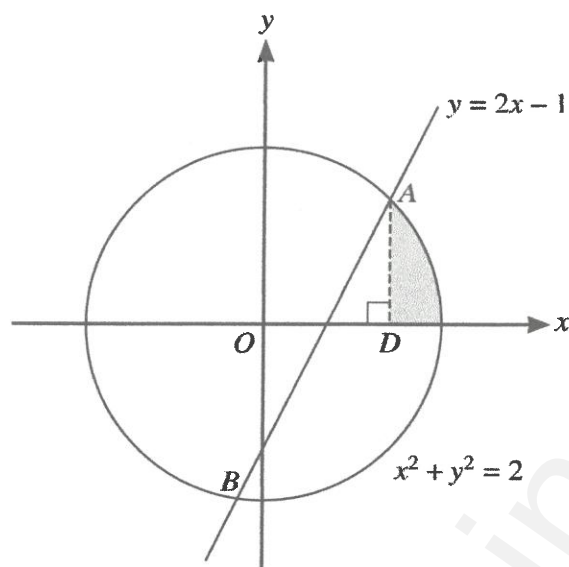




The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A.

[4]

Sub. $y = 2x - 1$ into eqⁿ of circle:

$$x^2 + (2x - 1)^2 = 2$$

$$x^2 + 4x^2 - 4x + 1 = 2$$

$$5x^2 - 4x - 1 = 0$$

$$(5x + 1)(x - 1) = 0$$

$$x = -\frac{1}{5} \text{ or } x = 1$$

point B

point A

$$\begin{aligned} x=1: \quad y &= 2(1) - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

$$\rightarrow \underline{\underline{A: (1, 1)}}$$

- 1 Points A and B have coordinates (5, 2) and (10, -1) respectively.

(a) Find the equation of the perpendicular bisector of AB.

[3]

$$\text{Mid-point of AB: } \left(\frac{5+10}{2}, \frac{2+(-1)}{2} \right) = \left(\frac{15}{2}, \frac{1}{2} \right)$$

$$\text{Gradient of AB: } \frac{-1-2}{10-5} = \frac{-3}{5}$$

$$\text{Gradient of perpendicular} = \frac{5}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = \frac{5}{3} \left(x - \frac{15}{2} \right)$$

$$y - \frac{1}{2} = \frac{5}{3}x - \frac{25}{2}$$

$$\underline{y = \frac{5}{3}x - 12}$$

(b) Find the equation of the circle with centre A which passes through B.

[3]

$$\begin{aligned} (x-5)^2 + (y-2)^2 &= r^2 \\ \text{Sub. B: } (10-5)^2 + (-1-2)^2 &= r^2 \\ 5^2 + (-3)^2 &= r^2 \\ 25 + 9 &= r^2 \\ r^2 &= 34 \end{aligned}$$

$$\underline{(x-5)^2 + (y-2)^2 = 34}$$

- 5 A circle has equation $(x-1)^2 + (y+4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection.

[4]

Sub. ② into ①: $(x-1)^2 + (x-9+4)^2 = 40$

$$(x-1)^2 + (x-5)^2 = 40$$

$$x^2 - 2x + 1 + x^2 - 10x + 25 = 40$$

$$2x^2 - 12x - 14 = 0$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$$x = 7, x = -1$$

$x = -1$: $y = -1 - 9$
 $y = -10$

$(-1, -10)$

$x = 7$: $y = 7 - 9$
 $y = -2$

$(7, -2)$

(b) Find an equation of the circle with diameter AB.

[3]

Centre = mid-point of AB = $\left(\frac{-1+7}{2}, \frac{-10-2}{2}\right)$
 $= (3, -6)$

$$(x-3)^2 + (y+6)^2 = r^2$$

Sub. in A or B: $(7, -2)$: $(7-3)^2 + (-2+6)^2 = r^2$

$$4^2 + (-4)^2 = r^2$$

$$16 + 16 = r^2$$

$$r^2 = 32$$

$\rightarrow (x-3)^2 + (y+6)^2 = 32$

- 6 Circles C_1 and C_2 have equations

$$x^2 + y^2 + 6x - 10y + 18 = 0 \quad (1) \text{ and } (x-9)^2 + (y+4)^2 - 64 = 0 \quad (2)$$

respectively.

↪ centre: $(9, -4)$, radius = 8

- (a) Find the distance between the centres of the circles.

[4]

(1): $x^2 + 6x + y^2 - 10y + 18 = 0$

$$(x+3)^2 - 9 + (y-5)^2 - 25 + 18 = 0$$

$$(x+3)^2 + (y-5)^2 = 16$$

↪ centre: $(-3, 5)$, radius = 4

Distance between centres:

$$d = \sqrt{(9 - (-3))^2 + (-4 - 5)^2}$$

$$= \sqrt{12^2 + (-9)^2}$$

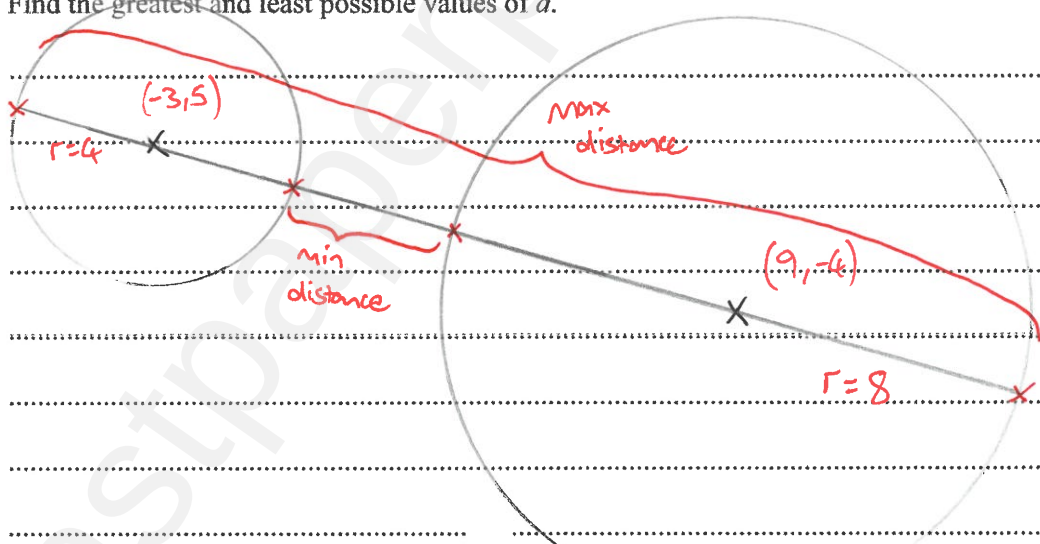
$$= \sqrt{225}$$

$$= \underline{15}$$

P and Q are points on C_1 and C_2 respectively. The distance between P and Q is denoted by d .

- (b) Find the greatest and least possible values of d .

[3]



Max distance = $15 + 8 + 4$ (radius of C_2 + distance between centres + radius of C_1)
 $= \underline{27}$

Min distance = $15 - 8 - 4$
 $= \underline{3}$



- 7 The equation of a circle is $(x-6)^2 + (y+a)^2 = 18$. The line with equation $y = 2a - x$ is a tangent to the circle.

(a) Find the two possible values of the constant a .

[5]

Solve simultaneously:

$$(x-6)^2 + (2a-x+a)^2 = 18$$

$$(x-6)^2 + (3a-x)^2 = 18$$

$$x^2 - 12x + 36 + 9a^2 - 6ax + x^2 = 18$$

$$2x^2 - 12x - 6ax + 18 + 9a^2 = 0$$

line is a tangent to the circle, so $b^2 - 4ac = 0$:

$$2x^2 + (-12-6a)x + 18 + 9a^2 = 0$$

$b^2 - 4ac = 0$:

$$(-12-6a)^2 - 4(2)(18 + 9a^2) = 0$$

$$144 + 144a + 36a^2 - 144 - 72a^2 = 0$$

$$-36a^2 + 144a = 0$$

$$-a^2 + 4a = 0$$

$$a^2 - 4a = 0$$

$$a(a-4) = 0$$

$$\underline{a=0} \quad \text{or} \quad \underline{a=4}$$

- (b) For the greater value of a , find the equation of the diameter which is perpendicular to the given tangent. [3]

$$a=4: (x-6)^2 + (y+4)^2 = 18$$

$$\rightarrow \text{Centre} = (6, -4)$$

$$\text{gradient of tangent: } y = 8 - x$$

$$\uparrow \text{gradient} = -1$$

$$\text{gradient of perpendicular} = \frac{-1}{-1} = 1$$

equation of diameter:

$$y - y_1 = m(x - x_1)$$

$$y - -4 = 1(x - 6)$$

$$y + 4 = x - 6$$

$$\underline{\underline{y = x - 10}}$$

- 7 A circle with centre (5, 2) passes through the point (7, 5).

(a) Find an equation of the circle.

[2]

$$\begin{aligned} (x-5)^2 + (y-2)^2 &= r^2 \\ \text{Sub. (7,5): } (7-5)^2 + (5-2)^2 &= r^2 \\ 2^2 + 3^2 &= r^2 \\ 4 + 9 &= r^2 \\ r^2 &= 13 \end{aligned}$$

$$\underline{(x-5)^2 + (y-2)^2 = 13}$$

The line $y = 5x - 10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB.

[7]

Sub. $y = 5x - 10$ into eqn of circle:

$$\begin{aligned} (x-5)^2 + (5x-10-2)^2 &= 13 \\ (x-5)^2 + (5x-12)^2 &= 13 \\ x^2 - 10x + 25 + 25x^2 - 120x + 144 &= 13 \\ 26x^2 - 130x + 169 &= 13 \\ 26x^2 - 130x + 156 &= 0 \\ x^2 - 5x + 6 &= 0 \\ (x-2)(x-3) &= 0 \\ x=2 \text{ or } x=3 \end{aligned}$$

continued....

$$\underline{x=2:} \quad y=5(2)-10 \\ =10-10 \\ =0$$

$$A: (2, 0)$$

$$\underline{x=3:} \quad y=5(3)-10 \\ =15-10 \\ =5$$

$$B: (3, 5)$$

$$\text{length of } AB = \sqrt{(3-2)^2 + (5-0)^2}$$

$$= \sqrt{1^2 + 5^2}$$

$$= \underline{\underline{\sqrt{26}}}$$

- 8 The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A(1, 1)$ and $B(2, -6)$ lie on the circle.

(a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

Sub. $(1, 1)$:

$$1^2 + 1^2 + a + b - 12 = 0$$

$$a + b - 10 = 0$$

$$a + b = 10 \quad (1)$$

Sub. $(2, -6)$: $2^2 + (-6)^2 + 2a - 6b - 12 = 0$

$$4 + 36 + 2a - 6b - 12 = 0$$

$$2a - 6b + 28 = 0$$

$$2a - 6b = -28 \quad (2)$$

$$(1) \times 2: 2a + 2b = 20 \quad (3)$$

$$(3) - (2): 8b = 48$$

$$b = 6$$

$$\text{Sub. into (1): } a + 6 = 10$$

$$a = 4$$

$$\rightarrow x^2 + y^2 + 4x + 6y - 12 = 0$$

$$x^2 + 4x + y^2 + 6y - 12 = 0$$

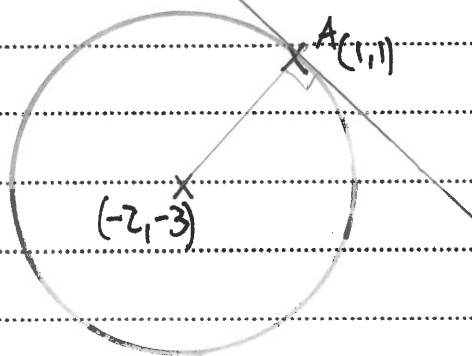
$$(x+2)^2 - 4 + (y+3)^2 - 9 - 12 = 0$$

$$(x+2)^2 + (y+3)^2 - 25 = 0$$

$$(x+2)^2 + (y+3)^2 = 25$$

$$\rightarrow \text{Centre: } \underline{\underline{(-2, -3)}}$$

- (b) Find the equation of the tangent to the circle at the point A, giving your answer in the form $px + qy = k$, where p , q and k are integers. [4]



$$\begin{aligned}\text{Gradient of radius} &= \frac{1 - (-3)}{1 - (-2)} \\ &= \frac{4}{3}\end{aligned}$$

$$\text{Gradient of tangent} = -\frac{3}{4}$$

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ y - 1 &= -\frac{3}{4}(x - 1)\end{aligned}$$

$$4y - 4 = -3(x - 1)$$

$$4y - 4 = -3x + 3$$

$$\underline{3x + 4y = 7}$$

10 The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C , for which AB is a diameter.

[4]

Mid-point = centre:

$$\left(\frac{-1+7}{2}, \frac{-2+4}{2} \right) = (3, 1)$$

$$(x-3)^2 + (y-1)^2 = r^2$$

Sub. $(7, 4)$:

$$(7-3)^2 + (4-1)^2 = r^2$$

$$4^2 + 3^2 = r^2$$

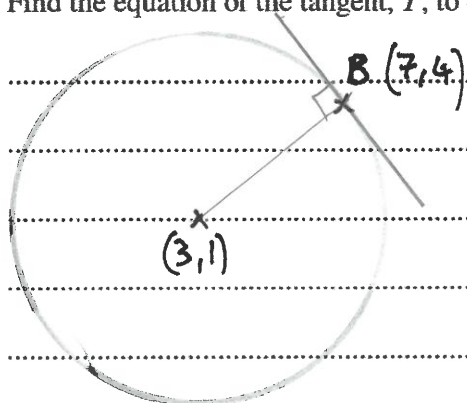
$$16 + 9 = r^2$$

$$r^2 = 25$$

$$\rightarrow \underline{(x-3)^2 + (y-1)^2 = 25}$$

- (b) Find the equation of the tangent,
- T
- , to circle
- C
- at the point
- B
- .

[4]



$$\begin{aligned}\text{Gradient of radius} &= \frac{4-1}{7-3} \\ &= \frac{3}{4}\end{aligned}$$

$$\text{Gradient of tangent} = -\frac{4}{3}$$

$$y - y_1 = m(x - x_1)$$

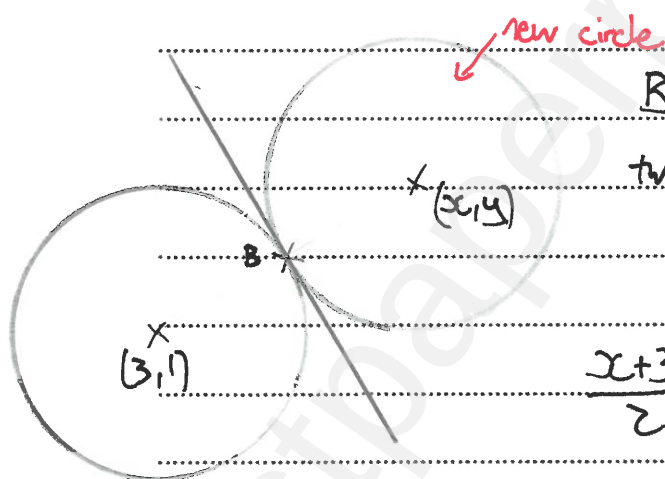
$$y - 4 = -\frac{4}{3}(x - 7)$$

$$y - 4 = -\frac{4}{3}x + \frac{28}{3}$$

$$y = -\frac{4}{3}x + \frac{40}{3}$$

- (c) Find the equation of the circle which is the reflection of circle
- C
- in the line
- T
- .

[3]



B is the mid-point of the two centres:

$$(7, 4) = \left(\frac{x+3}{2}, \frac{y+1}{2} \right)$$

$$\frac{x+3}{2} = 7$$

$$\frac{y+1}{2} = 4$$

$$x+3 = 14$$

$$y+1 = 8$$

$$x = 11$$

$$y = 7$$

radius is unchanged:

$$(x-11)^2 + (y-7)^2 = 25$$

8 The equation of a circle is $x^2 + y^2 + px + 2y + q = 0$, where p and q are constants.

- (a) Express the equation in the form $(x-a)^2 + (y-b)^2 = r^2$, where a is to be given in terms of p and r^2 is to be given in terms of p and q . [2]

$$x^2 + px + y^2 + 2y + q = 0$$

$$\left(x + \frac{p}{2}\right)^2 - \frac{p^2}{4} + (y+1)^2 - 1 + q = 0$$

$$\left(x + \frac{p}{2}\right)^2 + (y+1)^2 = 1 + \frac{p^2}{4} - q$$

$$\left(x - -\frac{p}{2}\right)^2 + (y - -1)^2 = 1 + \frac{p^2}{4} - q$$

The line with equation $x + 2y = 10$ is the tangent to the circle at the point $A(4, 3)$.

- (b) (i) Find the equation of the normal to the circle at the point A . [3]

Tangent: $2y = -x + 10$

$$y = -\frac{1}{2}x + 5$$

gradient of tangent = $-\frac{1}{2}$ gradient of normal = 2

Sub. (4,3): $y - y_1 = m(x - x_1)$

$$y - 3 = 2(x - 4)$$

$$y - 3 = 2x - 8$$

$$\underline{y = 2x - 5}$$

(ii) Find the values of p and q .*Normal to the circle goes through centre:*

$$y = 2x - 5 \text{ passes through } \left(-\frac{p}{2}, -1\right)$$

$$\text{Sub. } \left(-\frac{p}{2}, -1\right): -1 = 2\left(-\frac{p}{2}\right) - 5$$

$$-1 = -p - 5$$

$$4 = -p$$

$$\underline{p = -4}$$

$$\rightarrow \text{Eqn of circle: } \left(x + \frac{p}{2}\right)^2 + (y+1)^2 = 1 + \frac{p^2}{4} - q$$

$$\rightarrow (x - 2)^2 + (y+1)^2 = 1 + 4 - q$$

$$(x - 2)^2 + (y+1)^2 = 5 - q$$

Circle passes through (4,3) so sub this point:

$$(4 - 2)^2 + (3 + 1)^2 = 5 - q$$

$$2^2 + 4^2 = 5 - q$$

$$4 + 16 = 5 - q$$

$$20 = 5 - q$$

$$15 = -q$$

$$\underline{q = -15}$$

- 8 A circle with equation $x^2 + y^2 - 6x + 2y - 15 = 0$ meets the y -axis at the points A and B . The tangents to the circle at A and B meet at the point P .

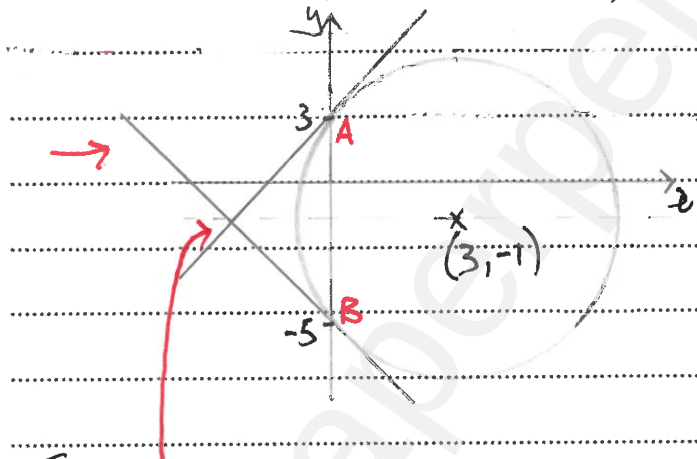
Find the coordinates of P .

[8]

$$\begin{aligned}
 x^2 - 6x + y^2 + 2y &= 15 \\
 (x-3)^2 - 9 + (y+1)^2 - 1 &= 15 \\
 (x-3)^2 + (y+1)^2 &= 25 \\
 \rightarrow \text{centre} &= (3, -1), \text{ radius} = 5
 \end{aligned}$$

Find points A & B : At y axis, $x = 0$:

$$\begin{aligned}
 0^2 + y^2 - 6(0) + 2y - 15 &= 0 \\
 y^2 + 2y - 15 &= 0 \\
 (y+5)(y-3) &= 0 \\
 y = -5 \text{ or } y = 3 \\
 \rightarrow (0, -5) \text{ and } (0, 3)
 \end{aligned}$$



Through symmetry, y -coordinate of point of intersection must be $y = -1$.

Find gradient of radius between centre and A :

$$m = \frac{-1 - 3}{3 - 0} = \frac{-4}{3}$$

gradient of tangent through A :

$$m_{\text{perp}} = \frac{3}{4}$$

Eqn of line through A:

$$y - 3 = \frac{3}{4}(x - 0)$$

$$y - 3 = \frac{3}{4}x$$

$$y = \frac{3}{4}x + 3$$

Find x -coord of point of intersection by substituting $y = -1$:

$$-1 = \frac{3}{4}x + 3$$

$$\frac{3}{4}x = -4$$

$$3x = -16$$

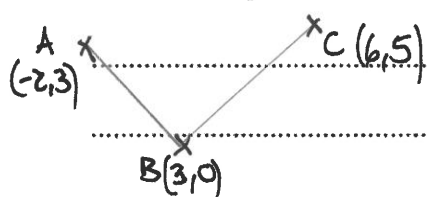
$$x = \frac{-16}{3}$$

$$\rightarrow \left(\frac{-16}{3}, -1 \right)$$

10 Points $A(-2, 3)$, $B(3, 0)$ and $C(6, 5)$ lie on the circumference of a circle with centre D .

(a) Show that angle $ABC = 90^\circ$.

[2]



$$\text{Gradient } AB = \frac{0-3}{3-(-2)} = \frac{-3}{5}$$

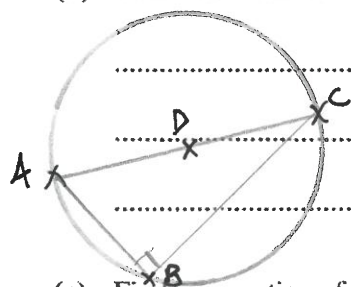
$$\text{Gradient } BC = \frac{5-0}{6-3} = \frac{5}{3}$$

$$\frac{-3}{5} \times \frac{5}{3} = -1$$

therefore they are perpendicular.

(b) Hence state the coordinates of D .

[1]



AC is a diameter, so D = mid-point of AC :

$$D = \left(\frac{-2+6}{2}, \frac{3+5}{2} \right) = \underline{(2, 4)}$$

(c) Find an equation of the circle.

[2]

$$(x-2)^2 + (y-4)^2 = r^2$$

Sub. any point: $(3, 0):$

$$(3-2)^2 + (0-4)^2 = r^2$$

$$1^2 + (-4)^2 = r^2$$

$$1 + 16 = r^2$$

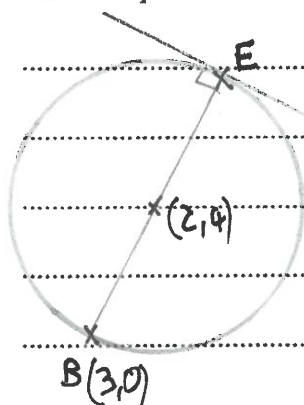
$$r^2 = 17$$

$$\rightarrow \underline{(x-2)^2 + (y-4)^2 = 17}$$

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E .

[5]



If BE is a diameter, $(2, 4)$ is mid-point of BE :

$$(2, 4) = \left(\frac{x+3}{2}, \frac{y+0}{2} \right)$$

$$\frac{x+3}{2} = 2 \quad \frac{y}{2} = 4$$

$$x+3 = 4 \quad y = 8$$

$$x = 1$$

$$\rightarrow E: (1, 8)$$

$$\text{Gradient of } BE = \frac{8-0}{1-3}$$

$$= \frac{8}{-2} = -4$$

$$\text{Gradient of tangent} = \underline{\underline{\frac{1}{4}}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = \frac{1}{4}(x - 1)$$

$$y - 8 = \frac{1}{4}x - \frac{1}{4}$$

$$\underline{\underline{y = \frac{1}{4}x + \frac{31}{4}}}$$

- 11 The coordinates of points A, B and C are (6, 4), (p, 7) and (14, 18) respectively, where p is a constant. The line AB is perpendicular to the line BC.

(a) Given that $p < 10$, find the value of p.

[4]

Gradient AB: $m_{AB} = \frac{7-4}{p-6}$

$$= \frac{3}{p-6}$$

Gradient BC: $m_{BC} = \frac{18-7}{14-p}$

$$= \frac{11}{14-p}$$

AB and BC are perpendicular:

$$m_{AB} \times m_{BC} = -1$$

$$\frac{3}{p-6} \times \frac{11}{14-p} = -1$$

$$\frac{33}{(p-6)(14-p)} = -1$$

$$33 = -(p-6)(14-p)$$

$$33 = -(14p - p^2 - 84 + 6p)$$

$$33 = p^2 - 20p + 84$$

$$p^2 - 20p + 51 = 0$$

$$(p-3)(p-17) = 0$$

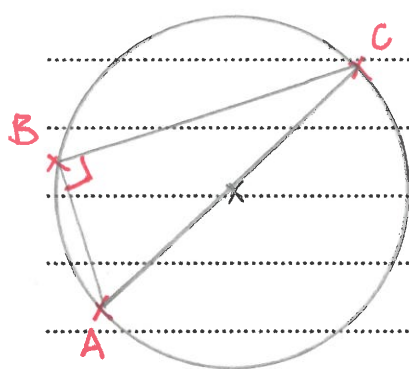
$$p = 3 \text{ or } p = 17$$

$$p < 10, \text{ so } \underline{p = 3}$$

A circle passes through the points A, B and C.

(b) Find the equation of the circle.

[3]



If ABC is a right-angle, AC must be a diameter (circle theorem)

Centre = mid-point of AC:

$$\left(\frac{6+14}{2}, \frac{4+18}{2} \right) = (10, 11)$$

$$\rightarrow (x-10)^2 + (y-11)^2 = r^2$$

Sub. A, B or C:

$$(6-10)^2 + (4-11)^2 = r^2$$

$$(-4)^2 + (-7)^2 = r^2$$

$$16 + 49 = r^2$$

$$(x-10)^2 + (y-11)^2 = 65$$

(c) Find the equation of the tangent to the circle at C, giving the answer in the form $dx + ey + f = 0$, where d , e and f are integers. [3]

Gradient of radius: $\frac{18-11}{14-10} = \frac{7}{4}$

Gradient of tangent: $m_{\text{tangent}} = \frac{-4}{7}$

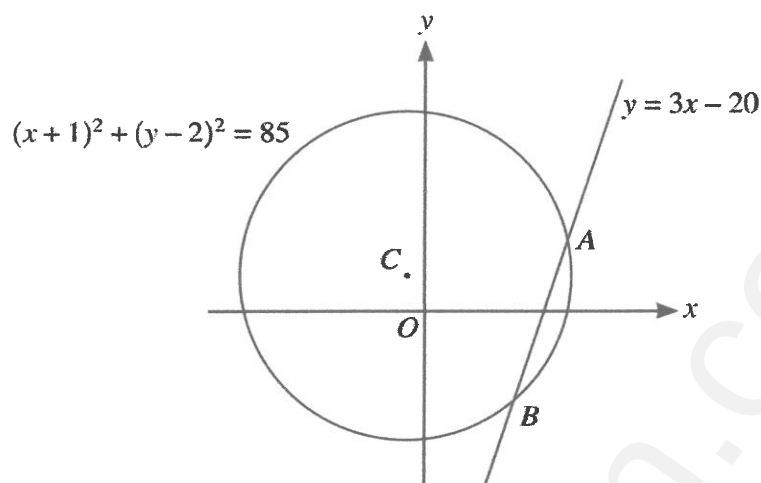
$$y - y_1 = m(x - x_1)$$

$$y - 18 = \frac{-4}{7}(x - 14) \quad \times 7$$

$$7y - 126 = -4(x - 14)$$

$$7y - 126 = -4x + 56$$

$$4x + 7y - 182 = 0$$



The circle with equation $(x+1)^2 + (y-2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

- (a) Find, by calculation, the coordinates of A and B.

[4]

sub. $y = 3x - 20$ into eqⁿ of circle:

$$(x+1)^2 + (3x-20-2)^2 = 85$$

$$(x+1)^2 + (3x-22)^2 = 85$$

$$x^2 + 2x + 1 + 9x^2 - 132x + 484 = 85$$

$$10x^2 - 130x + 485 = 85$$

$$10x^2 - 130x + 400 = 0$$

$$x^2 - 13x + 40 = 0$$

$$(x-8)(x-5) = 0$$

$$\underline{x=8} \text{ or } \underline{x=5}$$

$x=5$:

$$\begin{aligned} y &= 3(5) - 20 \\ &= -5 \end{aligned}$$

$x=8$:

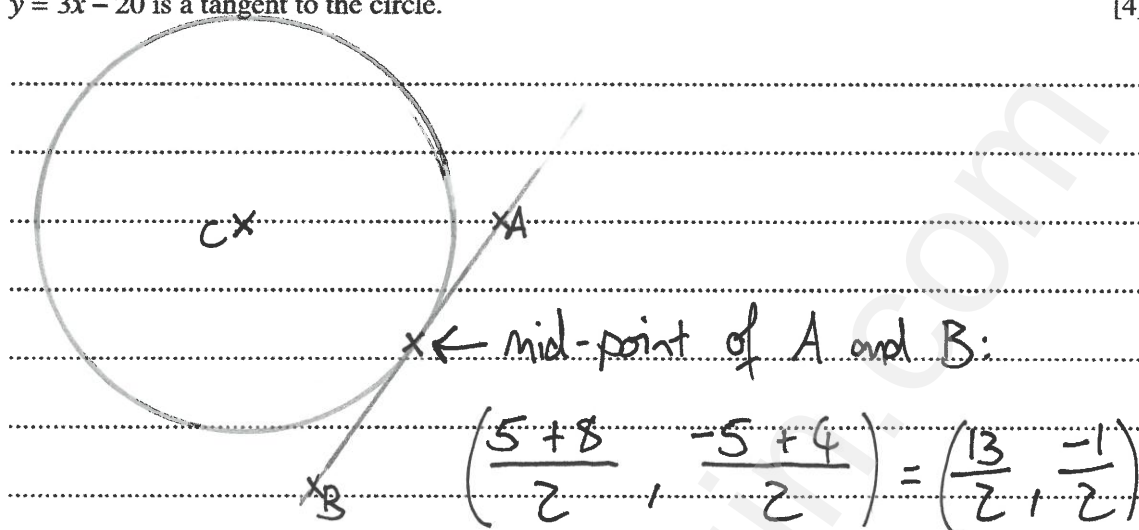
$$\begin{aligned} y &= 3(8) - 20 \\ &= 4 \end{aligned}$$

$$\underline{(5, -5)}$$

and

$$\underline{(8, 4)}$$

- (b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]



$$(x+1)^2 + (y-2)^2 = r^2$$

Sub. $\left(\frac{13}{2}, \frac{-1}{2} \right)$: $\left(\frac{13}{2} + 1 \right)^2 + \left(\frac{-1}{2} - 2 \right)^2 = r^2$

$$\left(\frac{15}{2} \right)^2 + \left(\frac{-5}{2} \right)^2 = r^2$$

$$\frac{225}{4} + \frac{25}{4} = r^2$$

$$\frac{250}{4} = r^2$$

$$r^2 = \frac{125}{2}$$

$$\rightarrow (x+1)^2 + (y-2)^2 = \frac{125}{2}$$

10 The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x -coordinates of the points A and B where the circle intersects the x -axis. [2]

$$x^2 - 4x + y^2 + 6y - 77 = 0$$

$$(x-2)^2 - 4 + (y+3)^2 - 9 - 77 = 0$$

$$(x-2)^2 + (y+3)^2 - 90 = 0$$

$$(x-2)^2 + (y+3)^2 = 90$$

At x -axis, $y=0$:

$$(x-2)^2 + (0+3)^2 = 90$$

$$(x-2)^2 + 9 = 90$$

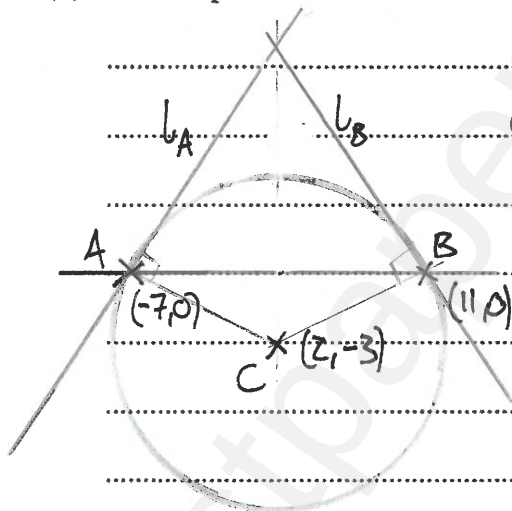
$$(x-2)^2 = 81$$

$$x-2 = \pm 9$$

$$x = 2 \pm 9$$

$$\underline{x = -7} \text{ or } \underline{x = 11}$$

(b) Find the point of intersection of the tangents to the circle at A and B . [6]



$$\text{Gradient of radius AC} = \frac{-3-0}{2-(-7)}$$

$$= \frac{-3}{9} = -\frac{1}{3}$$

$$\text{Gradient of line } l = 3$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 3(x - (-7))$$

$$y = 3(x + 7)$$

$$\underline{y = 3x + 21} \quad \textcircled{1}$$

$$\text{Gradient of radius } BC = \frac{0 - -3}{11 - 2}$$

$$= \frac{3}{9} = \frac{1}{3}$$

$$\text{Gradient of line } B = -3$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -3(x - 11)$$

$$\underline{y = -3x + 33} \quad (2)$$

Solve (1) & (2) simultaneously:

$$(1) = (2):$$

$$3x + 21 = -3x + 33$$

$$6x = 12$$

$$\underline{x = 2}$$

Sub into (1):

$$y = 3(2) + 21$$

$$= 6 + 21$$

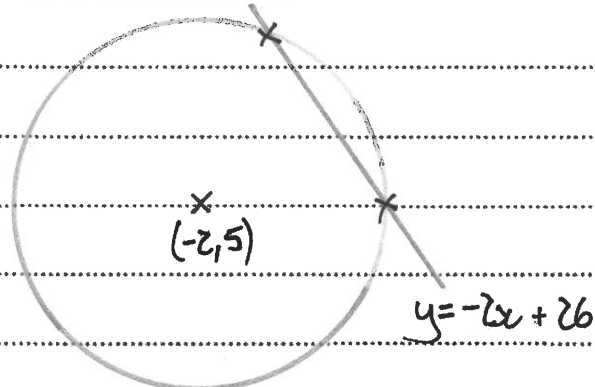
$$= \underline{27}$$

$$\rightarrow \underline{(2, 27)}$$

- 5 Points A (7, 12) and B lie on a circle with centre (-2, 5). The line AB has equation $y = -2x + 26$.

Find the coordinates of B.

[6]



Eqⁿ of circle: $(x + 2)^2 + (y - 5)^2 = r^2$

Sub. (7, 12): $(7 + 2)^2 + (12 - 5)^2 = r^2$

$$9^2 + 7^2 = r^2$$

$$130 = r^2$$

$$\rightarrow (x + 2)^2 + (y - 5)^2 = 130$$

Sub. $y = -2x + 26$ into eqⁿ of circle:

$$(x + 2)^2 + (-2x + 26 - 5)^2 = 130$$

$$(x + 2)^2 + (-2x + 21)^2 = 130$$

$$x^2 + 4x + 4 + 4x^2 - 84x + 441 = 130$$

$$5x^2 - 80x + 445 = 130$$

$$5x^2 - 80x + 315 = 0$$

$$x^2 - 16x + 63 = 0$$

$$(x - 9)(x - 7) = 0$$

$$x = 9 \text{ or } x = 7$$

← this is point A

$x = 9$: $y = -2(9) + 26$

$$= -18 + 26$$

$$= 8$$

$$\rightarrow \underline{\underline{(9, 8)}}$$

- 9 The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB.

[7]

sub. $y = 2x + 5$ into eqⁿ of circle:

$$x^2 + (2x + 5)^2 = 20$$

$$x^2 + 4x^2 + 20x + 25 = 20$$

$$5x^2 + 20x + 5 = 0$$

$$x^2 + 4x + 1 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)}$$

$$x = -2 + \sqrt{3} \quad \text{or} \quad x = -2 - \sqrt{3}$$

$$x = -2 + \sqrt{3}:$$

$$\begin{aligned} y &= 2(-2 + \sqrt{3}) + 5 \\ &= -4 + 2\sqrt{3} + 5 \\ &= 1 + 2\sqrt{3} \end{aligned}$$

$$x = -2 - \sqrt{3}:$$

$$\begin{aligned} y &= 2(-2 - \sqrt{3}) + 5 \\ &= -4 - 2\sqrt{3} + 5 \\ &= 1 - 2\sqrt{3} \end{aligned}$$

$$\underline{(-2 + \sqrt{3}, 1 + 2\sqrt{3})}$$

$$\underline{(-2 - \sqrt{3}, 1 - 2\sqrt{3})}$$

Length of chord:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[(-2 - \sqrt{3}) - (-2 + \sqrt{3})]^2 + [(1 - 2\sqrt{3}) - (1 + 2\sqrt{3})]^2}$$

$$= \sqrt{[-2 - \sqrt{3} + 2 - \sqrt{3}]^2 + [1 - 2\sqrt{3} - 1 - 2\sqrt{3}]^2}$$

$$= \sqrt{(-2\sqrt{3})^2 + (-4\sqrt{3})^2}$$

$$= \sqrt{12 + 48} = \underline{2\sqrt{15}}$$

A straight line through the point (10, 0) with gradient m is a tangent to the circle.

(b) Find the two possible values of m .

[5]

eqn of line: $y - y_1 = m(x - x_1)$

$$y - 0 = m(x - 10)$$

$$y = mx - 10m$$

sub. into eqn of circle:

$$x^2 + (mx - 10m)^2 = 20$$

$$x^2 + m^2x^2 - 20m^2x + 100m^2 = 20$$

$$x^2 + m^2x^2 - 20m^2x + 100m^2 - 20 = 0$$

$$(1 + m^2)x^2 - 20m^2x + 100m^2 - 20 = 0$$

If line is tangent to circle: $b^2 - 4ac = 0$:

$$(-20m^2)^2 - 4(1 + m^2)(100m^2 - 20) = 0$$

$$400m^4 - 4(100m^2 - 20 + 100m^4 - 20m^2) = 0$$

$$400m^4 - 4(100m^4 + 80m^2 - 20) = 0$$

$$400m^4 - 400m^4 - 320m^2 + 80 = 0$$

$$-320m^2 + 80 = 0$$

$$320m^2 = 80$$

$$m^2 = \frac{1}{4}$$

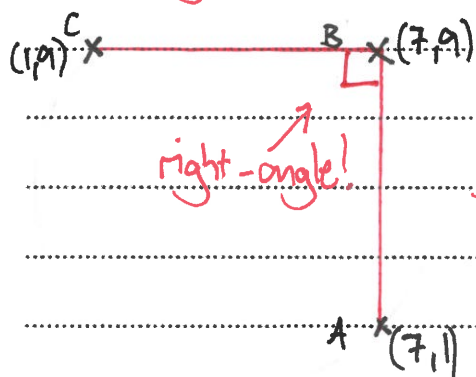
$$m = \pm \frac{1}{2}$$

- 8 The points $A(7, 1)$, $B(7, 9)$ and $C(1, 9)$ are on the circumference of a circle.

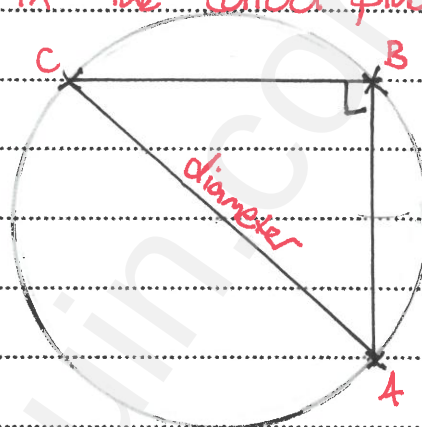
(a) Find an equation of the circle.

[5]

This question is simpler than it looks - draw a diagram with the points in the correct place:



SO:



$\rightarrow AC$ is a diameter, so mid-pt = Centre:

$$\left(\frac{7+1}{2}, \frac{1+9}{2} \right) = (4, 5)$$

$$\rightarrow (x-4)^2 + (y-5)^2 = r^2$$

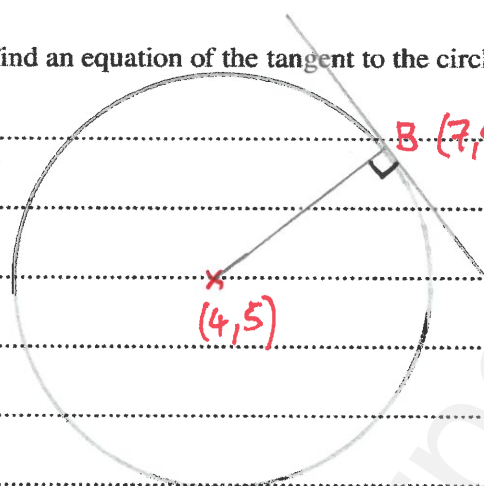
Sub. any point:

$$\begin{aligned} \text{sub. } (7, 9): & (7-4)^2 + (9-5)^2 = r^2 \\ & 3^2 + 4^2 = r^2 \\ & 9 + 16 = r^2 \\ & 25 = r^2 \end{aligned}$$

$$\rightarrow \underline{(x-4)^2 + (y-5)^2 = 25}$$

(b) Find an equation of the tangent to the circle at B .

[2]



$$\begin{aligned}\text{Gradient of radius} &= \frac{9-5}{7-4} \\ &= \frac{4}{3}\end{aligned}$$

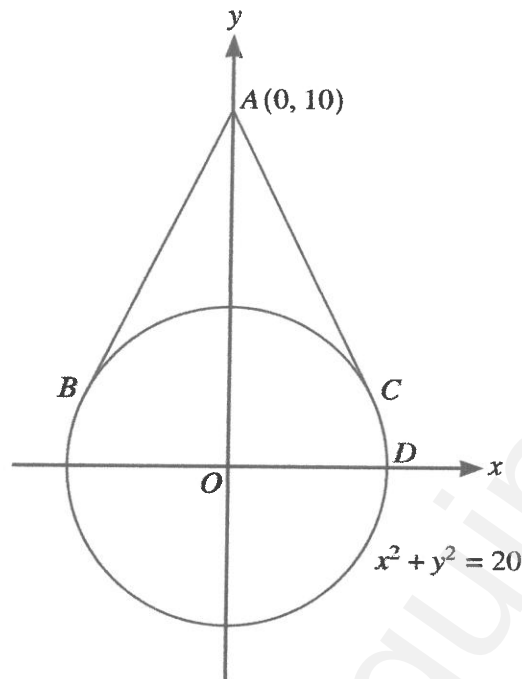
$$\therefore \text{Gradient of tangent} = -\frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 9 = -\frac{3}{4}(x - 7)$$

$$y - 9 = -\frac{3}{4}x + \frac{21}{4}$$

$$\underline{\underline{y = -\frac{3}{4}x + \frac{57}{4}}}$$



The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point $A(0, 10)$.

- (a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m . [4]

Sub. $y = mx + 10$ into eqⁿ of circle:

$$x^2 + (mx + 10)^2 = 20$$

$$x^2 + m^2x^2 + 20mx + 100 = 20$$

$$x^2 + m^2x^2 + 20mx + 80 = 0$$

$$(1 + m^2)x^2 + 20mx + 80 = 0$$

line is tangent, so $b^2 - 4ac = 0$:

$$(20m)^2 - 4(1 + m^2)(80) = 0$$

$$400m^2 - 320 - 320m^2 = 0$$

$$80m^2 - 320 = 0$$

$$80m^2 = 320$$

$$m^2 = 4$$

$$m = \pm 2$$

(b) Find the coordinates of B and C.

[3]

When $m=2$, $y=2x+10$. Sub into circle:

$$x^2 + (2x+10)^2 = 20$$

$$x^2 + 4x^2 + 40x + 100 = 20$$

$$5x^2 + 40x + 80 = 0$$

$$x^2 + 8x + 16 = 0$$

$$(x+4)^2 = 0$$

$$x = -4$$

$$y = 2(-4) + 10$$

$$= -8 + 10$$

$$= 2$$

$$\underline{(-4, 2)} \quad \swarrow B$$

When $m=-2$, $y=-2x+10$. Sub into circle:

$$x^2 + (-2x+10)^2 = 20$$

$$x^2 + 4x^2 - 40x + 100 = 20$$

$$5x^2 - 40x + 80 = 0$$

$$x^2 - 8x + 16 = 0$$

$$(x-4)^2 = 0$$

$$x = 4$$

$$y = -2(4) + 10$$

$$= -8 + 10$$

$$= 2$$

$$\underline{(4, 2)} \quad \swarrow C$$

(can also find second point using symmetry)

The point D is where the circle crosses the positive x-axis.

(c) Find angle BDC in degrees.

[3]

at D, $y=0$: $x^2 + 0^2 = 20$

$$x = \sqrt{20}$$

length BC: $BC = 8$

length CD: $CD = \sqrt{(\sqrt{20} - 4)^2 + (0 - 2)^2}$

$$= \sqrt{20 - 8\sqrt{20} + 16 + 4}$$

$$= \sqrt{40 - 8\sqrt{20}} = 2.05 \text{ (STO)}$$

length BD: $BD = \sqrt{(\sqrt{20} - -4)^2 + (0 - 2)^2}$

$$= \sqrt{20 + 8\sqrt{20} + 16 + 4}$$

$$= \sqrt{40 + 8\sqrt{20}} = 8.71 \text{ (STO)}$$

Cosine rule:

$$8^2 = (2.05)^2 + (8.71)^2 - 2(2.05)(8.71) \cos \theta$$

$$\cos \theta = \frac{64 - (2.05)^2 - (8.71)^2}{-2(2.05)(8.71)}$$

$$\theta = 63.4^\circ$$

