

- 1 (a) Expand $(1 + 3x)^6$ in ascending powers of x up to, and including, the term in x^2 . [2]

$$(1 + 3x)^6 = 1^6 + 6 \times 1^5 \times 3x + 15 \times 1^4 \times (3x)^2 + \dots$$

$$= 1 + 18x + 15 \times 9x^2 + \dots$$

$$= \underline{1 + 18x + 135x^2 + \dots}$$

- (b) Hence find the coefficient of x^2 in the expansion of $(1 - 7x + x^2)(1 + 3x)^6$. [2]

$$(1 - 7x + x^2)(1 + 18x + 135x^2)$$

$$x^2 \text{ term: } 135x^2 - 126x^2 + x^2$$

$$= 10x^2$$

$$\text{coeff} = \underline{10}$$

- 1 The coefficient of x^3 in the expansion of $\left(p + \frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant p .

$$\left(p + \frac{1}{p}x\right)^4 = p^4 + 4p^3\left(\frac{1}{p}x\right) + 6p^2\left(\frac{1}{p}x\right)^2 + 4p\left(\frac{1}{p}x\right)^3 + \dots$$

$$\rightarrow 4p \times \frac{1}{p^3}x^3$$

$$= \frac{4}{p^2}x^3$$

$$\text{coeff} = \frac{4}{p^2}$$

$$\frac{4}{p^2} = 144$$

$$4 = 144p^2$$

$$p^2 = \frac{1}{36}$$

$$p = \pm \frac{1}{6}$$

- 2 The coefficient of x^4 in the expansion of $(x+a)^6$ is p and the coefficient of x^2 in the expansion of $(ax+3)^4$ is q . It is given that $p+q=276$.

Find the possible values of the constant a .

[4]

$$(x+a)^6 = x^6 + 6x^5a + 15x^4a^2 + \dots$$

$$\rightarrow 15a^2x^4$$

$$\text{coeff} = 15a^2 = p$$

$$(ax+3)^4 = (ax)^4 + 4(ax)^3 \times 3 + 6(ax)^2 \times 3^2 + \dots$$

$$= a^4x^4 + 12a^3x^3 + 54a^2x^2$$

$$\rightarrow 54a^2x^2$$

$$\text{coeff} = 54a^2 = q$$

$$p+q=276:$$

$$15a^2 + 54a^2 = 276$$

$$69a^2 = 276$$

$$a^2 = 4$$

$$\underline{\underline{a = \pm 2}}$$

- 3 The coefficient of x^3 in the expansion of $(3+ax)^6$ is 160.

(a) Find the value of the constant a .

$$(3+ax)^6 = 3^6 + 6 \times 3^5 \times ax + 15 \times 3^4 \times (ax)^2 + \underline{20 \times 3^3 \times (ax)^3} + \dots$$

$$\rightarrow 20 \times 27 \times a^3 x^3$$

$$= 540 a^3 x^3$$

$$\text{coeff} = 540 a^3$$

$$540 a^3 = 160$$

$$a^3 = \frac{8}{27}$$

$$\underline{a = \frac{2}{3}}$$

(b) Hence find the coefficient of x^3 in the expansion of $(3+ax)^6(1-2x)$.

$$a = \frac{2}{3} :$$

$$\left(3 + \frac{2}{3}x\right)^6 = 3^6 + 6 \times 3^5 \times \frac{2}{3}x + 15 \times 3^4 \times \left(\frac{2}{3}x\right)^2 + 20 \times 3^3 \times \left(\frac{2}{3}x\right)^3 + \dots$$

$$= 729 + 972x + 540x^2 + 160x^3 + \dots$$

$$\rightarrow (729 + 972x + 540x^2 + 160x^3 + \dots)(1 - 2x)$$

$$\text{coeff of } x^3: 540x^2 \times -2x + 160x^3 \times 1$$

$$= -1080x^3 + 160x^3$$

$$= -920x^3$$

$$\underline{\text{coeff} = -920}$$

- 2 (a) Find the first three terms, in ascending powers of x , in the expansion of $(1+ax)^6$. [1]

$$(1+ax)^6 = 1^6 + 6 \times 1^5 \times (ax) + 15 \times 1^4 \times (ax)^2 + \dots$$

$$= \underline{1 + 6ax + 15a^2x^2 + \dots}$$

- (b) Given that the coefficient of x^2 in the expansion of $(1-3x)(1+ax)^6$ is -3 , find the possible values of the constant a . [4]

$$(1-3x)(1+6ax+15a^2x^2+\dots)$$

$$\text{coeff of } x^2: 1 \times 15a^2x^2 - 3x \times 6ax$$

$$= 15a^2x^2 - 18a^2x^2$$

$$\text{coeff} = 15a^2 - 18a$$

$$\rightarrow 15a^2 - 18a = -3$$

$$15a^2 - 18a + 3 = 0$$

$$5a^2 - 6a + 1 = 0$$

$$(5a-1)(a-1) = 0$$

$$\underline{a = \frac{1}{5}} \text{ or } \underline{a = 1}$$

- 1 (a) Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

$$\begin{aligned} \left(x - \frac{2}{x}\right)^6 &= x^6 + 6x^5 \times \left(\frac{-2}{x}\right) + \underline{15x^4 \times \left(\frac{-2}{x}\right)^2} \dots \\ &\dots + \underline{20x^3 \times \left(\frac{-2}{x}\right)^3} + 15x^2 \left(\frac{-2}{x}\right)^4 + 6x \left(\frac{-2}{x}\right)^5 \\ &\dots + \left(\frac{-2}{x}\right)^6 \end{aligned}$$

$$* 15x^4 \times \left(\frac{-2}{x}\right)^2 = 15x^4 \times \frac{4}{x^2}$$

$$= 60x^2$$

$$\text{coeff} = \underline{\underline{60}}$$

- (b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

$$* 20x^3 \left(\frac{-2}{x}\right)^3 = 20x^3 \times \frac{-8}{x^3}$$

$$= \underline{\underline{-160}}$$

$$\rightarrow (2 + 3x^2)(60x^2 - 160)$$

$$= 120x^2 - 480x^2$$

$$= -360x^2$$

$$\text{coeff} = \underline{\underline{-360}}$$

- 4 (a) Expand the following in ascending powers of x up to and including the term in x^2 .

(i) $(1+2x)^5$.

[1]

$$\begin{aligned}(1+2x)^5 &= 1^5 + 5 \times 1^4 \times (2x) + 10 \times 1^3 \times (2x)^2 + \dots \\ &= 1 + 5 \times 2x + 10 \times 4x^2 + \dots \\ &= \underline{1 + 10x + 40x^2 + \dots}\end{aligned}$$

(ii) $(1-ax)^6$, where a is a constant.

[2]

$$\begin{aligned}(1-ax)^6 &= 1^6 + 6 \times 1^5 \times (-ax) + 15 \times 1^4 \times (-ax)^2 + \dots \\ &= 1 + 6 \times -ax + 15 \times a^2 x^2 + \dots \\ &= \underline{1 - 6ax + 15a^2 x^2 + \dots}\end{aligned}$$

In the expansion of $(1+2x)^5(1-ax)^6$, the coefficient of x^2 is -5 .

- (b) Find the possible values of a .

[4]

$$(1 + 10x + 40x^2)(1 - 6ax + 15a^2x^2)$$

x^2 term: $15a^2x^2 - 60ax^2 + 40x^2$

coefficient: $15a^2 - 60a + 40$

coeff = -5 : $15a^2 - 60a + 40 = -5$

$$15a^2 - 60a + 45 = 0$$

$$a^2 - 4a + 3 = 0$$

$$(a-3)(a-1) = 0$$

$$\underline{a=3} \text{ or } \underline{a=1}$$

- 3 (a) Find the first three terms in the expansion of $(3 - 2x)^5$ in ascending powers of x . [3]

$$(3 - 2x)^5 = 3^5 + 5(3)^4(-2x) + 10(3)^3(-2x)^2 + \dots$$

$$= \underline{243 - 810x + 1080x^2 + \dots}$$

- (b) Hence find the coefficient of x^2 in the expansion of $(4 + x)^2(3 - 2x)^5$. [3]

$$\text{expand } (4 + x)^2 : 16 + 8x + x^2$$

$$\rightarrow (16 + 8x + x^2)(243 - 810x + 1080x^2)$$

$$= 17280x^2 - 6480x^2 + 243x^2$$

$$= 11043x^2$$

$$\text{coeff} = \underline{\underline{11043}}$$



- 3 (a) Find the complete expansion of $(2x - \frac{3}{x})^4$.

[4]

$$\begin{aligned}
 \left(2x - \frac{3}{x}\right)^4 &= (2x)^4 + 4(2x)^3\left(-\frac{3}{x}\right) + 6(2x)^2\left(-\frac{3}{x}\right)^2 \\
 &\quad + 4(2x)\left(-\frac{3}{x}\right)^3 + \left(-\frac{3}{x}\right)^4 \\
 &= 16x^4 + 4 \times 8x^3 \times \frac{-3}{x} + 6 \times 4x^2 \times \frac{9}{x^2} \\
 &\quad + 4 \times 2x \times \frac{-27}{x^3} + \frac{81}{x^4} \\
 &= \underline{16x^4 - 96x^2 + 216 - 216x^{-2} + 81x^{-4}}
 \end{aligned}$$

- (b) Hence determine the coefficient of x^2 in the expansion of $(x^2 + 5)(2x - \frac{3}{x})^4$.

[2]

$$(x^2 + 5)(16x^4 - 96x^2 + 216 \dots)$$

$$= 216x^2 - 480x^2$$

$$= -264x^2$$

$$\text{coeff} = \underline{-264}$$



3 Find the term independent of x in each of the following expansions.

(a) $\left(3x + \frac{2}{x^2}\right)^6$

[3]

$$\left(3x + \frac{2}{x^2}\right)^6 = (3x)^6 + 6(3x)^5\left(\frac{2}{x^2}\right) + \underline{15(3x)^4\left(\frac{2}{x^2}\right)^2} + \underline{20(3x)^3\left(\frac{2}{x^2}\right)^3} + 15(3x)^2\left(\frac{2}{x^2}\right)^4 + 6(3x)\left(\frac{2}{x^2}\right)^5 + \left(\frac{2}{x^2}\right)^6$$

$$* 15(3x)^4 \times \left(\frac{2}{x^2}\right)^2$$

$$= 15 \times 81x^4 \times \frac{4}{x^4}$$

$$= 15 \times 81 \times 4$$

$$= \underline{4860}$$

(b) $\left(3x + \frac{2}{x^2}\right)^6 (1 - x^3)$

[3]

$$* 20(3x)^3\left(\frac{2}{x^2}\right)^3$$

$$= 20 \times 27x^3 \times \frac{8}{x^6}$$

$$= 4320 \times \frac{1}{x^3}$$

$$\rightarrow \left(4860 + 4320 \times \frac{1}{x^3}\right)(1 - x^3)$$

$$= 4860 - 4320$$

$$= \underline{540}$$

- 5 In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a .

[4]

$$\left(2x^2 + \frac{a}{x}\right)^6 = (2x^2)^6 + 6(2x^2)^5\left(\frac{a}{x}\right) + \underline{15(2x^2)^4\left(\frac{a}{x}\right)^2} + \underline{20(2x^2)^3\left(\frac{a}{x}\right)^3} + \dots$$

$$\text{coeff of } x^6: 15(2x^2)^4 \times \left(\frac{a}{x}\right)^2$$

$$= 15 \times 16 x^8 \times \frac{a^2}{x^2}$$

$$= 240a^2x^6$$

$$\text{coeff} = 240a^2$$

$$\text{coeff of } x^3: 20(2x^2)^3 \times \left(\frac{a}{x}\right)^3$$

$$= 20 \times 8x^6 \times \frac{a^3}{x^3}$$

$$= 160a^3x^3$$

$$\text{coeff} = 160a^3$$

$$\rightarrow 240a^2 = 160a^3$$

$$240 = 160a$$

$$a = \frac{240}{160}$$

$$= \underline{\underline{\frac{3}{2}}}$$

- (b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$.

[1]

$$\text{coeff of } x^6: 240\left(\frac{3}{2}\right)^2 = 540$$

$$\text{coeff of } x^3: 160\left(\frac{3}{2}\right)^3 = 540$$

$$\rightarrow (1 - x^3)(\dots 540x^6 + 540x^3 \dots)$$

$$= 540x^6 - 540x^6 = \underline{\underline{0}}$$

- 1 (a) Find the first three terms in the expansion, in ascending powers of x , of $(1+x)^5$. [1]

$$(1+x)^5 = 1^5 + 5 \times 1^4 \times x + 10 \times 1^3 \times x^2 + \dots$$

$$= \underline{1 + 5x + 10x^2 + \dots}$$

- (b) Find the first three terms in the expansion, in ascending powers of x , of $(1-2x)^6$. [2]

$$(1-2x)^6 = 1^6 + 6 \times 1^5 \times (-2x) + 15 \times 1^4 \times (-2x)^2 + \dots$$

$$= 1 - 12x + 15 \times 4x^2 + \dots$$

$$= \underline{1 - 12x + 60x^2 + \dots}$$

- (c) Hence find the coefficient of x^2 in the expansion of $(1+x)^5(1-2x)^6$. [2]

$$(1 + 5x + 10x^2)(1 - 12x + 60x^2)$$

$$x^2 \text{ terms: } 60x^2 - 60x^2 + 10x^2$$

$$= 10x^2$$

$$\text{coeff} = \underline{10}$$

- 2 (a) Find the first three terms in the expansion, in ascending powers of x , of $(2 + 3x)^4$. [2]

$$(2 + 3x)^4 = 2^4 + 4 \times 2^3(3x) + 6 \times 2^2(3x)^2 + \dots$$

$$= 16 + 32 \times 3x + 24 \times 9x^2 + \dots$$

$$= \underline{16 + 96x + 216x^2 + \dots}$$

- (b) Find the first three terms in the expansion, in ascending powers of x , of $(1 - 2x)^5$. [2]

$$(1 - 2x)^5 = 1^5 + 5 \times 1^4 \times (-2x) + 10 \times 1^3 \times (-2x)^2 + \dots$$

$$= 1 + 5 \times -2x + 10 \times 4x^2$$

$$= \underline{1 - 10x + 40x^2 + \dots}$$

- (c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

$$\rightarrow (16 + 96x + 216x^2)(1 - 10x + 40x^2)$$

$$= 640x^2 - 960x^2 + 216x^2$$

$$= -104x^2 \quad \text{coeff} = -104$$

- 2 The coefficient of $\frac{1}{x}$ in the expansion of $(kx + \frac{1}{x})^5 + (1 - \frac{2}{x})^8$ is 74.

Find the value of the positive constant k .

[5]

①: $(kx + \frac{1}{x})^5 = (kx)^5 + 5(kx)^4(\frac{1}{x}) + 10(kx)^3(\frac{1}{x})^2 + \underline{10(kx)^2(\frac{1}{x})^3} + \dots$

$$10(kx)^2(\frac{1}{x})^3 = 10k^2x^2 \times \frac{1}{x^3}$$

$$= 10k^2 \times \frac{1}{x}$$

$$\text{coeff} = \underline{10k^2}$$

②: $(1 - \frac{2}{x})^8 = 1^8 + 8 \times 1^7 \times (\frac{-2}{x}) + \dots$

$$= 1 - \underline{16 \times \frac{1}{x}} + \dots$$

$$\text{coeff} = -16$$

coeff of $\frac{1}{x}$ for ① + ② = 74:

$$10k^2 - 16 = 74$$

$$10k^2 = 90$$

$$k^2 = 9$$

$$k = \pm 3$$

k is positive so $k = 3$

- 6 In the expansion of $\left(\frac{x}{a} + \frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant a .

[6]

$$\left(\frac{x}{a} + \frac{a}{x^2}\right)^7 = \left(\frac{x}{a}\right)^7 + 7\left(\frac{x}{a}\right)^6\left(\frac{a}{x^2}\right) + 21\left(\frac{x}{a}\right)^5\left(\frac{a}{x^2}\right)^2 + \dots$$

$$= \frac{x^7}{a^7} + 7 \times \frac{x^6}{a^6} \times \frac{a}{x^2} + 21 \times \frac{x^5}{a^5} \times \frac{a^2}{x^4} + \dots$$

$$= \frac{x^7}{a^7} + \frac{7}{a^5}x^4 + \frac{21}{a^3}x + \dots$$

$$\frac{\text{coeff. of } x^4}{\text{coeff. of } x} = 3: \frac{\frac{7}{a^5} \times a^5}{\frac{21}{a^3} \times a^5} = 3$$

$$\frac{7}{21a^2} = 3$$

$$7 = 63a^2$$

$$a^2 = \frac{1}{9}$$

$$a = \pm \frac{1}{3}$$

$$\underline{a = -\frac{1}{3}} \quad \text{or} \quad \underline{a = \frac{1}{3}}$$

- 8 (a) It is given that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is -15 .

Find the possible values of a .

[4]

$$(2 - ax)^5 = 2^5 + 5 \times 2^4 \times (-ax) + 10 \times 2^3 \times (-ax)^2 + \dots$$

$$= 32 - 80ax + 80a^2x^2 + \dots$$

$$\rightarrow (4 + 2x)(32 - 80ax + 80a^2x^2 + \dots)$$

$$\text{coeff of } x^2: 4 \times 80a^2x^2 + 2x \times -80ax$$

$$= 320a^2x^2 - 160ax$$

$$\text{coeff} = 320a^2 - 160a$$

$$\rightarrow 320a^2 - 160a = -15$$

$$320a^2 - 160a + 15 = 0$$

$$64a^2 - 32a + 3 = 0$$

$$a = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(64)(3)}}{2(64)}$$

$$\underline{a = \frac{3}{8}} \quad \text{or} \quad \underline{a = \frac{1}{8}}$$

- (b) It is given instead that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is k . It is also given that there is only one value of a which leads to this value of k .

Find the values of k and a .

[4]

coeff of $x^2 = k$:

from (a): $320a^2 - 160a = k$

$$320a^2 - 160a - k = 0$$

only one solution for a , so $b^2 - 4ac = 0$:

$$(-160)^2 - 4(320)(-k) = 0$$

$$25600 + 1280k = 0$$

$$1280k = -25600$$

$$\underline{k = -20}$$

Sub into quadratic:

$$320a^2 - 160a + 20 = 0$$

$$16a^2 - 8a + 1 = 0$$

$$(4a - 1)(4a - 1) = 0$$

$$\underline{a = \frac{1}{4}}$$