

- 4 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{xy}{1+x^2},$$

and $y = 2$ when $x = 0$.

Solve the differential equation, obtaining a simplified expression for y in terms of x .

[7]

$$\int \frac{1}{y} dy = \int \frac{x}{1+x^2} dx$$

$$\ln|y| = \frac{1}{2} \ln|1+x^2| + C$$

$$y=2, x=0:$$

$$\ln 2 = \frac{1}{2} \ln 1 + C$$

$$\ln 2 = 0 + C$$

$$C = \ln 2$$

$$\rightarrow \ln|y| = \frac{1}{2} \ln|1+x^2| + \ln 2$$

$$\ln|y| = \ln|1+x^2|^{\frac{1}{2}} + \ln 2$$

$$\ln y = \ln(2|1+x^2|^{\frac{1}{2}})$$

$$\ln y = \ln(2\sqrt{1+x^2})$$

$$y = 2\sqrt{1+x^2}$$

- 8 The variables x and y satisfy the differential equation

$$e^{4x} \frac{dy}{dx} = \cos^2 3y.$$

It is given that $y = 0$ when $x = 2$.

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

$$\int \frac{1}{\cos^2 3y} dy = \int \frac{1}{e^{4x}} dx$$

$$\int \sec^2 3y dy = \int e^{-4x} dx$$

$$\frac{1}{3} \tan 3y = -\frac{1}{4} e^{-4x} + C$$

$$y=0, x=2: \quad \frac{1}{3} \tan(0) = -\frac{1}{4} e^{-8} + C$$

$$0 = -\frac{1}{4} e^{-8} + C$$

$$C = \frac{1}{4} e^{-8}$$

$$\rightarrow \frac{1}{3} \tan 3y = -\frac{1}{4} e^{-4x} + \frac{1}{4} e^{-8}$$

$$\tan 3y = -\frac{3}{4} e^{-4x} + \frac{3}{4} e^{-8}$$

$$3y = \tan^{-1} \left(-\frac{3}{4} e^{-4x} + \frac{3}{4} e^{-8} \right)$$

$$\underline{y = \frac{1}{3} \tan^{-1} \left(-\frac{3}{4} e^{-4x} + \frac{3}{4} e^{-8} \right)}$$

- 4 The variables x and y satisfy the differential equation

$$(1 - \cos x) \frac{dy}{dx} = y \sin x.$$

It is given that $y = 4$ when $x = \pi$.

- (a) Solve the differential equation, obtaining an expression for y in terms of x .

[6]

$$\frac{dy}{dx} = \frac{y \sin x}{1 - \cos x}$$

$$\int \frac{1}{y} dy = \int \frac{\sin x}{1 - \cos x} dx$$

$$\ln|y| = \ln|1 - \cos x| + C$$

$$y=4, x=\pi:$$

$$\ln 4 = \ln|1 - \cos(\pi)| + C$$

$$\ln 4 = \ln 2 + C$$

$$C = \ln 4 - \ln 2$$

$$= \ln\left(\frac{4}{2}\right)$$

$$= \ln 2$$

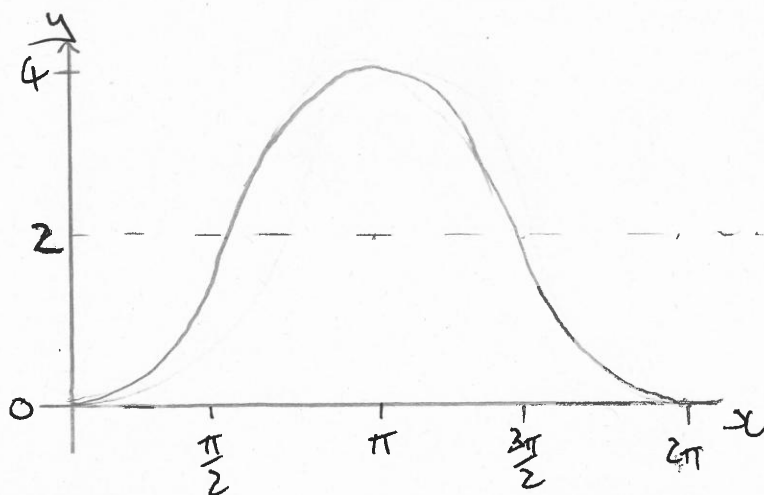
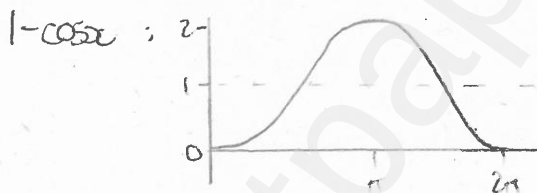
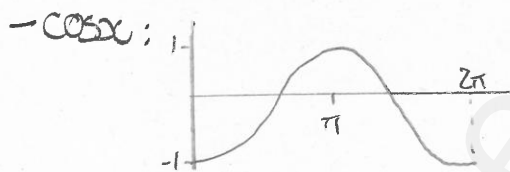
$$\ln|y| = \ln|1 - \cos x| + \ln 2$$

$$\ln|y| = \ln|2(1 - \cos x)|$$

$$\underline{y = 2(1 - \cos x)}$$

(b) Sketch the graph of y against x for $0 < x < 2\pi$.

[1]



- 9 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{3y} \sin^2 2x.$$

It is given that $y = 0$ when $x = 0$.

Solve the differential equation and find the value of y when $x = \frac{1}{2}$.

[7]

$$\int \frac{1}{e^{3y}} dy = \int \sin^2 2x dx$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\cos 4x = 1 - 2\sin^2 2x$$

$$2\sin^2 2x = 1 - \cos 4x$$

$$\sin^2 2x = \frac{1}{2} - \frac{1}{2} \cos 4x$$

$$\int e^{-3y} dy = \int \left(\frac{1}{2} - \frac{1}{2} \cos 4x \right) dx$$

$$-\frac{1}{3} e^{-3y} = \frac{1}{2} x - \frac{1}{8} \sin 4x + C$$

$$x=0, y=0:$$

$$-\frac{1}{3} e^0 = 0 - 0 + C$$

$$\underline{\underline{-\frac{1}{3} = C}}$$

$$\rightarrow -\frac{1}{3} e^{-3y} = \frac{1}{2} x - \frac{1}{8} \sin 4x - \frac{1}{3}$$

$$x = \frac{1}{2}:$$

$$-\frac{1}{3} e^{-3y} = \frac{1}{2} \left(\frac{1}{2} \right) - \frac{1}{8} \sin(2) - \frac{1}{3}$$

$$-\frac{1}{3} e^{-3y} = \frac{1}{4} - \frac{1}{3} - \frac{1}{8} \sin(2)$$

$$-\frac{1}{3} e^{-3y} = -\frac{1}{12} - \frac{1}{8} \sin(2)$$

$$e^{-3y} = \frac{1}{4} + \frac{3}{8} \sin(2)$$

$$-3y = \ln \left(\frac{1}{4} + \frac{3}{8} \sin(2) \right)$$

$$y = -\frac{1}{3} \ln \left(\frac{1}{4} + \frac{3}{8} \sin(2) \right)$$

$$\underline{\underline{y = 0.175}}$$

- 8 A certain curve is such that its gradient at a point (x, y) is proportional to $\frac{y}{x\sqrt{x}}$. The curve passes through the points with coordinates $(1, 1)$ and $(4, e)$.

(a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [8]

$$\frac{dy}{dx} \propto \frac{y}{x\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{ky}{x\sqrt{x}}$$

$$x' \times x^{\frac{1}{2}} = x^{\frac{3}{2}}$$

$$\int \frac{1}{y} dy = \int kx^{-\frac{3}{2}} dx$$

$$\ln|y| = -2kx^{-\frac{1}{2}} + C$$

$$\ln|y| = \frac{-2k}{\sqrt{x}} + C$$

sub. (1,1):

$$\ln 1 = \frac{-2k}{\sqrt{1}} + C$$

$$0 = -2k + C$$

$$C = 2k$$

$$\rightarrow \ln|y| = \frac{-2k}{\sqrt{x}} + 2k$$

sub. (4,e):

$$\ln e = \frac{-2k}{\sqrt{4}} + 2k$$

$$1 = -k + 2k$$

$$1 = k$$

$$\rightarrow \ln|y| = \frac{-2}{\sqrt{x}} + 2$$

$$y = e^{-\frac{2}{\sqrt{x}} + 2}$$

(b) Describe what happens to y as x tends to infinity.

[1]

$$\text{as } x \rightarrow \infty, \frac{-2}{\sqrt{x}} \rightarrow 0$$

$$\text{so } y = e^{\frac{-2}{\sqrt{x}}} + 2$$

$$\text{becomes } \underline{y = e^2}$$

- 7 (a) Given that $y = \ln(\ln x)$, show that

$$\frac{dy}{dx} = \frac{1}{x \ln x}. \quad [1]$$

$$y = \ln t \quad t = \ln x$$

$$\frac{dy}{dt} = \frac{1}{t} \quad \frac{dt}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1}{t} \times \frac{1}{x} = \frac{1}{\ln x} \times \frac{1}{x}$$

$$= \frac{1}{x \ln x}$$

The variables x and t satisfy the differential equation

$$x \ln x + t \frac{dx}{dt} = 0.$$

It is given that $x = e$ when $t = 2$.

- (b) Solve the differential equation obtaining an expression for x in terms of t , simplifying your answer. [7]

rearrange: $\frac{dx}{dt} = \frac{-x \ln x}{t}$

use part (a) $\rightarrow \int \frac{1}{x \ln x} dx = \int \frac{-1}{t} dt$

$$\ln(\ln x) = -\ln|t| + C$$

$x = e, t = 2:$

$$\ln(\ln e) = -\ln 2 + C$$

$$\ln 1 = -\ln 2 + C$$

$$0 = -\ln 2 + C$$

$$C = \ln 2$$

$$\rightarrow \ln(\ln x) = -\ln|t| + \ln 2 \rightarrow \ln x = \frac{2}{t}$$

$$\ln x = e^{-\ln t + \ln 2}$$

$$\ln x = e^{-\ln t} \times e^{\ln 2}$$

$$= e^{-\ln t} \times e^{\ln 2}$$

$$= t^{-1} \times 2$$

$$\underline{x = e^{\frac{2}{t}}}$$

- (c) Hence state what happens to the value of x as t tends to infinity.

[1]

$$\text{as } t \rightarrow \infty, \frac{2}{t} \rightarrow 0$$

$$\text{so } e^{\frac{2}{t}} \rightarrow e^0 = \underline{1}$$

- 7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y-1}{(x+1)(x+3)}$$

It is given that $y = 2$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x .

[9]

$$\int \frac{1}{y-1} dy = \int \frac{1}{(x+1)(x+3)} dx$$

$$\text{RHS: } \frac{1}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

$$1 = A(x+3) + B(x+1)$$

$$x = -3: 1 = 0 - 2B$$

$$B = -\frac{1}{2}$$

$$x = -1:$$

$$1 = 2A + 0$$

$$A = \frac{1}{2}$$

$$\rightarrow \int \frac{1}{y-1} dy = \int \left(\frac{\frac{1}{2}}{x+1} - \frac{\frac{1}{2}}{x+3} \right) dx$$

$$\int \frac{1}{y-1} dy = \frac{1}{2} \int \frac{1}{x+1} dx - \frac{1}{2} \int \frac{1}{x+3} dx$$

$$\ln|y-1| = \frac{1}{2} \ln|x+1| - \frac{1}{2} \ln|x+3| + C$$

$$\ln|y-1| = \ln|x+1|^{\frac{1}{2}} - \ln|x+3|^{\frac{1}{2}} + C$$

$$\ln|y-1| = \ln \left| \frac{x+1}{x+3} \right|^{\frac{1}{2}} + C$$

$$y-1 = e^{\ln \left| \frac{x+1}{x+3} \right|^{\frac{1}{2}} + C}$$

$$y-1 = e^c \times e^{\ln \left| \frac{x+1}{x+3} \right|^{\frac{1}{2}}}$$

$$y-1 = A e^{\ln \left| \frac{x+1}{x+3} \right|^{\frac{1}{2}}}$$

$$y-1 = A \times \sqrt{\frac{x+1}{x+3}}$$

$$y=2, x=0:$$

$$2-1 = A \times \sqrt{\frac{1}{3}}$$

$$1 = A \times \frac{1}{\sqrt{3}}$$

$$A = \sqrt{3}$$

$$\rightarrow y-1 = \sqrt{3} \times \sqrt{\frac{x+1}{x+3}}$$

$$y-1 = \sqrt{\frac{3(x+1)}{x+3}}$$

$$\underline{y = 1 + \sqrt{\frac{3(x+1)}{x+3}}}$$

- 7 The variables x and θ satisfy the differential equation

$$x \sin^2 \theta \frac{dx}{d\theta} = \tan^2 \theta - 2 \cot \theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

- (a) Show that $\frac{d}{d\theta}(\cot^2 \theta) = -\frac{2 \cot \theta}{\sin^2 \theta}$.

(You may assume without proof that the derivative of $\cot \theta$ with respect to θ is $-\operatorname{cosec}^2 \theta$.) [1]

$$\begin{aligned} \frac{d}{d\theta} (\cot \theta)^2 &= 2(\cot \theta)' x - \operatorname{cosec}^2 \theta \quad (\text{chain rule}) \\ &= -2 \cot \theta \times \frac{1}{\sin^2 \theta} \\ &= \frac{-2 \cot \theta}{\sin^2 \theta} \quad \text{QED} \end{aligned}$$

- (b) Solve the differential equation and find the value of x when $\theta = \frac{1}{6}\pi$. [7]

$$\int x dx = \int \frac{1}{\sin^2 \theta} (\tan^2 \theta - 2 \cot \theta) d\theta$$

$$\int x dx = \int \left(\frac{1}{\sin^2 \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{2 \cot \theta}{\sin^2 \theta} \right) d\theta$$

$$\int x dx = \int \frac{1}{\cos^2 \theta} d\theta - \int \frac{2 \cot \theta}{\sin^2 \theta} d\theta$$

$$\int x dx = \int \sec^2 \theta d\theta - \int \frac{2 \cot \theta}{\sin^2 \theta} d\theta \quad \checkmark \text{ use part (a)}$$

$$\frac{1}{2} x^2 = \tan \theta + \cot^2 \theta + C$$

$$x=2, \theta=\frac{\pi}{4}:$$

$$\frac{1}{2}x^2 = \tan\left(\frac{\pi}{4}\right) + \cot^2\left(\frac{\pi}{4}\right) + C$$

$$2 = 1 + 1 + C$$

$$\underline{C = 0}$$

$$\rightarrow \underline{\frac{1}{2}x^2 = \tan\theta + \cot^2\theta}$$

$$\text{When } \theta = \frac{\pi}{6}:$$

$$\frac{1}{2}x^2 = \tan\left(\frac{\pi}{6}\right) + \cot^2\left(\frac{\pi}{6}\right)$$

$$\frac{1}{2}x^2 = \frac{1}{\sqrt{3}} + (\sqrt{3})^2$$

$$\frac{1}{2}x^2 = \frac{1}{\sqrt{3}} + 3$$

$$x^2 = \frac{2}{\sqrt{3}} + 6$$

$$x = \sqrt{\frac{2}{\sqrt{3}} + 6}$$

$$= \underline{2.67} \text{ (3sf)}$$

- 8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x \frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x .

[6]

$$\int \frac{1}{y} dy = \int \frac{1 - 2x^2}{x} dx$$

$$\int \frac{1}{y} dy = \int \frac{1}{x} dx - \int \frac{2x^2}{x} dx$$

$$\int \frac{1}{y} dy = \int \frac{1}{x} dx - \int 2x dx$$

$$\ln|y| = \ln|x| - x^2 + C$$

$$y=1, x=1:$$

$$\ln 1 = \ln 1 - 1 + C$$

$$0 = 0 - 1 + C$$

$$C = 1$$

$$\rightarrow \ln|y| = \ln|x| - x^2 + 1$$

$$y = e^{\ln|x| - x^2 + 1}$$

$$y = e^{\ln|x|} \times e^{1-x^2}$$

$$\underline{y = xe^{1-x^2}}$$

- 8 (a) The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{4 + 9y^2}{e^{2x+1}}.$$

It is given that $y = 0$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

$$\int \frac{1}{4 + 9y^2} dy = \int \frac{1}{e^{2x+1}} dx$$

$$\int \frac{\frac{1}{9}}{\frac{4}{9} + y^2} dy = \int e^{-2x-1} dx$$

$$\frac{1}{9} \int \frac{1}{\frac{4}{9} + y^2} dy = \int e^{-2x-1} dx$$

$$\int \frac{1}{y^2 + a^2} \text{ with } a^2 = \frac{4}{9} \text{ so } a = \frac{2}{3}$$

$$\frac{1}{9} \times \frac{3}{2} \tan^{-1}\left(\frac{3}{2}y\right) = \frac{-1}{2}e^{-2x-1} + C$$

$$y=0, x=1:$$

$$\frac{1}{6} \tan^{-1}(0) = \frac{-1}{2}e^{-3} + C$$

$$0 = \frac{-1}{2}e^{-3} + C$$

$$C = \frac{1}{2}e^{-3}$$

$$\rightarrow \frac{1}{6} \tan^{-1}\left(\frac{3}{2}y\right) = \frac{-1}{2}e^{-2x-1} + \frac{1}{2}e^{-3}$$

$$\tan^{-1}\left(\frac{3y}{2}\right) = -3e^{-2x-1} + 3e^{-3}$$

$$\frac{3y}{2} = \tan(-3e^{-2x-1} + 3e^{-3})$$

cont..

$$3y = 2 \tan(-3e^{-2x-1} + 3e^{-3})$$

$$\underline{y = \frac{2}{3} \tan(-3e^{-2x-1} + 3e^{-3})}$$

- (b) State what happens to the value of y as x tends to infinity. Give your answer in an exact form.

[1]

$$\text{as } x \rightarrow \infty, -3e^{-2x-1} \rightarrow 0$$

$$\text{so } \underline{y \rightarrow \frac{2}{3} \tan(3e^{-3})}$$

- 11 The variables x and y satisfy the differential equation

$$x^2 \frac{dy}{dx} + y^2 + y = 0.$$

It is given that $x = 1$ when $y = 1$.

- (a) Solve the differential equation to obtain an expression for y in terms of x .

[8]

$$x^2 \frac{dy}{dx} = -y^2 - y$$

$$\int \frac{1}{-y^2 - y} dy = \int \frac{1}{x^2} dx$$

$$\int \frac{-1}{y^2 + y} dy = \int x^{-2} dx$$

LHS:

$$\frac{-1}{y^2 + y} = \frac{-1}{y(y+1)} = \frac{A}{y} + \frac{B}{y+1}$$

$$-1 = A(y+1) + By$$

$y = -1$:

$$-1 = 0 - B$$

$$-1 = -B$$

$$B = 1$$

$y = 0$:

$$-1 = A + 0$$

$$A = -1$$

$$\rightarrow \int \frac{-1}{y} dy + \int \frac{1}{y+1} dy = \int x^{-2} dx$$

$$-\ln|y| + \ln|y+1| = -x^{-1} + C$$

$$\ln\left|\frac{y+1}{y}\right| = -x^{-1} + C$$

$$x=1, y=1:$$

$$\ln\left|\frac{2}{1}\right| = -1 + C$$

$$\ln 2 + 1 = C$$

$$\rightarrow \ln\left|\frac{y+1}{y}\right| = -\frac{1}{x} + \ln 2 + 1$$

$$\frac{y+1}{y} = e^{-\frac{1}{x} + \ln 2 + 1}$$

$$\frac{y+1}{y} = e^{-\frac{1}{x} + 1} \times e^{\ln 2}$$

$$\frac{y+1}{y} = 2e^{-\frac{1}{x} + 1}$$

$$y+1 = 2ye^{-\frac{1}{x} + 1}$$

$$y+1 = 2ye^{1-\frac{1}{x}}$$

$$y - 2ye^{1-\frac{1}{x}} = -1$$

$$y(1 - 2e^{1-\frac{1}{x}}) = -1$$

$$y = \frac{-1}{1 - 2e^{1-\frac{1}{x}}} \quad \begin{matrix} x=1 \\ x=1 \end{matrix}$$

$$y = \frac{1}{2e^{1-\frac{1}{x}} - 1}$$

- (b) State what happens to the value of y when x tends to infinity. Give your answer in an exact form.

[1]

$$\text{as } x \rightarrow \infty, e^{1-\frac{1}{x}} \rightarrow e^1$$

$$\text{so } y \rightarrow \frac{1}{2e^1 - 1}$$

$$= \frac{1}{2e - 1}$$

- 8 In a certain chemical reaction the amount, x grams, of a substance is increasing. The differential equation satisfied by x and t , the time in seconds since the reaction began, is

$$\frac{dx}{dt} = kxe^{-0.1t},$$

where k is a positive constant. It is given that $x = 20$ at the start of the reaction.

- (a) Solve the differential equation, obtaining a relation between x , t and k .

[5]

$$\int \frac{1}{x} dx = \int ke^{-0.1t} dt$$

$$\ln|x| = -10ke^{-0.1t} + C$$

$$x=20, t=0:$$

$$\ln 20 = -10k + C$$

$$C = \ln 20 + 10k$$

$$\ln x = -10ke^{-0.1t} + 10k + \ln 20$$

- (b) Given that $x = 40$ when $t = 10$, find the value of k and find the value approached by x as t becomes large. [3]

$$x = 40, t = 10: \ln 40 = -10ke^{-1} + 10k + \ln 20$$

$$\ln 40 - \ln 20 = k(-10e^{-1} + 10)$$

$$\ln\left(\frac{40}{20}\right) = k(10 - 10e^{-1})$$

$$\ln 2 = k(10 - 10e^{-1})$$

$$k = \frac{\ln 2}{10 - 10e^{-1}}$$

$$\text{As } t \rightarrow \infty, e^{-0.1t} \rightarrow 0$$

$$\text{so } \ln x = -10ke^{-0.1t} + 10k + \ln 20$$

$$\text{becomes: } \ln x = 10k + \ln 20$$

$$\ln x = \frac{10 \ln 2}{10 - 10e^{-1}} + \ln 20$$

$$x = \exp\left(\frac{10 \ln 2}{10 - 10e^{-1}} + \ln 20\right)$$

$$\rightarrow \underline{x = 59.9 \text{ (3sf)}}$$

- 10 (a) By writing $y = \sec^3 \theta$ as $\frac{1}{\cos^3 \theta}$, show that $\frac{dy}{d\theta} = 3 \sin \theta \sec^4 \theta$. [2]

$$y = (\cos \theta)^{-3}$$

$$\frac{dy}{d\theta} = -3(\cos \theta)^{-4} \times -\sin \theta$$

$$= 3 \sin \theta \cos^{-4} \theta$$

$$= 3 \sin \theta \times \frac{1}{\cos^4 \theta}$$

$$= \underline{\underline{3 \sin \theta \sec^4 \theta}}$$

- (b) The variables x and θ satisfy the differential equation

$$(x^2 + 9) \sin \theta \frac{d\theta}{dx} = (x + 3) \cos^4 \theta.$$

It is given that $x = 3$ when $\theta = \frac{1}{3}\pi$.

Solve the differential equation to find the value of $\cos \theta$ when $x = 0$. Give your answer correct to 3 significant figures. [8]

$$\sin \theta \frac{d\theta}{dx} = \frac{(x+3) \cos^4 \theta}{x^2 + 9}$$

$$\int \frac{\sin \theta}{\cos^4 \theta} d\theta = \int \frac{x+3}{x^2+9} dx$$

$$\int \sin \theta \sec^4 \theta d\theta = \int \frac{x+3}{x^2+9} dx$$

$$\frac{1}{3} \int 3 \sin \theta \sec^4 \theta d\theta = \int \frac{x}{x^2+9} dx + \int \frac{3}{x^2+9} dx$$

$$\frac{1}{3} \int 3 \sin \theta \sec^4 \theta d\theta = \frac{1}{2} \int \frac{2x}{x^2+9} dx + 3 \int \frac{1}{x^2+9} dx$$

from part (a)

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$$\frac{1}{3} \times \frac{1}{\cos^3 \theta} = \frac{1}{2} \ln|x^2+9| + 3 \times \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\frac{1}{3\cos^3 \theta} = \frac{1}{2} \ln|x^2+9| + \tan^{-1}\left(\frac{x}{3}\right) + C$$

$x=3, \theta = \frac{\pi}{3}:$

$$\frac{1}{3\cos^3(\frac{\pi}{3})} = \frac{1}{2} \ln|3^2+9| + \tan^{-1}\left(\frac{3}{3}\right) + C$$

$$\frac{1}{3 \times (\frac{1}{2})^3} = \frac{1}{2} \ln|8| + \tan^{-1}(1) + C$$

$$\frac{1}{\frac{3}{8}} = \frac{1}{2} \ln|8| + \frac{\pi}{4} + C$$

$$\frac{8}{3} = \frac{1}{2} \ln|8| + \frac{\pi}{4} + C$$

$$C = \frac{8}{3} - \frac{1}{2} \ln|8| - \frac{\pi}{4}$$

$$\rightarrow \frac{1}{3\cos^3 \theta} = \frac{1}{2} \ln|x^2+9| + \tan^{-1}\left(\frac{x}{3}\right) + \frac{8}{3} - \frac{1}{2} \ln|8| - \frac{\pi}{4}$$

$x=0:$

$$\frac{1}{3\cos^3 \theta} = \frac{1}{2} \ln 9 + 0 + \frac{8}{3} - \frac{1}{2} \ln|8| - \frac{\pi}{4}$$

$$\frac{1}{\cos^3 \theta} = \frac{3}{2} \ln 9 + 8 - \frac{3}{2} \ln|8| - \frac{3\pi}{4}$$

$$\frac{1}{\cos \theta} = \left(\frac{3}{2} \ln 9 + 8 - \frac{3}{2} \ln|8| - \frac{3\pi}{4} \right)^{\frac{1}{3}}$$

$$\cos \theta = \left(\frac{3}{2} \ln 9 + 8 - \frac{3}{2} \ln|8| - \frac{3\pi}{4} \right)^{-\frac{1}{3}}$$

$$= \underline{\underline{0.601}}$$