac{1}{2}\ln 10$$

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2 \quad \times 2$

$$20x^2 - 2x^3 = -\ln t + \frac{19}{4} - \ln 10$$

$$\ln t + \ln 10 = 2x^3 - 20x^2 + \frac{19}{4}$$

$$\ln(10t) = 2x^3 - 20x^2 + \frac{19}{4}$$

$$10t = e^{2x^3 - 20x^2 + \frac{19}{4}}$$

$$t = \frac{1}{10}e^{2x^3 - 20x^2 + \frac{19}{4}}$$

10 A large plantation of area 20 km^2 is becoming infected with a plant disease. At time t years the area infected is x km^2 and the rate of increase of x is proportional to the ratio of the area infected to the area not yet infected. *

When $t = 0$, $x = 1$ and $\frac{dx}{dt} = 1$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{19x}{20 - x}.$$

[2]

Area infected $= x$ Area not infected $= 20 - x$

* $\frac{dx}{dt} \propto \frac{x}{20-x}$

$\frac{dx}{dt} = \frac{kx}{20-x}$

$t=0, x=1, \frac{dx}{dt}=1$:

$1 = \frac{k}{20-1}$

$k = 19$

$\rightarrow \frac{dx}{dt} = \frac{19x}{20-x}$ QED

(b) Solve the differential equation and show that when $t = 1$ the value of x satisfies the equation $x = e^{0.9+0.05x}$. [5]

$\int \frac{20-x}{19x}\,dx = \int 1\,dt$

$\int \left(\frac{20}{19x} - \frac{x}{19x}\right) dx = \int 1\,dt$

$\frac{20}{19}\int \frac{1}{x}\,dx - \int \frac{1}{19}\,dx = \int 1\,dt$

$\frac{20}{19}\ln|x| - \frac{1}{19}x = t + C$

$20\ln|x| - x = 19t + 19C$

$t=0, x=1$:

$20\ln 1 - 1 = 0 + 19C$

$-1 = 19C$

$20\ln|x| - x = 19t - 1$

$t=1$:

$20\ln|x| - x = 19 - 1$

$20\ln|x| - x = 18$

$20\ln|x| = 18 + x$

$\ln|x| = 0.9 + 0.05x$

$x = e^{0.9+0.05x}$

QED

(c) Use an iterative formula based on the equation in part **(b)**, with an initial value of 2, to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

$x_{n+1} = e^{0.9 + 0.05x_n}$

$x_1 = 2.0000$

$x_2 = 2.7183$

$x_3 = 2.8177$

$x_4 = 2.8317$

$x_5 = 2.8337$

$x_6 = 2.8340$

x_4, x_5 and x_6 all round to 2.83 to 2dp, so

$x = 2.83$

(d) Calculate the value of t at which the entire plantation becomes infected. [1]

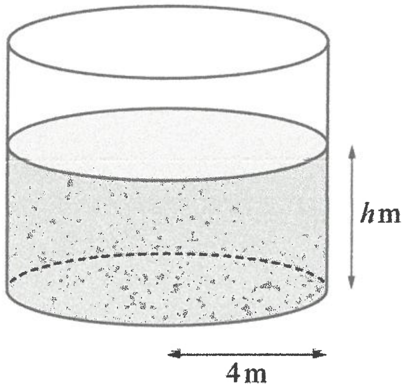
$20\ln|x| - x = 19t - 1$

$x = 20$: $20\ln 20 - 20 + 1 = 19t$

$40.9.. = 19t$

$t = 2.15$ years (3sf)

10



A large cylindrical tank is used to store water. The base of the tank is a circle of radius 4 metres. At time t minutes, the depth of the water in the tank is h metres. There is a tap at the bottom of the tank. When the tap is open, water flows out of the tank at a rate proportional to the square root of the volume of water in the tank. *

(a) Show that $\frac{dh}{dt} = -\lambda\sqrt{h}$, where λ is a positive constant. [4]

$V = \pi \times 4^2 \times h$

$= 16\pi h$

$\frac{dV}{dh} = 16\pi$

* $\frac{dV}{dt} \propto -\sqrt{V}$ water flowing out so volume decreasing.

$\frac{dV}{dt} = -k\sqrt{V}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{1}{16\pi} \times -k\sqrt{V}$ $= 16\pi h$

$= \frac{1}{16\pi} \times -k\sqrt{16\pi h}$

$= \frac{-k\sqrt{16\pi}}{16\pi} \times \sqrt{h}$

$= \frac{-4k\sqrt{\pi}}{16\pi} \times \sqrt{h}$

$= \frac{-k\sqrt{\pi}}{4\pi} \times \sqrt{h}$

$= \frac{-k}{4\sqrt{\pi}} \times \sqrt{h}$

QED

$\left(\lambda = \frac{k}{4\sqrt{\pi}}\right)$

(b) At time $t = 0$ the tap is opened. It is given that $h = 4$ when $t = 0$ and that $h = 2.25$ when $t = 20$.

Solve the differential equation to obtain an expression for t in terms of h, and hence find the time taken to empty the tank. [6]

$$\frac{dh}{dt} = -\lambda\sqrt{h}$$

$$\int \frac{1}{\sqrt{h}}\,dh = \int -\lambda\,dt$$

$$\int h^{-\frac{1}{2}}\,dh = -\int \lambda\,dt$$

$$2h^{\frac{1}{2}} = -\lambda t + C$$

Sub. $h=4$, $t=0$:

$$2(4)^{\frac{1}{2}} = 0 + C$$

$$4 = C$$

Sub. $h=2.25$, $t=20$:

$$2(2.25)^{\frac{1}{2}} = -\lambda(20) + 4 \quad C$$

$$2(1.5) = -20\lambda + 4$$

$$3 = -20\lambda + 4$$

$$20\lambda = 1$$

$$\lambda = \frac{1}{20}$$

$$\rightarrow \quad 2h^{\frac{1}{2}} = -\frac{1}{20}t + 4$$

$$\underset{\times 20}{2\sqrt{h}} = \underset{\times 20}{\frac{-1}{20}t} + \underset{\times 20}{4}$$

$$40\sqrt{h} = -t + 80$$

$$t = 80 - 40\sqrt{h}$$

When tank is empty, $h=0$:

$t = 80 - 0 = $ 80 minutes to empty

11 In a field there are 300 plants of a certain species, all of which can be infected by a particular disease. At time t after the first plant is infected there are x infected plants. The rate of change of x is proportional to the product of the number of plants infected and the number of plants that are **not** yet infected. The variables x and t are treated as continuous, and it is given that $\frac{dx}{dt} = 0.2$ and $x = 1$ when $t = 0$. *

(a) Show that x and t satisfy the differential equation

$$1495\frac{dx}{dt} = x(300 - x).$$ [2]

Infected plants = x Plants not infected: $300 - x$

* $\frac{dx}{dt} \propto x(300-x)$

$\frac{dx}{dt} = kx(300-x)$

$\frac{dx}{dt} = 0.2,\ x=1,\ t=0$:

$0.2 = k \times 1 \times (300-1)$

$0.2 = 299k$

$k = \frac{1}{1495}$

→ $\frac{dx}{dt} = \frac{1}{1495}x(300-x)$

$1495\frac{dx}{dt} = x(300-x)$ QED

(b) Using partial fractions, solve the differential equation and obtain an expression for t in terms of a single logarithm involving x. [9]

$\int \frac{1495}{x(300-x)}\,dx = \int 1\,dt$

÷1495 will make partial fractions a bit easier:

$\int \frac{1}{x(300-x)}\,dx = \int \frac{1}{1495}\,dt$

$\frac{1}{x(300-x)} \equiv \frac{A}{x} + \frac{B}{300-x}$

$1 \equiv A(300-x) + Bx$

$\underline{x = 300:}$ $1 = 0 + 300B$

$B = \dfrac{1}{300}$

$\underline{x = 0:}$ $1 = 300A + 0$

$A = \dfrac{1}{300}$

$$\int \left(\frac{\frac{1}{300}}{x} + \frac{\frac{1}{300}}{300 - x} \right) dx = \int \frac{1}{1495}\, dt$$

$$\frac{1}{300} \int \left(\frac{1}{x} + \frac{1}{300 - x} \right) dx = \int \frac{1}{1495}\, dt$$

$$\frac{1}{300} \int \left(\frac{1}{x} - \frac{-1}{300 - x} \right) dx = \int \frac{1}{1495}\, dt$$

$$\frac{1}{300} \left[\ln|x| - \ln|300 - x| \right] = \frac{1}{1495} t + C$$

$\underline{x = 1,\ t = 0:}$

$$\frac{1}{300} \left[\ln(1) - \ln(299) \right] = 0 + C$$

$$\frac{1}{300} \times -\ln 299 = C$$

$$C = \frac{-\ln 299}{300}$$

$$\frac{1}{300} \left(\ln x - \ln(300 - x) \right) = \frac{1}{1495} t - \frac{\ln 299}{300}$$

$$\ln x - \ln(300 - x) = \frac{300}{1495} t - \ln 299$$

$$\ln\left(\frac{x}{300 - x} \right) = \frac{300}{1495} t - \ln 299$$

Cont...

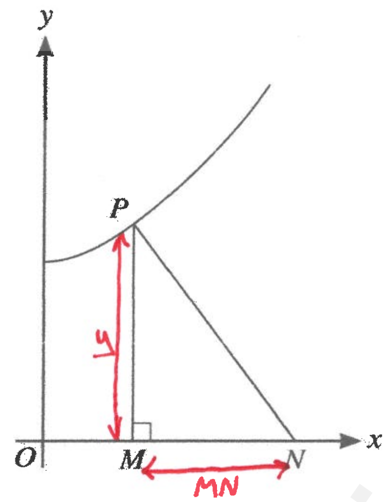
$$\ln\left(\frac{x}{300-x}\right) + \ln 299 = \frac{300}{1495}t$$

$$\ln\left(\frac{299x}{300-x}\right) = \frac{300}{1495}t$$

$$\ln\left(\frac{299x}{300-x}\right) = \frac{60}{299}t$$

$$t = \frac{299}{60}\ln\left(\frac{299x}{300-x}\right)$$

7



For the curve shown in the diagram, the normal to the curve at the point P with coordinates (x, y) meets the x-axis at N. The point M is the foot of the perpendicular from P to the x-axis.

The curve is such that for all values of x in the interval $0 \leqslant x < \frac{1}{2}\pi$, the area of triangle PMN is equal to $\tan x$. *

(a) **(i)** Show that $\dfrac{MN}{y} = \dfrac{dy}{dx}$. [1]

gradient of normal at P = $\dfrac{\Delta y}{\Delta x} = \dfrac{-y}{MN}$

gradient of curve at P = $\dfrac{-1}{\left(\frac{-y}{MN}\right)}$ (gradient of normal $= \frac{-1}{m}$)

$\dfrac{dy}{dx} = \dfrac{-1}{\frac{-y}{MN}} \rightarrow \dfrac{dy}{dx} = \dfrac{MN}{y}$

(ii) Hence show that x and y satisfy the differential equation $\frac{1}{2}y^2\dfrac{dy}{dx} = \tan x$. [2]

Area of triangle PMN $= \frac{1}{2} \times MN \times y$

from (a): $MN = y\dfrac{dy}{dx}$

$\rightarrow$ Area $= \frac{1}{2}y^2\dfrac{dy}{dx}$

* $\tan x = \frac{1}{2}y^2\dfrac{dy}{dx}$

(b) Given that $y = 1$ when $x = 0$, solve this differential equation to find the equation of the curve, expressing y in terms of x. [6]

$$\int \tan x \, dx = \int \frac{1}{2} y^2 \, dy$$

$$\int \frac{\sin x}{\cos x} \, dx = \frac{1}{2} \int y^2 \, dy$$

$$-\ln|\cos x| = \frac{1}{6} y^3 + C$$

$y = 1, x = 0:$

$$-\ln 1 = \frac{1}{6} + C$$

$$0 = \frac{1}{6} + C$$

$$C = -\frac{1}{6}$$

$$\rightarrow -\ln|\cos x| = \frac{1}{6} y^3 - \frac{1}{6}$$

$$\frac{1}{6} y^3 = \frac{1}{6} - \ln|\cos x|$$

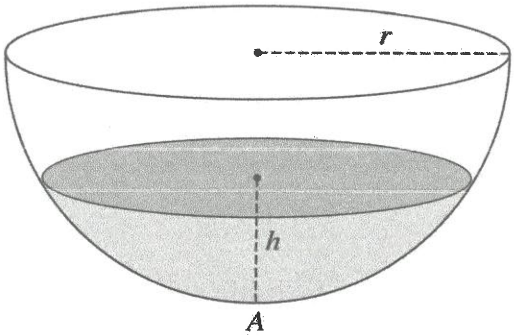
$$y^3 = 1 - 6\ln|\cos x|$$

$\cos x$ is positive when $0 \leq x \leq \frac{\pi}{2}$

$$y^3 = 1 - 6\ln\cos x$$

$$y = \sqrt[3]{1 - 6\ln\cos x}$$

10



A tank containing water is in the form of a hemisphere. The axis is vertical, the lowest point is A and the radius is r, as shown in the diagram. The depth of water at time t is h. At time $t = 0$ the tank is full and the depth of the water is r. At this instant a tap at A is opened and water begins to flow out at a rate proportional to $\sqrt{h}$.* The tank becomes empty at time $t = 14$.

The volume of water in the tank is V when the depth is h. It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

(a) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

where B is a positive constant. [4]

$V = \frac{1}{3}\pi(3rh^2 - h^3)$

$= \pi r h^2 - \frac{1}{3}\pi h^3$

$\frac{dV}{dh} = 2\pi r h - \pi h^2$

water flowing out, so volume decreasing

* $\frac{dV}{dt} \propto -\sqrt{h}$

$\frac{dV}{dt} = -k\sqrt{h}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{1}{2\pi r h - \pi h^2} \times -k\sqrt{h}$

$= -\frac{kh^{\frac{1}{2}}}{2\pi r h - \pi h^2}$

$= -\frac{k}{2\pi r h^{\frac{1}{2}} - \pi h^{3/2}}$

$= -\frac{k/\pi}{2rh^{\frac{1}{2}} - h^{3/2}}$

QED (where $B = \frac{k}{\pi}$)

(b) Solve the differential equation and obtain an expression for t in terms of h and r. [8]

$$\frac{dh}{dt} = -\frac{B}{2rh^{1/2} - h^{3/2}}$$

$$\int \left(2rh^{\frac{1}{2}} - h^{\frac{3}{2}}\right) dh = -\int B\,dt$$

$$\frac{2}{3} \times 2rh^{3/2} - \frac{2}{5}h^{5/2} = -Bt + C$$

$$\frac{4}{3}rh^{3/2} - \frac{2}{5}h^{5/2} = -Bt + C$$

At $t = 14$, $V = 0$ so $h = 0$:

$$0 - 0 = -14B + C$$

$$\underline{C = 14B}$$

$$\rightarrow \frac{4}{3}rh^{3/2} - \frac{2}{5}h^{5/2} = -Bt + 14B$$

At $t = 0$, $r = h$:

$$\frac{4}{3}h \times h^{3/2} - \frac{2}{5}h^{5/2} = -Bt + 14B$$

$$\frac{4}{3}h^{5/2} - \frac{2}{5}h^{5/2} = -\underbrace{Bt}_{=0} + 14B$$

$$\frac{14}{15}h^{5/2} = 14B$$

$$B = \frac{1}{15}h^{5/2}$$

$$\rightarrow \frac{4}{3}rh^{3/2} - \frac{2}{5}h^{5/2} = \frac{-1}{15}h^{5/2}t + \frac{14}{15}h^{5/2}$$

$$\frac{1}{15}h^{5/2}t = \frac{2}{5}h^{5/2} - \frac{4}{3}rh^{3/2} + \frac{14}{15}h^{5/2}$$

$$h^{5/2}t = 6h^{5/2} - 20rh^{3/2} + 14h^{5/2}$$

$$t = 6 - 20rh^{-1} + 14$$

$$\underline{t = 20 - 20rh^{-1}}$$