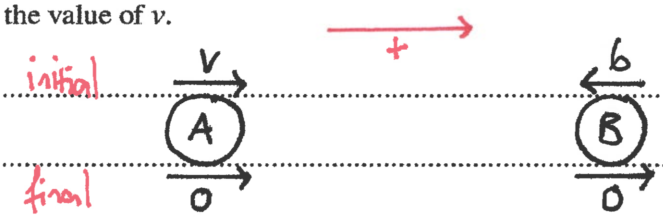
2 Two particles A and B, of masses 3.2 kg and 2.4 kg respectively, lie on a smooth horizontal table. A moves towards B with a speed of $v\,\mathrm{m\,s^{-1}}$ and collides with B, which is moving towards A with a speed of $6\,\mathrm{m\,s^{-1}}$. In the collision the two particles come to rest.

(a) Find the value of v. [2]



$$M_A U_A + M_B U_B = M_A V_A + M_B V_B$$

$$3.2 \times V + 2.4 \times -6 = 0 + 0$$

$$3.2V - 14.4 = 0$$

$$3.2V = 14.4$$

$$V = \underline{4.5\,\mathrm{ms^{-1}}}$$

(b) Find the loss of kinetic energy of the system due to the collision. [2]

Initial Kinetic Energy (both A and B moving):

$$KE_{init} = \frac{1}{2} \times 3.2 \times 4.5^2 + \frac{1}{2} \times 2.4 \times 6^2$$

$$= 32.4 + 43.2$$

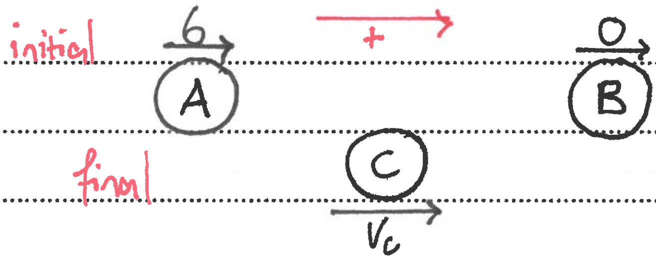
$$= 75.6\,\mathrm{J}$$

Final KE = 0 (both particles stationary)

So loss of KE = $\underline{75.6\,\mathrm{J}}$

1 A particle B of mass 5 kg is at rest on a smooth horizontal table. A particle A of mass 2.5 kg moves on the table with a speed of $6\,\mathrm{m\,s^{-1}}$ and collides directly with B. In the collision the two particles coalesce.

(a) Find the speed of the combined particle after the collision. [2]



$$m_A u_A + m_B u_B = m_C V_C$$

$$2.5 \times 6 + 0 = 7.5 \times V_C$$

$$15 = 7.5 V_C$$

$$V_C = \underline{2\,\mathrm{ms^{-1}}}$$

(b) Find the loss of kinetic energy of the system due to the collision. [3]

Initial Kinetic Energy (only A moving):

$$KE_{init} = \frac{1}{2} \times 2.5 \times 6^2$$

$$= \underline{45\,J}$$

Final Kinetic Energy (only C moving):

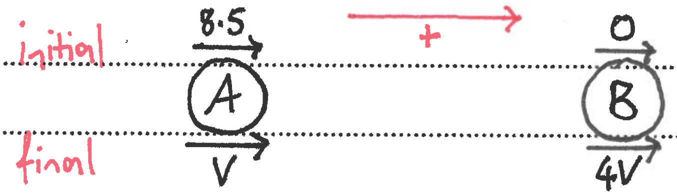
$$KE_{fin} = \frac{1}{2} \times 7.5 \times 2^2$$

$$= \underline{15\,J}$$

$$\text{Loss in KE} = 45 - 15 = \underline{30\,J}$$

1 Small smooth spheres A and B, of equal radii and of masses 5 kg and 3 kg respectively, lie on a smooth horizontal plane. Initially B is at rest and A is moving towards B with speed $8.5\,\mathrm{m\,s^{-1}}$. The spheres collide and after the collision A continues to move in the same direction but with a quarter of the speed of B.

(a) Find the speed of B after the collision. [3]



$m_A u_A + m_B u_B = m_A V_A + m_B V_B$

$5 \times 8.5 + 0 = 5V + 3 \times 4V$

$42.5 = 17V$

$V = 2.5\,\mathrm{ms^{-1}}$

$V_B = 4V = \underline{10\,\mathrm{ms^{-1}}}$

(b) Find the loss of kinetic energy of the system due to the collision. [2]

$KE_{initial} = \frac{1}{2} \times 5 \times 8.5^2$

$= \underline{180.625\,\mathrm{J}}$

$KE_{final} = \frac{1}{2} \times 5 \times 2.5^2 + \frac{1}{2} \times 3 \times 10^2$

$= 15.625 + 150$

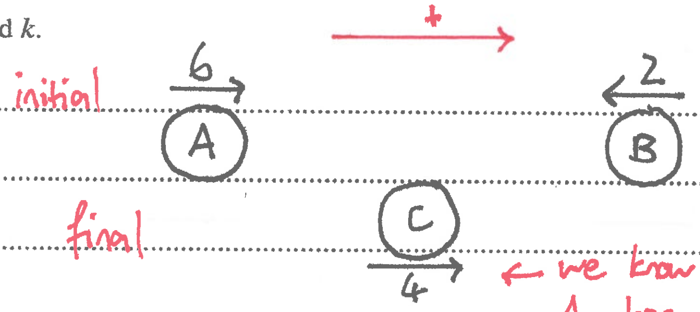
$= 165.625\,\mathrm{J}$

Loss in $KE = 180.625 - 165.625$

$= \underline{15\,\mathrm{J}}$

2 Two small smooth spheres A and B, of equal radii and of masses km kg and m kg respectively, where $k > 1$, are free to move on a smooth horizontal plane. A is moving towards B with speed $6\,\text{m s}^{-1}$ and B is moving towards A with speed $2\,\text{m s}^{-1}$. After the collision A and B coalesce and move with speed $4\,\text{m s}^{-1}$.

(a) Find k. [3]



← we know C is moving right because A has more momentum than B (6 > 2 and km > m)

$$m_A u_A + m_B u_B = m_C v_C$$
$$km \times 6 + m \times -2 = (km + m) \times 4$$
$$6km - 2m = 4km + 4m \quad \div m$$
$$6k - 2 = 4k + 4$$
$$2k = 6$$
$$\underline{k = 3}$$

(b) Find, in terms of m, the loss of kinetic energy due to the collision. [2]

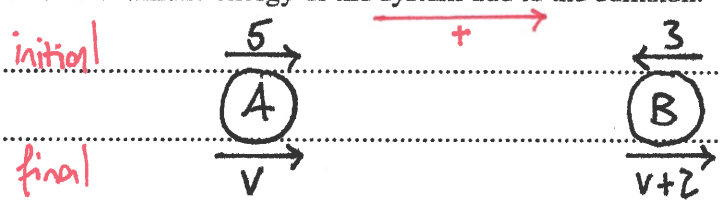
$$KE_{initial} = \frac{1}{2} \times 3m \times 6^2 + \frac{1}{2} \times m \times 2^2$$
$$= 54m + 2m$$
$$= \underline{56m\ J}$$

$$KE_{final} = \frac{1}{2} \times 4m \times 4^2$$
$$= \underline{32m\ J}$$

Loss in KE $= 56m - 32m = \underline{24m\ J}$

2 Small smooth spheres A and B, of equal radii and of masses 6 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially A is moving towards B with speed $5\,\mathrm{m\,s^{-1}}$ and B is moving towards A with speed $3\,\mathrm{m\,s^{-1}}$. After the spheres collide, both A and B move in the same direction and the difference in the speeds of the spheres is $2\,\mathrm{m\,s^{-1}}$.

Find the loss of kinetic energy of the system due to the collision. [5]



Spheres must be moving right because the momentum of A is greater than B. B must be $2\,\mathrm{ms^{-1}}$ faster than A - if it was the other way around they would coalesce.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$6 \times 5 + 2 \times -3 = 6 \times v + 2(v+2)$$
$$30 - 6 = 6v + 2v + 4$$
$$24 = 8v + 4$$
$$20 = 8v$$
$$\underline{v = 2.5\,\mathrm{ms^{-1}}}$$

$\rightarrow v_A = 2.5\,\mathrm{ms^{-1}}, \quad v_B = 4.5\,\mathrm{ms^{-1}}$

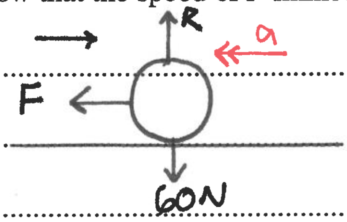
$$KE_{initial} = \tfrac{1}{2} \times 6 \times 5^2 + \tfrac{1}{2} \times 2 \times 3^2$$
$$= 75 + 9$$
$$= \underline{84\,\mathrm{J}}$$

$$KE_{final} = \tfrac{1}{2} \times 6 \times 2.5^2 + \tfrac{1}{2} \times 2 \times 4.5^2$$
$$= 18.75 + 20.25$$
$$= \underline{39\,\mathrm{J}}$$

Loss of KE $= 84 - 39 = \underline{45\,\mathrm{J}}$

4 Two particles P and Q, of masses 6 kg and 2 kg respectively, lie at rest 12.5 m apart on a rough horizontal plane. The coefficient of friction between each particle and the plane is 0.4. Particle P is projected towards Q with speed $20\,\mathrm{m\,s^{-1}}$.

(a) Show that the speed of P immediately before the collision with Q is $10\sqrt{3}\,\mathrm{m\,s^{-1}}$. [3]



$R(\rightarrow): \quad F = ma$

$-\mu R = 6a$

$-0.4 \times 60 = 6a$

$-24 = 6a$

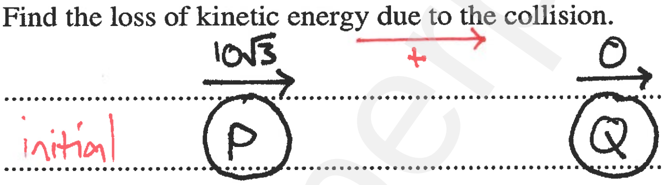
$\underline{a = -4\,\mathrm{ms^{-2}}}$

$S = 12.5$	$v^2 = u^2 + 2as$
$u = 20$	$= 20^2 + 2(-4)(12.5)$
$v =$	$= 400 - 100$
$a = -4$	$v^2 = 300$
$t =$	$v = \sqrt{300}$

$= \underline{10\sqrt{3}\ \mathrm{ms^{-1}}}$ QED

In the collision P and Q coalesce to form particle R.

(b) Find the loss of kinetic energy due to the collision. [4]



$M_P U_P + M_Q U_Q = M_R V$

$6 \times 10\sqrt{3} + 0 = 8V$

$60\sqrt{3} = 8V$

$V = \underline{7.5\sqrt{3}\ \mathrm{ms^{-1}}}$

continued ...

Initial KE (only P moving):

$KE_{init} = \frac{1}{2} \times 6 \times (10\sqrt{3})^2$
$= \frac{1}{2} \times 6 \times 300$
$= \underline{900\text{ J}}$

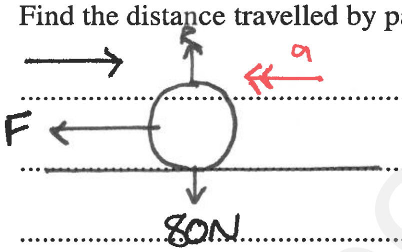
Final KE (only R moving):

$KE_{fin} = \frac{1}{2} \times 8 \times (7.5\sqrt{3})^2$
$= \frac{1}{2} \times 8 \times 168.75$
$= \underline{675\text{ J}}$

Loss of KE $= 900 - 675$
$= \underline{225\text{ J}}$

The coefficient of friction between *R* and the plane is 0.4.

(c) Find the distance travelled by particle *R* before coming to rest. [2]



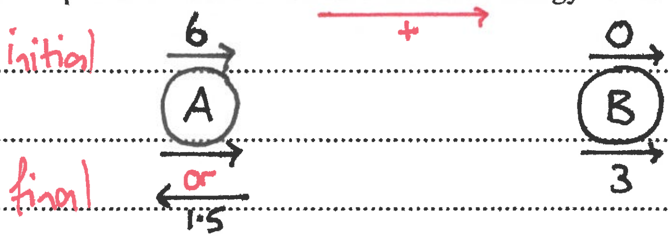
$R(\rightarrow)$: $F = ma$
$-\mu R = 8a$
$-0.4 \times 80 = 8a$
$-32 = 8a$
$\underline{a = -4\text{ ms}^{-2}}$

$s =$
$u = 7.5\sqrt{3}$
$v = 0$
$a = -4$
$t =$

$v^2 = u^2 + 2as$
$0^2 = (7.5\sqrt{3})^2 + 2(-4)s$
$0 = 168.75 - 8s$
$8s = 168.75$
$\underline{s = 21.1\text{ m}}$

4 Two small smooth spheres A and B, of equal radii and of masses 4 kg and m kg respectively, lie on a smooth horizontal plane. Initially, sphere B is at rest and A is moving towards B with speed $6\,\mathrm{m\,s^{-1}}$. After the collision A moves with speed $1.5\,\mathrm{m\,s^{-1}}$ and B moves with speed $3\,\mathrm{m\,s^{-1}}$.

Find the two possible values of the loss of kinetic energy due to the collision. [6]



Scenario 1: A continues to move to the right after collision:

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$4 \times 6 + 0 = 4 \times 1.5 + m \times 3$

$24 = 6 + 3m$

$18 = 3m$

$\underline{m = 6\text{kg}}$

Scenario 2: A rebounds and moves to the left after collision:

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$4 \times 6 + 0 = 4 \times -1.5 + m \times 3$

$24 = -6 + 3m$

$30 = 3m$

$\underline{m = 10\text{kg}}$

Scenario 1:

$KE_{init} = \frac{1}{2} \times 4 \times 6^2 = 72\text{J}$ $\quad KE_{final} = \frac{1}{2} \times 4 \times 1.5^2 + \frac{1}{2} \times 6 \times 3^2 = 31.5\text{J}$

Loss of KE $= 72 - 31.5 = \underline{40.5\text{J}}$

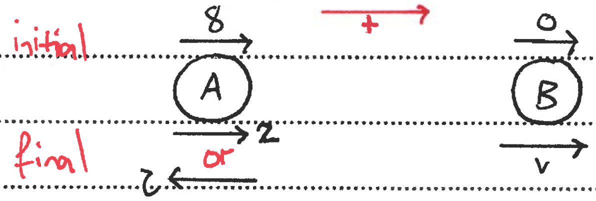
Scenario 2:

$KE_{init} = \frac{1}{2} \times 4 \times 6^2 = 72\text{J}$ $\quad KE_{final} = \frac{1}{2} \times 4 \times 1.5^2 + \frac{1}{2} \times 10 \times 3^2 = 49.5\text{J}$

Loss of KE $= 72 - 49.5 = \underline{22.5\text{J}}$

4 Two particles, A and B, of masses 3 kg and 6 kg respectively, lie on a smooth horizontal plane. Initially, B is at rest and A is moving towards B with speed $8\,\mathrm{m\,s^{-1}}$. After A and B collide, A moves with speed $2\,\mathrm{m\,s^{-1}}$.

Find the greater of the two possible total losses of kinetic energy due to the collision. [6]



Find the two possible values of v:

① If A moves right after collision:

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$3 \times 8 + 0 = 3 \times 2 + 6v_B$$
$$24 = 6 + 6v_B$$
$$6v_B = 18$$
$$v_B = 3\,\mathrm{ms^{-1}}$$

② If A moves left after collision:

$$3 \times 8 + 0 = 3 \times -2 + 6v_B$$
$$24 = -6 + 6v_B$$
$$6v_B = 30$$
$$v_B = 5\,\mathrm{ms^{-1}}$$

Greatest loss of KE is when final KE is as small as possible, so we need to pick the case where $v_B = 3\,\mathrm{ms^{-1}}$:

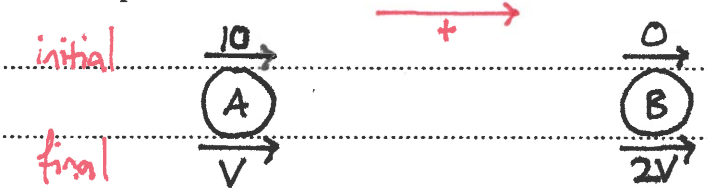
Initial KE (only A moving): $KE = \frac{1}{2} \times 3 \times 8^2$
$= 96\,\mathrm{J}$

Final KE (A and B moving): $KE = \frac{1}{2} \times 3 \times 2^2 + \frac{1}{2} \times 6 \times 3^2$
$= 6 + 27$
$= 33\,\mathrm{J}$

Loss in KE $= 96 - 33 = 63\,\mathrm{J}$

4 Small smooth spheres A and B, of equal radii and of masses 4 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially B is at rest and A is moving towards B with speed $10\,\text{m}\,\text{s}^{-1}$. After the spheres collide A continues to move in the same direction but with half the speed of B.

(a) Find the speed of B after the collision. [2]

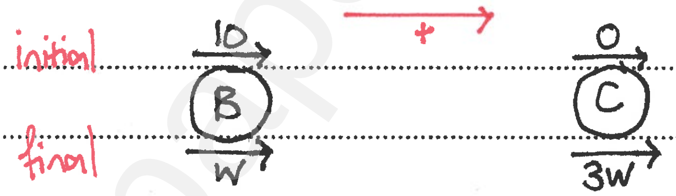


$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$4\times 10 + 0 = 4\times V + 2\times 2V$$
$$40 = 4V + 4V$$
$$40 = 8V$$
$$V = 5\,\text{ms}^{-1}$$

$$V_B = 2V$$
$$= 10\,\text{ms}^{-1}$$

A third small smooth sphere C, of mass 1 kg and with the same radius as A and B, is at rest on the plane. B now collides directly with C. After this collision B continues to move in the same direction but with one third the speed of C.

(b) Show that there is another collision between A and B. [3]

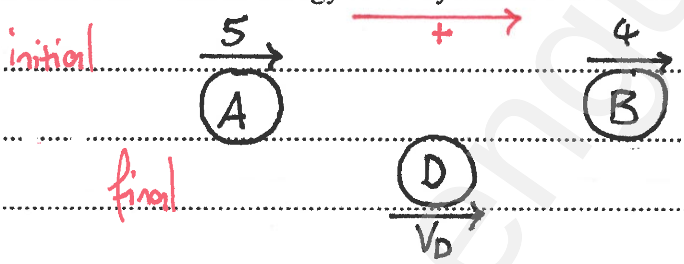


$$m_B u_B + m_C u_C = m_B v_B + m_C v_C$$
$$2\times 10 + 0 = 2\times w + 1\times 3w$$
$$20 = 2w + 3w$$
$$20 = 5w$$
$$w = 4\,\text{ms}^{-1}$$

A is travelling right at $5\,\text{ms}^{-1}$, B is travelling right at $4\,\text{ms}^{-1}$, so A will catch up with B and they will collide.

(c) *A* and *B* coalesce during this collision.

Find the total loss of kinetic energy in the system due to the three collisions. [5]



$$m_A u_A + m_B u_B = m_D V_D$$
$$4 \times 5 + 2 \times 4 = 6 \times V_D$$
$$20 + 8 = 6V_D$$
$$28 = 6V_D$$
$$V_D = \frac{14}{3}\ ms^{-1}$$

Initial KE (only A is moving):

$$KE_{init} = \frac{1}{2} \times 4 \times 10^2$$
$$= 200\,J$$

Final KE (C and D are moving):

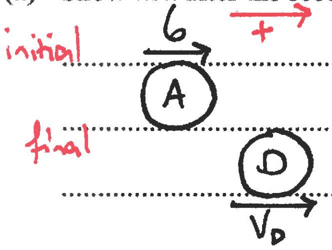
$$KE_{fin} = \frac{1}{2} \times 1 \times 12^2 + \frac{1}{2} \times 6 \times \left(\frac{14}{3}\right)^2$$
$$= 72 + \frac{196}{3}$$
$$= \frac{412}{3}\,J$$

Change in KE $= 200 - \frac{412}{3}$

$= \frac{188}{3}\,J$ (decrease)

6 Three particles A, B and C of masses 5 kg, 1 kg and 2 kg respectively lie at rest in that order on a straight smooth horizontal track XYZ. Initially A is at X, B is at Y and C is at Z. Particle A is projected towards B with a speed of $6\,\text{m s}^{-1}$ and at the same instant C is projected towards B with a speed of $v\,\text{m s}^{-1}$. In the subsequent motion, A collides and coalesces with B to form particle D. Particle D then collides and coalesces with C to form particle E and E moves towards Z.

(a) Show that after the second collision the speed of E is $\frac{15-v}{4}\,\text{m s}^{-1}$. [3]

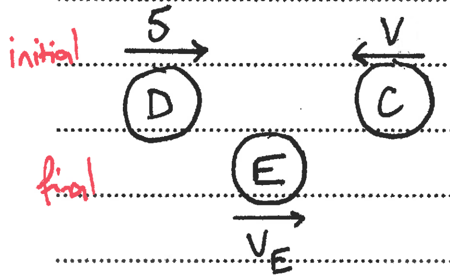


$m_A u_A + m_B u_B = m_D V_D$

$5 \times 6 + 0 = 6V_D$

$30 = 6V_D$

$V_D = 5\,\text{ms}^{-1}$



$m_D u_D + m_C u_C = m_E V_E$

$6 \times 5 + 2 \times -V = 8V_E$

$30 - 2V = 8V_E$

$V_E = \dfrac{30-2V}{8}$

$= \dfrac{15-V}{4}\,\text{ms}^{-1}$ QED

(b) The total loss of kinetic energy of the system due to the two collisions is 63 J.

Use the result from **(a)** to show that $v = 3$. [3]

Initial KE (A and C moving):

$KE_{init} = \frac{1}{2} \times 5 \times 6^2 + \frac{1}{2} \times 2 \times V^2$

$= 90 + V^2$

Final KE (only E moving):

$KE_{fin} = \frac{1}{2} \times 8 \times \left(\frac{15-V}{4}\right)^2$

$= 4 \times \dfrac{225 - 30V + V^2}{16}$

$= \frac{1}{4}(225 - 30v + V^2)$

Loss of KE = 63J :

$KE_{init} - KE_{fin} = 63$

$90 + v^2 - \frac{1}{4}(225 - 30v + v^2) = 63$ $\times 4$

$360 + 4v^2 - 225 + 30v - v^2 = 252$

$3v^2 + 30v - 117 = 0$

$v = \frac{-30 \pm \sqrt{30^2 - 4(3)(-117)}}{2(3)}$

$v = 3$ or $v = -13$

V has to be positive, or C would be moving away from B:

$v = 3 ms^{-1}$

(c) It is given that the distance *XY* is 36 m and the distance *YZ* is 98 m.

(i) Find the time between the two collisions. [4]

Collision A & B: time $= \frac{\text{distance}}{\text{speed}} = \frac{36}{6} = 6s$

In these 6 seconds, how far has C travelled from Z?

distance $= 6 \times 3 = 18m$

Collision D & C: So there is $98 - 18 = 80m$ between D & C

distance$_D$ = speed$_D$ × time   distance$_C$ = speed$_C$ × time

$d_D = 5t$ $d_C = 3t$

$\rightarrow 80 = 5t + 3t$

$80 = 8t$

$t = 10s$ between collisions

(ii) Find the time between the instant that *A* is projected from *X* and the instant that *E* reaches *Z*. [1]

How far from Z do D & C collide? → sub. into d_C (and add 18)

$3t + 18 = 30 + 18 = 48m$

How long does it take E to travel 48m? ($V_E = 3ms^{-1}$)

$t = \frac{48}{3} = 16s$

Time from start $= 6 + 10 + 16$

$= 32s$

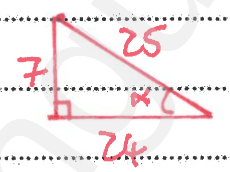
7 A bead, A, of mass 0.1 kg is threaded on a long straight rigid wire which is inclined at $\sin^{-1}(\frac{7}{25})$ to the horizontal. A is released from rest and moves down the wire. The coefficient of friction between A and the wire is μ. When A has travelled 0.45 m down the wire, its speed is $0.6\,\mathrm{m\,s^{-1}}$.

(a) Show that $\mu = 0.25$. [6]

Find acceleration:

$s = 0.45$	$v^2 = u^2 + 2as$
$u = 0$	$0.6^2 = 0^2 + 2 \times a \times 0.45$
$v = 0.6$	$0.36 = 0.9a$
$a =$	$\underline{a = 0.4\,\mathrm{ms^{-2}}}$
$t =$	

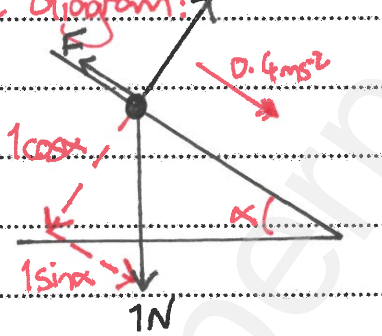
$\alpha = \sin^{-1}\left(\frac{7}{25}\right) \rightarrow$



$\cos\alpha = \frac{24}{25}$

$\tan\alpha = \frac{7}{24}$

Force diagram:



$R(\nearrow)$: $R - 1\cos\alpha = 0$

$R - 1 \times \frac{24}{25} = 0$

$\underline{R = 0.96\,N}$

$R(\searrow)$: $1\sin\alpha - F = ma$

$1 \times \frac{7}{25} - F = 0.1 \times 0.4$

$0.28 - F = 0.04$

$-F = -0.24$

$F = 0.24$

$\mu R = 0.24$

$\mu \times 0.96 = 0.24$

$\underline{\mu = 0.25}$ QED

Another bead, B, of mass 0.5 kg is also threaded on the wire. At the point where A has travelled 0.45 m down the wire, it hits B which is instantaneously at rest on the wire. A is brought to instantaneous rest in the collision. The coefficient of friction between B and the wire is 0.275.

(b) Find the time from when the collision occurs until A collides with B again. **[6]**

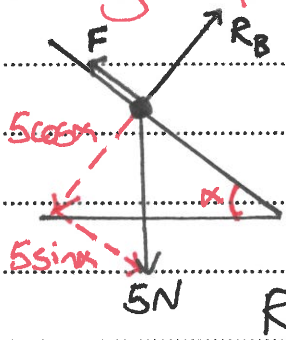
$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$0.1 \times 0.6 + 0 = 0 + 0.5 v_B$$

$$0.06 = 0.5 v_B$$

$$v_B = 0.12 \text{ ms}^{-1}$$

Force diagram for B:



$$R(\nearrow): R_B - 5\cos\alpha = 0$$

$$R_B - 5 \times \tfrac{24}{25} = 0$$

$$R_B = 4.8 \text{ N}$$

$$R(\searrow): 5\sin\alpha - F = ma$$

$$5 \times \tfrac{7}{25} - \mu R = 0.5a$$

$$1.4 - 0.275 \times 4.8 = 0.5a$$

$$1.4 - 1.32 = 0.5a$$

$$0.08 = 0.5a$$

$$a = 0.16 \text{ ms}^{-2}$$

Suvat for B:

$+$ $s = s_B$	$s_B = ut + \frac{1}{2}at^2$
$u = 0.12$	$= 0.12t + \frac{1}{2} \times 0.16t^2$
$v =$	$s_B = 0.12t + 0.08t^2$
$a = 0.16$	continued...
$t =$	

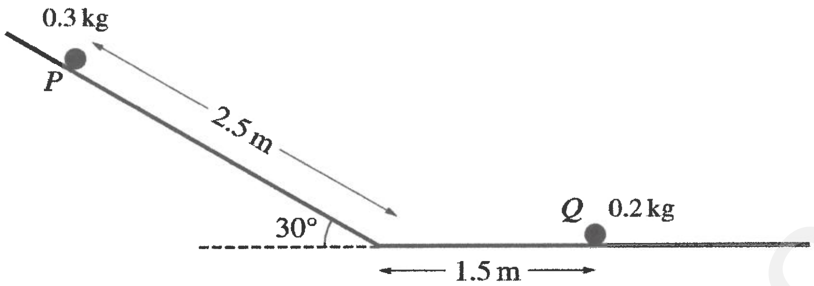
Suvat for A:

$s = s_A$ | $s_A = ut + \frac{1}{2}at^2$
$u = 0$ | $= 0 + \frac{1}{2} \times 0.4 \times t^2$
$v =$ | $s_A = 0.2t^2$
$a = 0.4$
$t =$

When they collide, $s_A = s_B$:

$$0.2t^2 = 0.12t + 0.08t^2$$
$$0.12t^2 = 0.12t$$
$$0.12t^2 - 0.12t = 0 \quad \div 0.12$$
$$t^2 - t = 0$$
$$t(t-1) = 0$$
$$t = 0 \times \quad \text{or} \quad \underline{t = 1s} \checkmark$$

7

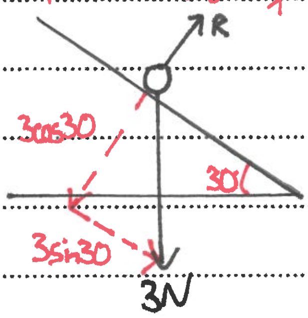


A particle P of mass 0.3 kg, lying on a smooth plane inclined at 30° to the horizontal, is released from rest. P slides down the plane for a distance of 2.5 m and then reaches a horizontal plane. There is no change in speed when P reaches the horizontal plane. A particle Q of mass 0.2 kg lies at rest on the horizontal plane 1.5 m from the end of the inclined plane (see diagram). P collides directly with Q.

(a) It is given that the horizontal plane is smooth and that, after the collision, P continues moving in the same direction, with speed $2\,\text{m}\,\text{s}^{-1}$.

Find the speed of Q after the collision. [5]

Find acceleration of P using force diagram:



R(↘): $F = ma$

$3\sin 30 = 0.3a$

$1.5 = 0.3a$

$a = 5\,\text{ms}^{-2}$

Find velocity of P at bottom of inclined plane:

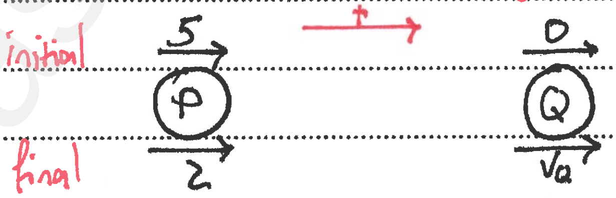
$s = 2.5$ | $v^2 = u^2 + 2as$

$u = 0$ | $= 0^2 + 2 \times 5 \times 2.5$

$v =$ | $v^2 = 25$

$a = 5$ | $v = 5\,\text{ms}^{-1}$ → horizontal plane is smooth, so P remains at this velocity until it hits Q.

$t =$



$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$

$0.3 \times 5 + 0 = 0.3 \times 2 + 0.2 \times v_Q$

$1.5 = 0.6 + 0.2v_Q$

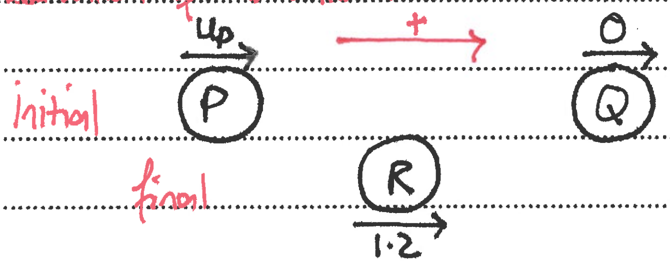
$0.9 = 0.2v_Q$

$v_Q = 4.5\,\text{ms}^{-1}$

(b) It is given instead that the horizontal plane is rough and that when P and Q collide, they coalesce and move with speed $1.2\,\mathrm{m\,s^{-1}}$.

Find the coefficient of friction between P and the horizontal plane. [5]

Conservation of momentum:

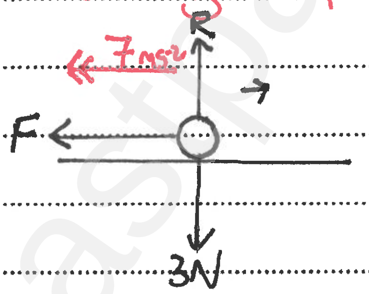


$$M_P U_P + M_Q U_Q = M_R V_R$$
$$0.3 \times U_P + 0 = 0.5 \times 1.2$$
$$0.3 u_P = 0.6$$
$$\underline{U_P = 2\,\mathrm{ms^{-1}}}$$

Suvat for P on horizontal plane:

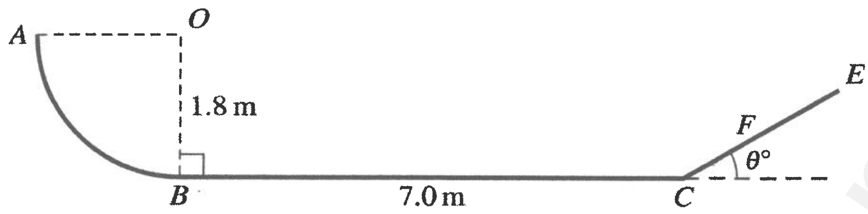
$S = 1.5$	$V^2 = u^2 + 2as$
$u = 5$	$2^2 = 5^2 + 2 \times a \times 1.5$
$V = 2$	$4 = 25 + 3a$
$a =$	$-21 = 3a$
$t =$	$\underline{a = -7\,\mathrm{ms^{-2}}}$

Force diagram for P:



$$R(\uparrow):\ R - 3 = 0$$
$$R = 3N$$
$$R(\rightarrow):\ -F = ma$$
$$-\mu R = 0.3 \times -7$$
$$-\mu \times 3 = -2.1$$
$$-\mu = -0.7$$
$$\underline{\mu = 0.7}$$

7



The diagram shows a smooth track which lies in a vertical plane. The section AB is a quarter circle of radius 1.8 m with centre O. The section BC is a horizontal straight line of length 7.0 m and OB is perpendicular to BC. The section CFE is a straight line inclined at an angle of $\theta°$ above the horizontal.

A particle P of mass 0.5 kg is released from rest at A. Particle P collides with a particle Q of mass 0.1 kg which is at rest at B. Immediately after the collision, the speed of P is 4 m s^{-1} in the direction BC. You should assume that P is moving horizontally when it collides with Q.

(a) Show that the speed of Q immediately after the collision is 10 m s^{-1}. [4]

P: $\text{Work}_{W} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$

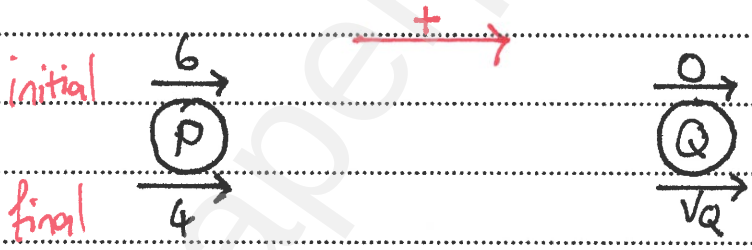
$0 + 0 + 0 = \frac{1}{2} \times 0.5 \times V^2 + 0.5 \times 10 \times -1.8 + 0$

$0 = 0.25V^2 - 9$

$0.25V^2 = 9$

$V^2 = 36$

$\underline{V = 6\ \text{ms}^{-1}}$



$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$

$0.5 \times 6 + 0 = 0.5 \times 4 + 0.1 \times V_Q$

$3 = 2 + 0.1 V_Q$

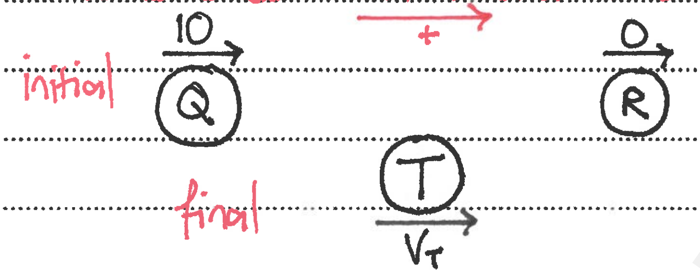
$1 = 0.1 V_Q$

$V_Q = \underline{10\ \text{ms}^{-1}}$ QED

When Q reaches C, it collides with a particle R of mass 0.4 kg which is at rest at C. The two particles coalesce. The combined particle comes instantaneously to rest at F. You should assume that there is no instantaneous change in speed as the combined particle leaves C, nor when it passes through C again as it returns down the slope.

(b) Given that the distance CF is 0.4 m, find the value of θ. [4]

B→C: Track is smooth and no other horizontal forces, so acceleration = 0



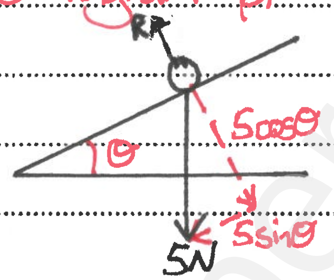
$M_Q U_Q + M_R U_R = M_T V_T$

$0.1 \times 10 + 0 = 0.5 V_T$

$1 = 0.5 V_T$

$V_T = 2 ms^{-1}$

Force diagram for T on incline:



$R(\nearrow)$: $F = ma$

$-5\sin\theta = 0.5a$

$a = -10\sin\theta$

SUVAT on incline for T:

+

$s = 0.4$	$v^2 = u^2 + 2as$
$u = 2$	$0^2 = 2^2 + 2 \times -10\sin\theta \times 0.4$
$v = 0$	$0 = 4 - 8\sin\theta$
$a = -10\sin\theta$	$8\sin\theta = 4$
$t =$	$\sin\theta = \frac{1}{2}$
	$\theta = 30°$

[Question 7 continues on the next page.]

(c) Find the distance from B at which P collides with the combined particle. [5]

Time taken for Q to go from B to C:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{7}{10}$$

$$= 0.7\text{s}$$

Time taken for coalesced particle (T) to go from C→F→C:

($a = -10\sin 30 = -5\text{ms}^{-2}$)

+↗

$s = 0$	$s = ut + \frac{1}{2}at^2$
$u = 2$	$0 = 2t + \frac{1}{2} \times -5 \times t^2$
$v =$	$0 = 2t - 2.5t^2$
$a = -5$	$2.5t^2 - 2t = 0$
$t =$	$5t^2 - 4t = 0$

$$t(5t - 4) = 0$$

$$t = 0^{\times}, \quad 5t - 4 = 0$$

$$5t = 4$$

$$t = 0.8\text{s} \checkmark$$

So after 1.5s, T is at C and travelling towards B with $v = 2\text{ms}^{-1}$.

How far along BC is P after 1.5s?

distance = speed × time

= 4 × 1.5

= 6m

→ They are 1m apart.

P: distance = speed × time

$d_P = 4t$

T: distance = speed × time

$d_T = 2t$

$$4t + 2t = 1$$

$$6t = 1$$

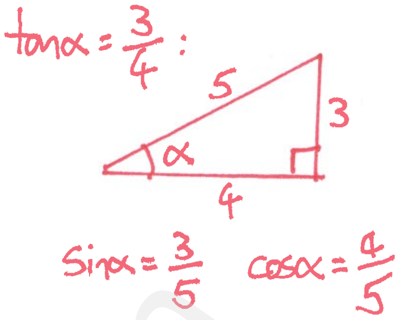
$$t = \frac{1}{6}$$

sub into d_P: $d_P = 4 \times \frac{1}{6}$

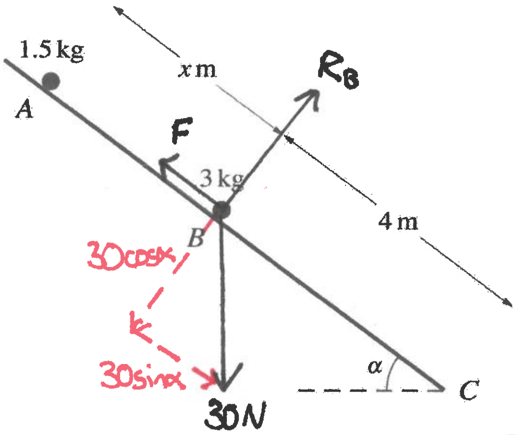
$= \frac{2}{3}\text{m}$

distance from B $= 6 + \frac{2}{3}$

$= 6\frac{2}{3}\text{m}$



7



Particles of masses 1.5 kg and 3 kg lie on a plane which is inclined at an angle of α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The section of the plane from A to B is smooth and the section of the plane from B to C is rough. The 1.5 kg particle is held at rest at A and the 3 kg particle is in limiting equilibrium at B. The distance AB is x m and the distance BC is 4 m (see diagram).

(a) Show that the coefficient of friction between the particle at B and the plane is 0.75. [3]

B: $R(\nearrow)$: $R_B - 30\cos\alpha = 0$

$R_B - 30 \times \frac{4}{5} = 0$

$R_B = 24N$

$R(\searrow)$: $30\sin\alpha - F = 0$

$30 \times \frac{3}{5} - \mu R = 0$

$18 - \mu \times 24 = 0$

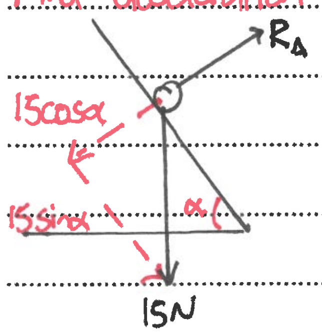
$24\mu = 18$

$\mu = 0.75$

The 1.5 kg particle is released from rest. In the subsequent motion the two particles collide and coalesce. The time taken for the combined particle to travel from B to C is 2 s. The coefficient of friction between the combined particle and the plane is still 0.75.

(b) Find x. **[6]**

Find acceleration of A using force diagram:



$R(\swarrow)$: $15\sin\alpha = ma$

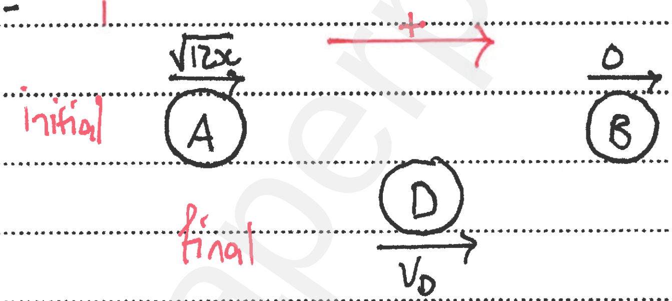
$15 \times \frac{3}{5} = 1.5a$

$9 = 1.5a$

$\underline{a = 6\,\text{ms}^{-2}}$

Suvat for $A \to B$:

$s = x$	$v^2 = u^2 + 2as$
$u = 0$	$= 0^2 + 2 \times 6 \times x$
$v =$	$v^2 = 12x$
$a = 6$	$v = \sqrt{12x}$
$t =$	



$m_A u_A + m_B u_B = m_D V_D$

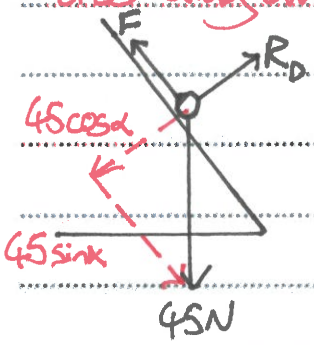
$1.5 \times \sqrt{12x} + 0 = 4.5 V_D$

$4.5 V_D = 1.5 \times \sqrt{12x}$

$\underline{V_D = \frac{1}{3}\sqrt{12x}}$

Continued..

Force diagram for coalesced particle on surface B→C:



$R(\nearrow)$: $R_D - 45\cos\alpha = 0$

$R_D = 45 \times \frac{4}{5}$

$R_D = 36N$

$R(\searrow)$: $45\sin\alpha - F = ma$

$45 \times \frac{3}{5} - \mu R = 4.5a$

$27 - 0.75 \times 36 = 4.5a$

$27 - 27 = 4.5a$

$0 = 4.5a$

$a = 0$

Since $a=0$, we can use distance = speed × time:

$4 = \frac{1}{3}\sqrt{12x} \times 2$

$12 = \sqrt{12x} \times 2$

$6 = \sqrt{12x}$

$36 = 12x$

$x = 3m$ → $V_D = \frac{1}{3}\sqrt{36} = 2ms^{-1}$

(c) Find the total loss of energy of the particles from the time the 1.5 kg particle is released until the combined particle reaches *C*. [3]

Change in potential energy of A:

$\Delta PE = 1.5 \times 10 \times -3\sin\alpha$

$= 1.5 \times 10 \times -1.8$

$= -27J$

Change in potential energy of D:

$\Delta PE = 4.5 \times 10 \times -4\sin\alpha$

$= 4.5 \times 10 \times -2.4$

$= -108J$

Initial KE = 0 because everything starts from rest.

Final KE $= \frac{1}{2} \times 4.5 \times 2^2$

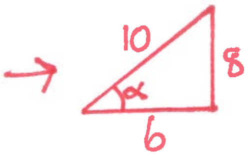
$= 9J$

Loss of energy $= -27 - 108 + 9 = -126J$

So a 126J loss

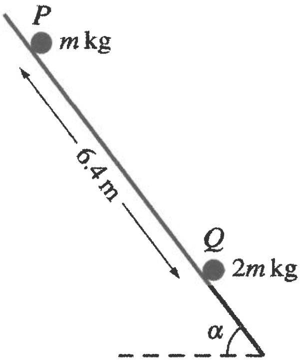
7

$\sin\alpha = 0.8$
$= \frac{8}{10}$



$\cos\alpha = \frac{6}{10} = 0.6$

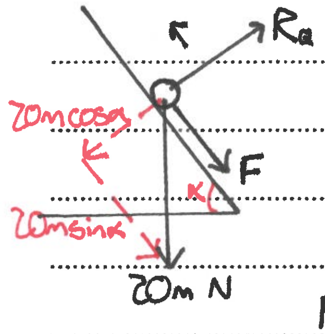
$\tan\alpha = \frac{8}{6}$



Particles P and Q have masses m kg and $2m$ kg respectively. The particles are initially held at rest 6.4 m apart on the same line of greatest slope of a rough plane inclined at an angle α to the horizontal, where $\sin\alpha = 0.8$ (see diagram). Particle P is released from rest and slides down the line of greatest slope. Simultaneously, particle Q is projected up the same line of greatest slope at a speed of $10\,\mathrm{m\,s^{-1}}$. The coefficient of friction between each particle and the plane is 0.6.

(a) Show that the acceleration of Q up the plane is $-11.6\,\mathrm{m\,s^{-2}}$. [4]

Q:



$R(\nearrow)$: $R_Q - 20m\cos\alpha = 0$

$R_Q - 20m \times 0.6 = 0$

$R_Q - 12m = 0$

$R_Q = 12m$

$R(\nwarrow)$: $-20m\sin\alpha - F = 2ma$

$-20m \times 0.8 - \mu R = 2ma$

$-16m - 0.6 \times 12m = 2ma$

$-16m - 7.2m = 2ma$

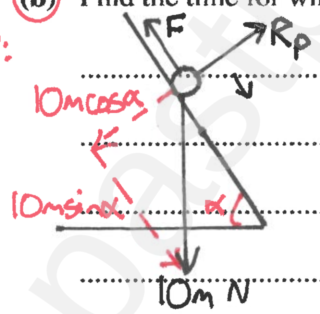
$-23.2m = 2ma \quad \div 2m$

$a = -11.6\,\mathrm{ms^{-2}}$ QED

incredibly difficult (recommend skipping)

(b) Find the time for which the particles are in motion before they collide. [5]

P:



$R(\nearrow)$: $R_P - 10m\cos\alpha = 0$

$R_P - 10m \times 0.6 = 0$

$R_P - 6m = 0$

$R_P = 6m$

continued

$R(\searrow)$: $10m\sin\alpha - F = ma$

$10m \times 0.8 - \mu R = ma$

$8m - 0.6 \times 6m = ma$

$4.4m = ma$

$a = 4.4 ms^{-2}$

* At this point we are expected to realise that Q goes up, stops, and begins to fall back down before P and Q collide. This makes the question incredibly difficult, especially for 5 marks. I can only assume a mistake was made when setting the Q. *

Note that friction changes direction when Q is falling down, so the acceleration changes.

Q: On the way up:

$\nwarrow +$		
$s =$	$v = u + at$	$v^2 = u^2 + 2as$
$u = 10$	$0 = 10 - 11.6t$	$0^2 = 10^2 + 2(-11.6) \times s$
$v = 0$	$11.6t = 10$	$0 = 100 - 23.2s$
$a = -11.6$	$t = 0.862s$ STO	$23.2s = 100$
$t =$		$s = 4.31$ m STO

So after 0.862s, Q is 4.31m up the plane.

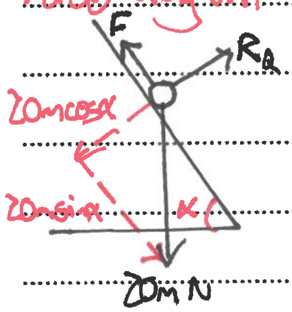
Where is P after 0.862s?

$\searrow +$		
$s =$	$s = ut + \frac{1}{2}at^2$	$v = u + at$
$u = 0$	$= 0 + \frac{1}{2} \times 4.4 \times 0.862^2$	$= 0 + 4.4 \times 0.862$
$v =$	$= 1.635$ m down the plane. STO	$= 3.79 ms^{-1}$ STO
$a = 4.4$		
$t = 0.862$		

→ Distance between P and Q: $d = 6.4 - 4.31 - 1.635$

$= 0.455m$ STO

Force diagram for Q on the way down:



$R(\searrow)$: $20m\sin\alpha - \mu R = 2ma$

$16m - 7.2m = 2ma$

$8.8m = 2ma$

$\underline{a = 4.4\text{ms}^{-2}}$

$\searrow + Q$: $s = x$, $u = 0$, $v =$, $a = 4.4$, $t =$

$s = ut + \frac{1}{2}at^2$

$x = 0 + \frac{1}{2} \times 4.4t^2$

$\underline{x = 2.2t^2}$ ①

$\searrow + P$: $s = x + 0.455$, $u = 3.79$, $v =$, $a = 4.4$, $t =$

$s = ut + \frac{1}{2}at^2$

$\underline{x + 0.455 = 3.79t + 2.2t^2}$ ②

Sub. ① → ②:

$\cancel{2.2t^2} + 0.455 = 3.79t + \cancel{2.2t^2}$

$3.79t = 0.455$

$t = 0.120$ STO

Time elapsed $= 0.862 + 0.120$

$= \underline{0.982\text{s}}$

(c) The particles coalesce on impact.

Find the speed of the combined particle immediately after the impact. [4]

Find velocities of P and Q at moment of impact:

$\searrow + P$: $s =$, $u = 0$, $v =$, $a = 4.4$, $t = 0.982$

$v = u + at$

$= 0 + 4.4(0.982)$

$= \underline{4.32\text{ ms}^{-1}}$ STO

$\searrow + Q$: $s =$, $u = 0$, $v =$, $a = 4.4$, $t = 0.120$

$v = u + at$

$= 0 + 4.4(1.20)$

$= \underline{0.527\text{ms}^{-1}}$ STO

initial: P $\xrightarrow{4.32}$ $\xrightarrow{+}$ Q $\xrightarrow{0.527}$

final: R $\xrightarrow{V_R}$

$m_P u_P + m_Q u_Q = m_R V_R$

$\cancel{m} \times 4.32 + 2\cancel{m} \times 0.527 = 3\cancel{m} V_R$

$5.375 = 3V_R$

$\underline{V_R = 1.79\text{ms}^{-1}}$