



- 1 An arithmetic progression has fourth term 15 and eighth term 25.

Find the 30th term of the progression.

[3]

$$u_4 = 15: a + 3d = 15 \quad (1)$$

$$u_8 = 25: a + 7d = 25 \quad (2)$$

$$(2) - (1): 4d = 10$$

$$d = 2.5$$

$$\rightarrow (1): a + 3(2.5) = 15$$

$$a + 7.5 = 15$$

$$a = 7.5$$

$$\text{Sequence: } 7.5, 10, 12.5, 15, \dots$$

$$u_{30}: u_{30} = a + 29d$$

$$= 7.5 + 29(2.5)$$

$$= 7.5 + 72.5$$

$$= \underline{80}$$



- 2 The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is -15 .

Find the sum of the first 50 terms of the progression.

[5]

$$u_{13} = 12: a + 12d = 12 \quad (1)$$

$$S_{30} = -15: \frac{30}{2} [2a + 29d] = -15$$

$$2a + 29d = -1 \quad (2)$$

$$(1) \times 2: 2a + 24d = 24$$

$$- (2): 2a + 29d = -1$$

$$-5d = 25$$

$$d = -5$$

$$\rightarrow (1): a + 12(-5) = 12$$

$$a - 60 = 12$$

$$a = 72$$

$$S_{50}: S_{50} = \frac{50}{2} [2(72) + 49(-5)]$$

$$= 25 [144 - 245]$$

$$= 25 \times -101$$

$$= \underline{\underline{-2525}}$$

- 1 The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression.

[4]

Sum of first nine terms = 117

$$\frac{1}{2} \times 9 [2a + 8d] = 117$$

$$9(2a + 8d) = 234$$

$$2a + 8d = 26$$

$$a + 4d = 13 \quad (1)$$

Sum of first thirteen terms = 117 + 91 (=208)

$$\frac{1}{2} \times 13 [2a + 12d] = 208$$

$$13(2a + 12d) = 416$$

$$2a + 12d = 32$$

$$a + 6d = 16 \quad (2)$$

$$(2) - (1): 2d = 3$$

$$d = 1.5$$

$$\text{Sub. into (1): } a + 4(1.5) = 13$$

$$a + 6 = 13$$

$$a = 7$$

- 5 The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

$$\theta = \frac{\pi}{4}: \quad u_1 = \tan\left(\frac{\pi}{4}\right)$$

$$= 1$$

$$u_2 = \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2}$$

$$d = u_2 - u_1$$

$$= \frac{\sqrt{2}}{2} - 1$$

$$S_{40} = \frac{40}{2} \left[2(1) + (39)\left(\frac{\sqrt{2}}{2} - 1\right) \right]$$

$$= 20 \left[2 + \frac{39\sqrt{2}}{2} - 39 \right]$$

$$= 20 \left(\frac{39\sqrt{2}}{2} - 37 \right)$$

$$= \underline{\underline{390\sqrt{2} - 740}}$$

- 4 The first, second and third terms of an arithmetic progression are k , $6k$ and $k + 6$ respectively.

(a) Find the value of the constant k .

[2]

Sequence: $k, 6k, k + 6$

$+5k$ $-5k + 6$

common difference

so: $5k = -5k + 6$

$10k = 6$

$k = 0.6$

(b) Find the sum of the first 30 terms of the progression.

[3]

from (a): $a = \underline{0.6}$, $d = 5 \times 0.6$
 $= \underline{3}$

$S_{30} = \frac{30}{2} [2(0.6) + 29(3)]$

$= 15 [1.2 + 87]$

$= 15 \times 88.2$

$= 1323$

- 2 The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression.

[5]

Sum of first 20 terms is 405:

$$\frac{20}{2} [2a + 19d] = 405$$

$$10(2a + 19d) = 405$$

$$2a + 19d = 40.5 \quad (1)$$

sum of first 40 terms is 1410:

$$\frac{40}{2} [2a + 39d] = 1410$$

$$20(2a + 39d) = 1410$$

$$2a + 39d = 70.5 \quad (2)$$

(2) - (1):

$$20d = 30$$

$$d = \frac{3}{2}$$

Sub. into (1): $2a + 19\left(\frac{3}{2}\right) = 40.5$

$$2a + 28.5 = 40.5$$

$$2a = 12$$

$$a = 6$$

60th term:

$$u_{60} = 6 + 59\left(\frac{3}{2}\right)$$

$$= \underline{\underline{94.5}}$$

- 7 The first term of an arithmetic progression is 1.5 and the sum of the first ten terms is 127.5 .

(a) Find the common difference.

[2]

$$a = 1.5 \quad S_{10} = \frac{10}{2} [2(1.5) + 9 \times d]$$

$$127.5 = 5 [3 + 9d]$$

$$127.5 = 15 + 45d$$

$$45d = 112.5$$

$$d = \underline{\underline{2.5}}$$

(b) Find the sum of all the terms of the arithmetic progression whose values are between 25 and 100.

[5]

$$U_n = a + (n-1)d$$

Find first term over 25:

$$1.5 + (n-1) \times 2.5 > 25$$

$$(n-1) \times 2.5 > 23.5$$

$$n-1 > 9.4$$

$$n > 10.4 \rightarrow 11^{\text{th}} \text{ term}$$

Find last term less than 100:

$$1.5 + (n-1) \times 2.5 < 100$$

$$(n-1) \times 2.5 < 98.5$$

$$n-1 < 39.4$$

$$n < 40.4 \rightarrow 40^{\text{th}} \text{ term}$$

Sum between 11th and 40th terms (inclusive):

$$\text{Sum} = S_{40} - S_{10}$$

$$= \frac{40}{2} [2(1.5) + 39(2.5)] - \frac{10}{2} [2(1.5) + 9(2.5)]$$

$$= 20(100.5) - 5(25.5)$$

$$= 2010 - 127.5$$

$$= \underline{\underline{1882.5}}$$

- 4 The n th term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84.

[5]

first term: $n=1$:

$$\frac{1}{2}(3(1) - 15)$$

$$= \frac{1}{2}(-12)$$

$$= -6$$

second term: $n=2$:

$$\frac{1}{2}(3(2) - 15)$$

$$= \frac{1}{2}(-9)$$

$$= -4.5$$

sequence starts like this: $-6, -4.5, \dots$
 $+1.5$

so common difference = 1.5

$S_n = 84$:

$$\rightarrow S_n = \frac{n}{2} [2(-6) + (n-1)(1.5)]$$

$$\frac{n}{2} [-12 + 1.5n - 1.5] = 84$$

$$n \left[\frac{-27}{2} + \frac{3}{2}n \right] = 168$$

$$-\frac{27}{2}n + \frac{3}{2}n^2 = 168$$

$$3n^2 - 27n - 336 = 0$$

$$n^2 - 9n - 112 = 0$$

$$-27n + 3n^2 = 336$$

$$(n-16)(n+7) = 0$$

$$n=16 \quad \text{or} \quad n=-7$$

$\rightarrow n=16$

- 4 The sum, S_n , of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The k th term in the progression is greater than 200.

Find the smallest possible value of k .

[5]

Set $n=1$ to find S_1 (ie. first term):

$$S_1 = 1^2 + 4(1)$$

$$= 5 \quad \leftarrow \text{first term} = 5$$

Set $n=2$: $S_2 = 2^2 + 4(2)$

$$= 4 + 8$$

$$= 12$$

$\uparrow$ sum of first two terms

$$\text{second term} = 12 - 5$$

$$= 7$$

$\rightarrow$ Sequence: $5, 7, \dots$

$+2 \leftarrow$ common difference

$$k^{\text{th}} \text{ term: } a + (n-1)d > 200$$

$$5 + (k-1) \times 2 > 200$$

$$2(k-1) > 195$$

$$2k - 2 > 195$$

$$2k > 197$$

$$k > 98.5$$

$$\text{so } \underline{k = 99}$$

- 2 The first, second and third terms of an arithmetic progression are a , $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression.

[5]

sequence: $a, 2a, a^2$

common difference between a and $2a$:

$$d = 2a - a$$

$$d = a \quad (1)$$

common difference between $2a$ and a^2 :

$$d = a^2 - 2a \quad (2)$$

solve simultaneously: $(1) = (2)$: $a^2 - 2a = a$

$$a^2 - 3a = 0$$

$$a(a - 3) = 0$$

$$a = 0 \quad \text{or} \quad \underline{a = 3}$$

$\times$ $\checkmark$

$\rightarrow \underline{a = 3}, \underline{d = 3}$

$$S_{50} = \frac{50}{2} [2(3) + 49(3)]$$

$$= 25 (6 + 147)$$

$$= 25 \times 153$$

$$= \underline{\underline{3825}}$$

- 8 (a) The first three terms of an arithmetic progression are 25 , $4p - 1$ and $13 - p$, where p is a constant.

Find the value of the tenth term of the progression.

[4]

$$u_1 = 25, \quad u_2 = 4p - 1, \quad u_3 = 13 - p$$

find d :

$$\begin{aligned} u_2 - u_1: \quad d &= 4p - 1 - 25 \\ &= 4p - 26 \quad (1) \end{aligned}$$

$$\begin{aligned} u_3 - u_2: \quad d &= 13 - p - (4p - 1) \\ &= 13 - p - 4p + 1 \\ &= 14 - 5p \quad (2) \end{aligned}$$

$$\begin{aligned} (1) = (2): \quad 4p - 26 &= 14 - 5p \\ 9p - 26 &= 14 \\ 9p &= 40 \\ p &= \frac{40}{9} \end{aligned}$$

$$\begin{aligned} d &= 4\left(\frac{40}{9}\right) - 26 \\ &= \frac{-74}{9} \end{aligned}$$

$$\begin{aligned} u_{10}: \quad u_{10} &= a + 9d \\ &= 25 + 9\left(\frac{-74}{9}\right) \\ &= 25 - 74 \\ &= \underline{\underline{-49}} \end{aligned}$$

- 2 The first term of an arithmetic progression is -20 and the common difference is 5 .

- (a) Find the sum of the first 20 terms of the progression.

[2]

Sequence: $-20, -15, -10, -5, \dots$

$$S_{20} = \frac{20}{2} [2(-20) + 19(5)]$$

$$= 10 [-40 + 95]$$

$$= 10 \times 55 = \underline{550}$$

It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

- (b) Find the value of k .

[3]

Sum of first $2k$ terms:

$$S_{2k} = \frac{2k}{2} [2(-20) + (2k-1) \times 5]$$

$$= k [-40 + 10k - 5]$$

$$= k (10k - 45)$$

$$= \underline{10k^2 - 45k}$$

Sum of first k terms:

$$S_k = \frac{k}{2} [2(-20) + (k-1) \times 5]$$

$$= \frac{k}{2} [-40 + 5k - 5]$$

$$= \frac{k}{2} (5k - 45)$$

$$= \underline{2.5k^2 - 22.5k}$$

$S_{2k} = 10 \times S_k$:

$$10k^2 - 45k = 10(2.5k^2 - 22.5k)$$

$$10k^2 - 45k = 25k^2 - 225k$$

$$15k^2 - 180k = 0$$

$$k^2 - 12k = 0$$

$$\rightarrow k(k-12) = 0$$

$$k=0 \text{ or } \underline{k=12}$$

$\uparrow$ $\times$
0 terms.

- 4 The first term of an arithmetic progression is 84 and the common difference is -3 .

(a) Find the smallest value of n for which the n th term is negative.

[2]

$$\begin{aligned} u_n &= a + (n-1)d \\ &= 84 + (n-1)(-3) \\ &= 84 - 3n + 3 \\ &= 87 - 3n \end{aligned}$$

$$u_n < 0:$$

$$87 - 3n < 0$$

$$87 < 3n$$

$$3n > 87$$

$$n > 29$$

$$\underline{n = 30}$$

It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k .

[3]

$$S_{2k}: S_{2k} = \frac{2k}{2} [2(84) + (2k-1)(-3)]$$

$$= k(168 - 6k + 3)$$

$$= 171k - 6k^2$$

$$S_k: S_k = \frac{k}{2} [2(84) + (k-1)(-3)]$$

$$= \frac{k}{2} (168 - 3k + 3)$$

$$= \frac{171}{2}k - \frac{3}{2}k^2$$

$$9k^2 - 171k = 0$$

$$k^2 - 19k = 0$$

$$k(k-19) = 0$$

$$k=0 \text{ or } \underline{k=19}$$

$$S_{2k} = S_k: 171k - 6k^2 = \frac{171}{2}k - \frac{3}{2}k^2$$

$$342k - 12k^2 = 171k - 3k^2$$

- 5 An arithmetic progression has first term 5 and common difference 6.

For this progression, find the sum of all the terms that lie between 150 and 400.

[6]

$$a = 5 \quad d = 6$$

Find first term greater than 150:

$$a + (n-1)d > 150$$

$$5 + (n-1)6 > 150$$

$$5 + 6n - 6 > 150$$

$$6n > 151$$

$$n > 25.16\ldots$$

→ 26th term

Find last term smaller than 400:

$$5 + (n-1)6 < 400$$

$$5 + 6n - 6 < 400$$

$$6n < 401$$

$$n < 66.83\ldots$$

→ 66th term

So we want the sum of the terms between u_{26} and u_{66} (inclusive)

$$= S_{66} - S_{25}$$

$$= \frac{66}{2} [2(5) + 65(6)] - \frac{25}{2} [2(5) + 24(6)]$$

$$= 33 \times 400 - 12.5 \times 154$$

$$= 13200 - 1925$$

$$= \underline{\underline{11275}}$$



- 3 An arithmetic progression has first term 4 and common difference d . The sum of the first n terms of the progression is 5863.

(a) Show that $(n-1)d = \frac{11726}{n} - 8$.

[1]

$$S_n = \frac{n}{2} [2(4) + (n-1)d]$$

$$5863 = \frac{n}{2} [8 + (n-1)d]$$

$$11726 = n [8 + (n-1)d]$$

$$\frac{11726}{n} = 8 + (n-1)d$$

$$(n-1)d = \frac{11726}{n} - 8$$

- (b) Given that the n th term is 139, find the values of n and d , giving the value of d as a fraction. [4]

$$u_n = 139: 4 + (n-1)d = 139$$

$$(n-1)d = 135$$

sub. this value for $(n-1)d$ into eqn from part (a):

$$135 = \frac{11726}{n} - 8$$

$$143 = \frac{11726}{n}$$

$$143n = 11726$$

$$n = \frac{11726}{143}$$

$$n = 82$$

$$(n-1)d = 135$$

$$81d = 135$$

$$d = \frac{5}{3}$$

- 8 The first term of a progression is $\sin^2 \theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2 \theta \cos^2 \theta$.

It is now given instead that the progression is arithmetic.

- (b) (i) Find the common difference of the progression in terms of $\sin \theta$.

[3]

$$\begin{aligned}
 \text{Common difference} &= \sin^2 \theta \cos^2 \theta - \sin^2 \theta \\
 &= \sin^2 \theta (\cos^2 \theta - 1) \\
 &\quad \quad \quad \uparrow = 1 - \sin^2 \theta \\
 &= \sin^2 \theta (1 - \sin^2 \theta - 1) \\
 &= \sin^2 \theta \times -\sin^2 \theta \\
 &= \underline{\underline{-\sin^4 \theta}}
 \end{aligned}$$

- (ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$.

[3]

$$\begin{aligned}
 a &= \sin^2\left(\frac{\pi}{3}\right) & d &= -\sin^4\left(\frac{\pi}{3}\right) \\
 &= \frac{3}{4} & &= -\frac{9}{16}
 \end{aligned}$$

$$S_{16} = \frac{16}{2} \left[2\left(\frac{3}{4}\right) + 15\left(-\frac{9}{16}\right) \right]$$

$$= 8 \left(\frac{3}{2} - \frac{135}{16} \right)$$

$$= 8 \times -\frac{111}{16}$$

$$= \underline{\underline{-\frac{111}{2}}}$$

- 7 The first and second terms of an arithmetic progression are $\frac{1}{\cos^2 \theta}$ and $-\frac{\tan^2 \theta}{\cos^2 \theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4 \theta}$. [4]

Sequence: $\frac{1}{\cos^2 \theta}, -\frac{\tan^2 \theta}{\cos^2 \theta}$

$$\text{common difference} = \frac{-\tan^2 \theta}{\cos^2 \theta} - \frac{1}{\cos^2 \theta}$$

$$= \frac{-\tan^2 \theta - 1}{\cos^2 \theta}$$

$$= \frac{-\frac{\sin^2 \theta}{\cos^2 \theta} - 1}{\cos^2 \theta} \quad \begin{matrix} \times \cos^2 \theta \\ \times \cos^2 \theta \end{matrix}$$

$$= \frac{-\sin^2 \theta - \cos^2 \theta}{\cos^4 \theta}$$

$$= \frac{-(\sin^2 \theta + \cos^2 \theta)}{\cos^4 \theta} \quad \leftarrow = 1$$

$$= \frac{-1}{\cos^4 \theta} \quad \text{QED}$$

- (b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$.

[3]

$$u_{13} = a + 12d$$

$$= \frac{1}{\cos^2 \theta} + 12 \times \frac{-1}{\cos^4 \theta}$$

$\theta = \frac{\pi}{6}$: $u_{13} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \frac{12}{\left(\frac{\sqrt{3}}{2}\right)^4}$

$$= \frac{1}{\frac{3}{4}} - \frac{12}{\frac{9}{16}}$$

$$= \frac{4}{3} - \frac{192}{9}$$

$$= \frac{12}{9} - \frac{192}{9}$$

$$= \frac{-180}{9}$$

$$= \underline{\underline{-20}}$$

- 5 The first, third and fifth terms of an arithmetic progression are $2 \cos x$, $-6\sqrt{3} \sin x$ and $10 \cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x .

[3]

$$u_1 = 2 \cos x = a$$

$$u_3: u_3 = a + 2d$$

$$= 2 \cos x + 2d$$

$$\rightarrow -6\sqrt{3} \sin x = 2 \cos x + 2d \quad (1)$$

$$u_5: u_5 = a + 4d$$

$$= 2 \cos x + 4d$$

$$\rightarrow 10 \cos x = 2 \cos x + 4d$$

$$8 \cos x = 4d$$

$$d = 2 \cos x$$

$$\frac{-\sqrt{3} \sin x}{\cos x} = 1$$

$$\frac{\sin x}{\cos x} = \frac{-1}{\sqrt{3}}$$

$$\tan x = \frac{-1}{\sqrt{3}}$$

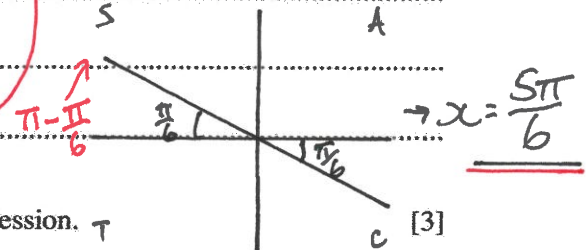
Sub. into (1):

$$-6\sqrt{3} \sin x = 2 \cos x + 2(2 \cos x)$$

$$-6\sqrt{3} \sin x = 6 \cos x$$

$$-\sqrt{3} \sin x = \cos x$$

$$x = \frac{-\pi}{6}$$



(b) Hence find the exact sum of the first 25 terms of the progression.

[3]

$$a = 2 \cos\left(\frac{5\pi}{6}\right)$$

$$= -\sqrt{3}$$

$$d = 2 \cos\left(\frac{5\pi}{6}\right)$$

$$= -\sqrt{3}$$

$$S_{25} = \frac{25}{2} [2(-\sqrt{3}) + 24(-\sqrt{3})]$$

$$= \frac{25}{2} [-26\sqrt{3}] = -325\sqrt{3}$$

$$= 25 \times -13\sqrt{3}$$

9. (b) An arithmetic progression P has first term a and common difference d . An arithmetic progression Q has first term $2(a+1)$ and common difference $(d+1)$. It is given that

$$\textcircled{A} \quad \frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \textcircled{B} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d .

[6]

5th term of P : $a + 4d$

12th term of Q : $2(a+1) + 11(d+1)$
 $= 2a + 2 + 11d + 11$
 $= 2a + 11d + 13$

$\rightarrow \textcircled{A}$: $\frac{a + 4d}{2a + 11d + 13} = \frac{1}{3}$

$$3(a + 4d) = 2a + 11d + 13$$

$$3a + 12d = 2a + 11d + 13$$

$$a + 12d = 11d + 13$$

$$a + d = 13 \quad \textcircled{1}$$

Sum of first 5 terms of P : $\frac{5}{2}[2a + 4d]$
 $= 5a + 10d$

Sum of first 5 terms of Q : $\frac{5}{2}[2 \times 2(a+1) + 4(d+1)]$
 $= \frac{5}{2}[4a + 4 + 4d + 4]$
 $= \frac{5}{2}[4a + 4d + 8]$
 $= 10a + 10d + 20$

$\rightarrow \textcircled{1} - \textcircled{2}$:

$$3d = 21$$

$$d = 7$$

$\rightarrow \textcircled{1}$:

$$a + 7 = 13$$

$$a = 6$$

$\rightarrow \textcircled{B}$: $\frac{5a + 10d}{10a + 10d + 20} = \frac{2}{3}$

$$15a + 30d = 20a + 20d + 40$$

$$5a - 10d = -40$$

$$a - 2d = -8 \quad \textcircled{2}$$