



- 1 A curve has equation $y = 5 + 3x - 2x^2$ and a straight line has equation $y = kx + 13$, where k is a constant.

Find the set of values of k for which the curve and the line do **not** meet.

[4]

Solve simultaneously:

$$-2x^2 + 3x + 5 = kx + 13$$

$$-2x^2 + 3x - kx - 8 = 0$$

$$-2x^2 + (3-k)x - 8 = 0$$

Curve and line do not intersect, so $b^2 - 4ac < 0$:

$$(3-k)^2 - 4(-2)(-8) < 0$$

$$9 - 6k + k^2 - 64 < 0$$

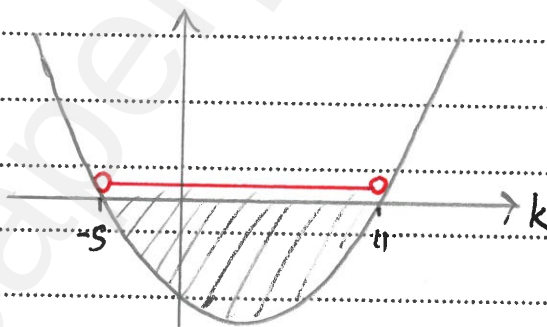
$$k^2 - 6k - 55 < 0$$

$$(k-11)(k+5) < 0$$

Critical values:

$$(k-11)(k+5) = 0$$

$$k=11 \text{ or } k=-5$$



$$\underline{\underline{-5 < k < 11}}$$



- 2 A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points.

[5]

Solve simultaneously:

$$x^2 + 2cx + 4 = 4x + c$$

$$x^2 + 2cx - 4x + 4 - c = 0$$

$$x^2 + (2c - 4)x + 4 - c = 0$$

Two distinct points of intersection, so $b^2 - 4ac > 0$

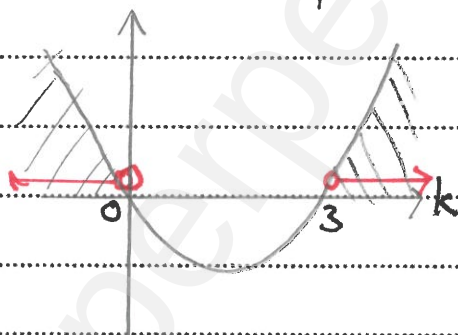
$$(2c - 4)^2 - 4(1)(4 - c) > 0$$

$$4c^2 - 16c + 16 - 16 + 4c > 0$$

$$4c^2 - 12c > 0$$

$$4c(c - 3) > 0$$

critical values: $c = 0$, $c = 3$



so $k < 0$, $k = 3$

- 2 A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect.

[5]

Solve simultaneously:

$$kx^2 + 2x - k = kx - 2$$

$$kx^2 + 2x - kx + 2 - k = 0$$

$$kx^2 + (2 - k)x + 2 - k = 0$$

Curve and line do not intersect, so $b^2 - 4ac < 0$:

$$(2 - k)^2 - 4(k)(2 - k) < 0$$

$$4 - 4k + k^2 - 4k(2 - k) < 0$$

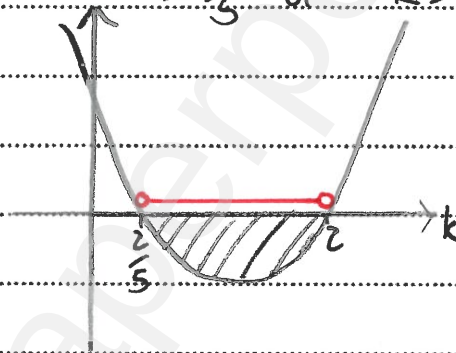
$$k^2 - 4k + 4 - 8k + 4k^2 < 0$$

$$5k^2 - 12k + 4 < 0$$

$$(5k - 2)(k - 2) < 0$$

Critical values: $5k - 2 = 0$ or $k - 2 = 0$

$$k = \frac{2}{5} \text{ or } k = 2$$



$$\underline{\underline{\frac{2}{5} < k < 2}}$$

9 The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

- (b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

Solve simultaneously:

$$x^2 + (kx - 5)^2 + 6x - 2(kx - 5) - 26 = 0$$

$$x^2 + k^2x^2 - 10kx + 25 + 6x - 2kx + 10 - 26 = 0$$

$$x^2 + k^2x^2 + 6x - 12kx + 9 = 0$$

$$(1 + k^2)x^2 + (6 - 12k)x + 9 = 0$$

Intersect at two distinct points, so $b^2 - 4ac > 0$:

$$(6 - 12k)^2 - 4(1 + k^2)(9) > 0$$

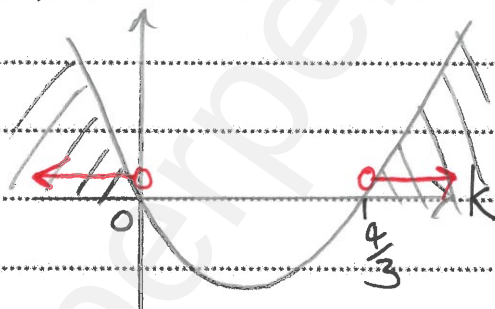
$$36 - 144k + 144k^2 - 36 - 36k^2 > 0$$

$$108k^2 - 144k > 0$$

$$3k^2 - 4k > 0$$

$$k(3k - 4) > 0$$

$$\text{critical values: } k = 0 \quad k = \frac{4}{3}$$



$$\underline{k < 0} \quad \text{or} \quad \underline{k > \frac{4}{3}}$$

- 2 A line has equation $y = 2cx + 3$ and a curve has equation $y = cx^2 + 3x - c$, where c is a constant.

Showing all necessary working, determine which of the following statements is correct.

- A The line and curve intersect only for a particular set of values of c .
 B The line and curve intersect for all values of c .
 C The line and curve do not intersect for any values of c .

[4]

Solve simultaneously:

$$cx^2 + 3x - c = 2cx + 3$$

$$cx^2 + 3x - 2cx - c - 3 = 0$$

$$cx^2 + (3 - 2c)x + (-c - 3) = 0$$

$b^2 - 4ac$:

$$(3 - 2c)^2 - 4 \times c \times (-c - 3)$$

$$= 9 - 12c + 4c^2 + 4c^2 + 12c$$

$$= \underline{8c^2 + 9}$$

$8c^2 + 9$ is positive for all values of c , so the discriminant is > 0 . This means the line and curve intersect for all values of c .

- 3 A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m , and the corresponding coordinates of the points at which the line touches the curve. [6]

Solve simultaneously:

$$x^2 - 4x + 3 = mx - 6$$

$$x^2 - 4x - mx + 9 = 0$$

$$x^2 + (-4-m)x + 9 = 0$$

Line is a tangent to the curve, so $b^2 - 4ac = 0$

$$(-4-m)^2 - 4(1)(9) = 0$$

$$16 + 8m + m^2 - 36 = 0$$

$$m^2 + 8m - 20 = 0$$

$$(m+10)(m-2) = 0$$

$$m = -10 \quad \text{or} \quad m = 2$$

$m = -10$:

$$x^2 - 4x + 3 = -10x - 6$$

$$x^2 + 6x + 9 = 0$$

$$(x+3)^2 = 0$$

$$x = -3$$

→ $y = -10x - 6$:

$$y = -10(-3) - 6$$

$$= 30 - 6$$

$$= 24$$

$$\underline{(-3, 24)}$$

$m = 2$:

$$x^2 - 4x + 3 = 2x - 6$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3$$

→ $y = 2x - 6$:

$$y = 2(3) - 6$$

$$= 6 - 6$$

$$= 0$$

$$\underline{(3, 0)}$$

- 7 The straight line $y = x + 5$ meets the curve $2x^2 + 3y^2 = k$ at a single point P .

(a) Find the value of the constant k .

[4]

Solve simultaneously:

$$2x^2 + 3(x+5)^2 = k$$

$$2x^2 + 3(x^2 + 10x + 25) = k$$

$$2x^2 + 3x^2 + 30x + 75 = k$$

$$5x^2 + 30x + 75 - k = 0$$

They meet at a single point, so $b^2 - 4ac = 0$:

$$30^2 - 4(5)(75 - k) = 0$$

$$900 - 1500 + 20k = 0$$

$$-600 + 20k = 0$$

$$20k = 600$$

$$\underline{k = 30}$$

(b) Find the coordinates of P .

[2]

sub. $k = 30$: $2x^2 + 3y^2 = 30$

solve simultaneously:

$$2x^2 + 3(x+5)^2 = 30$$

$$2x^2 + 3(x^2 + 10x + 25) = 30$$

$$2x^2 + 3x^2 + 30x + 75 = 30$$

$$5x^2 + 30x + 45 = 0 \quad \div 5$$

$$x^2 + 6x + 9 = 0$$

$$(x + 3)^2 = 0$$

$$\underline{x = -3}$$

sub. into $y = x + 5$: $y = -3 + 5$
 $\underline{y = 2}$

- 5 The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c .

[3]

rearrange $xy = 16 : y = \frac{16}{x}$

solve simultaneously: $\frac{16}{x} = mx + c$

$$16 = mx^2 + cx$$

$$mx^2 + cx - 16 = 0$$

Line is a tangent to curve, so $b^2 - 4ac = 0$:

$$c^2 - 4(m)(-16) = 0$$

$$c^2 + 64m = 0$$

$$64m = -c^2$$

$$m = \frac{-c^2}{64}$$

- (b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points.

[3]

Solve simultaneously: $\frac{16}{x} = -4x + c$

$$16 = -4x^2 + cx$$

$$4x^2 - cx + 16 = 0$$

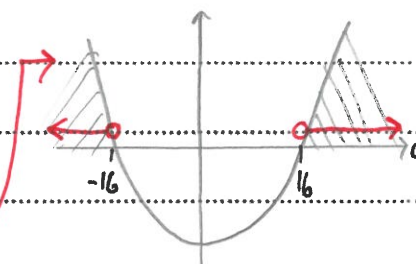
Intersect at two distinct points, so $b^2 - 4ac > 0$:

$$(-c)^2 - 4(4)(16) > 0$$

$$c^2 - 256 > 0$$

$$(c + 16)(c - 16) > 0$$

Critical values: $c = -16$ and $c = 16$



so $c < -16$ or $c > 16$

- 1 A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k .

[4]

Solve simultaneously:

$$x^2 - kx + 2 = 3x - 2k$$

$$x^2 - kx - 3x + 2 + 2k = 0$$

$$x^2 + (-k-3)x + 2 + 2k = 0$$

Find discriminant: $b^2 - 4ac$

$$(-k-3)^2 - 4(1)(2+2k)$$

$$k^2 + 6k + 9 - 8 - 8k$$

$$k^2 - 2k + 1$$

$$= (k-1)^2$$

↪ This is positive for all values of k because it is being squared.

$$\text{since } (k-1)^2 \geq 0$$

$b^2 - 4ac \geq 0$, so the curve and the line meet for all values of k .

- 5 The equation of a curve is $y = 4x^2 - kx + \frac{1}{2}k^2$ and the equation of a line is $y = x - a$, where k and a are constants.

- (a) Given that the curve and the line intersect at the points with x -coordinates 0 and $\frac{3}{4}$, find the values of k and a . [4]

Solve simultaneously:

$$4x^2 - kx + \frac{1}{2}k^2 = x - a$$

sub. $x=0$:

$$4(0)^2 - k(0) + \frac{1}{2}k^2 = 0 - a$$

$$\frac{1}{2}k^2 = -a$$

$$a = -\frac{1}{2}k^2 \quad (1)$$

sub. $x = \frac{3}{4}$:

$$4\left(\frac{3}{4}\right)^2 - k\left(\frac{3}{4}\right) + \frac{1}{2}k^2 = \frac{3}{4} - a$$

$$4 \times \frac{9}{16} - \frac{3k}{4} + \frac{k^2}{2} = \frac{3}{4} - a$$

$$\frac{9}{4} - \frac{3k}{4} + \frac{k^2}{2} = \frac{3}{4} - a$$

$$9 - 3k + 2k^2 = 3 - 4a$$

$$2k^2 - 3k + 6 = -4a \quad (2)$$

sub. (1) into (2):

$$2k^2 - 3k + 6 = -4\left(-\frac{1}{2}k^2\right)$$

$$2k^2 - 3k + 6 = 2k^2$$

$$-3k + 6 = 0$$

$$6k = 12$$

$$k = 2$$

sub. into (1): $a = -\frac{1}{2}(2)^2$

$$= -\frac{1}{2} \times 4$$

$$= -2$$

- (b) Given instead that $a = -\frac{7}{2}$, find the values of k for which the line is a tangent to the curve. [5]

$$\rightarrow y = x + \frac{7}{2}$$

solve simultaneously:

$$4x^2 - kx + \frac{1}{2}k^2 = x + \frac{7}{2}$$

$$8x^2 - 2kx + k^2 = 2x + 7$$

$$8x^2 - 2kx - 2x + k^2 - 7 = 0$$

$$8x^2 + (-2k - 2)x + k^2 - 7 = 0$$

Line is a tangent to the curve, so $b^2 - 4ac = 0$:

$$(-2k - 2)^2 - 4(8)(k^2 - 7) = 0$$

$$4k^2 + 8k + 4 - 32(k^2 - 7) = 0$$

$$4k^2 + 8k + 4 - 32k^2 + 224 = 0$$

$$-28k^2 + 8k + 228 = 0$$

$$28k^2 - 8k - 228 = 0$$

$$7k^2 - 2k - 57 = 0$$

$$k = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(-57)}}{2(7)}$$

$$\underline{k = 3} \quad \text{or} \quad \underline{k = -\frac{19}{7}}$$

- 11 The point P lies on the line with equation $y = mx + c$, where m and c are positive constants. A curve has equation $y = -\frac{m}{x}$. There is a single point P on the curve such that the straight line is a tangent to the curve at P .

(a) Find the coordinates of P , giving the y -coordinate in terms of m .

[6]

Solve simultaneously:

$$mx + c = -\frac{m}{x}$$

$$mx^2 + cx = -m$$

$$mx^2 + cx + m = 0 \quad (1)$$

Line is a tangent to the curve so $b^2 - 4ac = 0$:

$$c^2 - 4(m)(m) = 0$$

$$c^2 - 4m^2 = 0$$

$$c^2 = 4m^2$$

$$c = \pm \sqrt{4m^2}$$

$$= \pm 2m$$

c and m are both positive, so $c = 2m$ only

sub. into (1):

$$mx^2 + 2mx + m = 0$$

$$x^2 + 2x + 1 = 0$$

$$(x + 1)^2 = 0$$

$$x = -1$$

sub. into $y = mx + 2m \rightarrow (c = 2m)$

$$y = m(-1) + 2m$$

$$= -m + 2m$$

$$= m$$

$$\rightarrow (-1, m)$$

The normal to the curve at P intersects the curve again at the point Q .

(b) Find the coordinates of Q in terms of m .

[4]

$$\text{Gradient of normal} = -\frac{1}{m}$$

$$y - y_1 = m(x - x_1)$$

$$y - m = -\frac{1}{m}(x + 1)$$

$$y - m = -\frac{1}{m}x - \frac{1}{m}$$

$$y = -\frac{1}{m}x - \frac{1}{m} + m$$

This intersects curve again, so solve simultaneously with $y = -\frac{m}{x}$:

$$\frac{-m}{x} = -\frac{1}{m}x - \frac{1}{m} + m$$

$$\frac{-m^2}{x} = -x - 1 + m^2$$

$$-m^2 = -x^2 - x + m^2x$$

$$x^2 + x - m^2x - m^2 = 0$$

$$x(x+1) - m^2(x+1) = 0$$

$$(x - m^2)(x + 1) = 0$$

$$x = m^2 \quad \text{or} \quad x = -1$$

Sub into $y = -\frac{m}{x}$:

$$y = -\frac{m}{m^2}$$

$$= -\frac{1}{m}$$

$$\rightarrow (m^2, -\frac{1}{m})$$