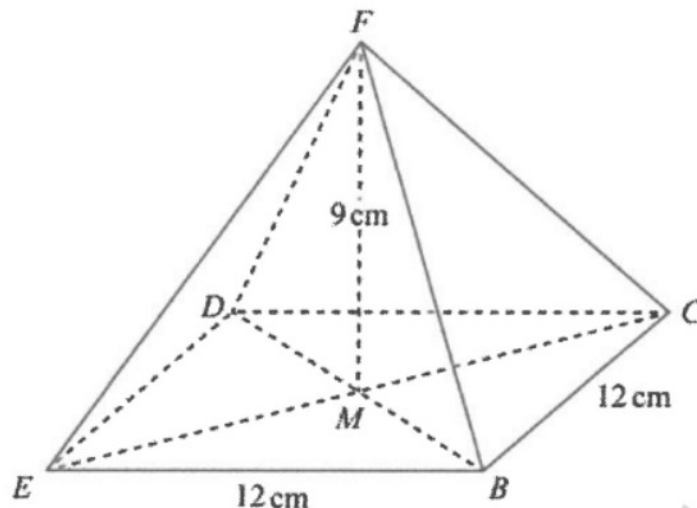


4 (a)



The diagram shows a pyramid with a square base $BCDE$.
The diagonals CE and BD intersect at M , and the vertex F is directly above M .
 $BE = 12$ cm and $FM = 9$ cm.

(i) Calculate the volume of the pyramid.

[The volume, V , of a pyramid with base area A and height h is $V = \frac{1}{3}Ah$.]

$$\begin{aligned} V &= \frac{1}{3} \times \underbrace{12 \times 12}_{\text{base}} \times 9 \\ &= \frac{1}{3} \times 144 \times 9 \\ &= \underline{432} \end{aligned}$$

432 cm³ [2]

6 Calculate the volume of a sphere with diameter 4.8 cm.

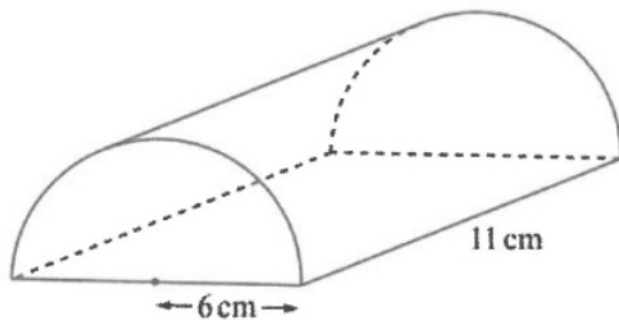
[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$$\text{radius} = 2.4$$

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \pi \times 2.4^3 \\ &= \underline{57.9} \end{aligned}$$

57.9 cm³ [2]

(b)



NOT TO
SCALE

The diagram shows a half-cylinder of radius 6 cm and length 11 cm.

Calculate the volume of the half-cylinder.

Semi-Circle:

$$\begin{aligned} \text{Area} &= \frac{\pi r^2}{2} \\ &= \frac{\pi \times 6^2}{2} \\ &= \frac{36\pi}{2} \\ &= 18\pi \end{aligned}$$

Volume:

$$\begin{aligned} V &= 18\pi \times 11 \\ &= 198\pi \\ &= \underline{622} \end{aligned}$$

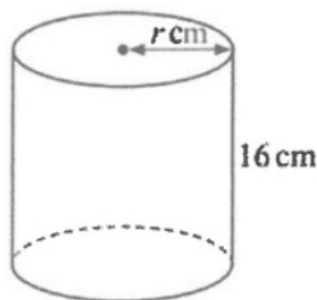
..... 622 cm³ [2]

- 5 Find the volume of a cone with radius 3 cm and perpendicular height 8 cm.
Give your answer in terms of π .

$$V = \frac{1}{3} \pi r^2 h$$

$$\begin{aligned} V &= \frac{1}{3} \times \pi \times 3^2 \times 8 \\ &= \frac{1}{3} \times \pi \times 9 \times 8 \\ &= \pi \times 3 \times 8 \\ &= \underline{24\pi} \end{aligned}$$

..... 24 π cm³ [2]



NOT TO
SCALE

The diagram shows a cylinder with radius r cm and height 16 cm.

A sphere has radius 3 cm.

The volume of the cylinder is equal to the volume of the sphere.

Find the value of r .

Sphere:

$$\begin{aligned}
 V &= \frac{4}{3} \pi r^3 \\
 &= \frac{4}{3} \times \pi \times 3^3 \\
 &= \frac{4}{3} \times \pi \times 27 \\
 &= \underline{36\pi}
 \end{aligned}$$

Cylinder:

$$\begin{aligned}
 V &= \pi r^2 h \\
 36\pi &= \pi r^2 \times 16 \\
 \div \pi &\quad \div \pi \\
 36 &= r^2 \times 16 \\
 \div 16 &\quad \div 16 \\
 \frac{9}{4} &= r^2 \\
 r &= \sqrt{\frac{9}{4}} \\
 &= \frac{3}{2} \quad r = \underline{1.5 \text{ cm}} \quad [4]
 \end{aligned}$$

- 7 The volume of a hemisphere with radius 3 cm is $k\pi \text{ cm}^3$.

Find the value of k .

Volume of hemisphere:

$$\begin{aligned}
 V &= \frac{1}{2} \times \frac{4}{3} \pi r^3 \\
 &= \frac{2}{3} \pi \times 3^3 \\
 &= \frac{2}{3} \pi \times 27 \\
 &= \underline{18\pi \text{ cm}^3}
 \end{aligned}$$

$$k = \underline{18} \quad [2]$$

6 A solid metal cuboid has a volume of 600 cm^3 .

(a) The base of the cuboid is 10 cm by 12 cm .

Calculate the height of the cuboid.

$$V = 10 \times 12 \times h$$

$$600 = 120h$$

$\div 120 \quad \div 120$

$$h = 5$$

$$\underline{5} \text{ cm [2]}$$

(b) The solid metal cuboid is melted and made into 1120 spheres, each with radius 0.45 cm .

Find the volume of metal **not** used in making these spheres.

Volume of 1 sphere:

$$V = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \pi \times 0.45^3$$

$$= 0.3817 \dots$$

STO

1120 spheres:

$$1120 \times 0.3817 \dots$$

$$= 427.51$$

Volume not used:

$$600 - 427.51$$

$$= 172.49$$

$$\underline{172} \text{ cm}^3 \text{ [2]}$$

(3sf)

6 A cone has base radius 5 cm and height $\frac{5}{4} \text{ cm}$.

A hemisphere has radius $r \text{ cm}$.

The volume of the hemisphere is equal to the volume of the cone.

Find the value of r .

Volume of cone:

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 5^2 \times \frac{5}{4}$$

$$= \frac{125}{12} \pi$$

Volume of hemisphere:

$$V = \frac{1}{2} \times \frac{4}{3} \pi r^3$$

$$\frac{125}{12} \pi = \frac{2}{3} \pi r^3$$

$\div \pi \quad \div \pi$

$$\frac{125}{12} = \frac{2}{3} r^3$$

$$375 = 24 r^3$$

$\div 24 \quad \div 24$

$$r^3 = \frac{375}{24}$$

$$r^3 = \frac{125}{8}$$

$$r = \sqrt[3]{\frac{125}{8}}$$
$$= \frac{5}{2}$$

$$r = \underline{2.5 \text{ cm}} \text{ [3]}$$

- 17 Find the radius of a hemisphere of volume 80 cm^3 .

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$$V = \frac{1}{2} \times \frac{4}{3} \pi r^3$$

$$80 = \frac{2}{3} \times \pi \times r^3$$

$$\frac{240}{\div 2\pi} = \frac{2\pi r^3}{\div 2\pi}$$

$$r^3 = \frac{240}{2\pi}$$

$$r^3 = \frac{120}{\pi}$$

$$r = \sqrt[3]{\frac{120}{\pi}} \dots\dots\dots 3.37 \text{ cm [3]}$$

- (b) A cylinder with radius 6 cm and height h cm has the same volume as a sphere with radius 4.5 cm.

Find the value of h .

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Sphere:

$$V = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \pi \times 4.5^3$$

$$= \frac{243}{2} \pi$$

Cylinder:

$$V = \pi r^2 h$$

$$\frac{243}{2} \pi = \pi \times 6^2 \times h$$

$$\frac{243}{2} = 36h$$

$$h = \dots\dots\dots 3.375 \text{ cm [3]}$$

$$\frac{121.5}{\div 36} = \frac{36h}{\div 36}$$

$$h = 3.375 \text{ cm}$$

- (c) A solid metal cube of side 20 cm is melted down and made into 40 solid spheres, each of radius r cm.

Find the value of r .

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Cube:

$$V = 20 \times 20 \times 20$$

$$= 8000$$

40 spheres:

$$V = 40 \times \frac{4}{3} \pi r^3$$

$$\frac{8000}{\times 3} = \frac{160}{3 \times 3} \pi r^3$$

$$\frac{24000}{\div 160\pi} = \frac{160\pi r^3}{\div 160\pi}$$

$$r^3 = \frac{24000}{160\pi}$$

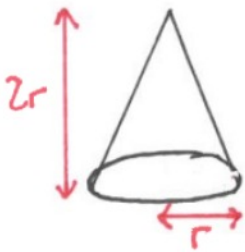
$$r^3 = \frac{150}{\pi}$$

$$r = \sqrt[3]{\frac{150}{\pi}}$$

$$r = \dots\dots\dots 3.62 \text{ cm [3]}$$

- 11 The volume of a cone is $18\pi \text{ cm}^3$.
The height of the cone is the same as the diameter of its base.

Find the radius of the base.



$$V = \frac{1}{3} \pi r^2 h$$

$$18\pi = \frac{1}{3} \times \pi \times r^2 \times 2r$$

$$18\pi = \frac{2}{3} \pi r^3$$

$$\begin{array}{cc} \times 3 & \times 3 \\ 54\pi = 2\pi r^3 \end{array}$$

$$\begin{array}{cc} \div 2\pi & \div 2\pi \\ 27 = r^3 \end{array}$$

$$27 = r^3$$

$$r = \sqrt[3]{27}$$

$$r = 3 \text{ cm}$$

$$\underline{\underline{3}} \text{ cm [3]}$$

- (c) 2000 cm^3 of iron is melted down and some of it is used to make 50 spheres with radius 2 cm.

- (i) Calculate the percentage of iron that is left over.
[The volume, V , of a sphere with radius r is $V = \frac{4}{3} \pi r^3$.]

50 spheres:

$$V = 50 \times \frac{4}{3} \pi \times 2^3$$

$$= \frac{1600}{3} \pi$$

iron left over:

$$2000 - \frac{1600}{3} \pi$$

$$= 324.48 \text{ cm}^3$$

% left over:

$$\frac{324.48}{2000} \times 100$$

$$\underline{\underline{16.2}} \% \text{ [3]}$$

- (ii) The iron left over is then made into a cube.

Calculate the length of an edge of the cube.

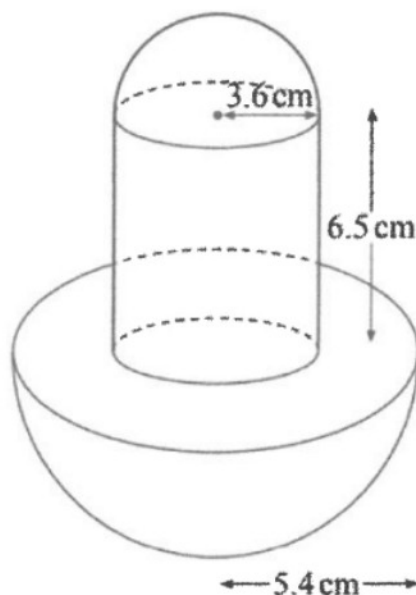
$$V = x^3$$

$$324.48 = x^3$$

$$x = \sqrt[3]{324.48}$$

$$= \underline{\underline{6.87 \text{ cm}}}$$

$$\underline{\underline{6.87}} \text{ cm [1]}$$

NOT TO
SCALE

The diagram shows a solid formed by joining two hemispheres and a cylinder.
The radius of the large hemisphere is 5.4 cm.
The radius of the small hemisphere and the radius of the cylinder are both 3.6 cm.
The height of the cylinder is 6.5 cm.

- (i) Show that the volume of the solid is 692 cm^3 , correct to the nearest cubic centimetre.

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Small hemisphere:

$$\begin{aligned} V &= \frac{1}{2} \times \frac{4}{3} \pi r^3 \\ &= \frac{2}{3} \times \pi \times 3.6^3 \\ &= \underline{97.716} \end{aligned}$$

Large hemisphere:

$$\begin{aligned} V &= \frac{1}{2} \times \frac{4}{3} \pi r^3 \\ &= \frac{2}{3} \times \pi \times 5.4^3 \\ &= \underline{329.79} \end{aligned}$$

Cylinder:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 3.6^2 \times 6.5 \\ &= \underline{264.65} \end{aligned}$$

$$\begin{aligned} \text{Total: } &97.716 + 329.79 + 264.65 \\ &= \underline{692 \text{ cm}^3} \quad [4] \end{aligned}$$

- 16 The volume of a hemisphere with radius r cm is $\frac{16}{3}\pi \text{ cm}^3$.

Find the value of r .

$$V = \frac{1}{2} \times \frac{4}{3} \pi r^3$$

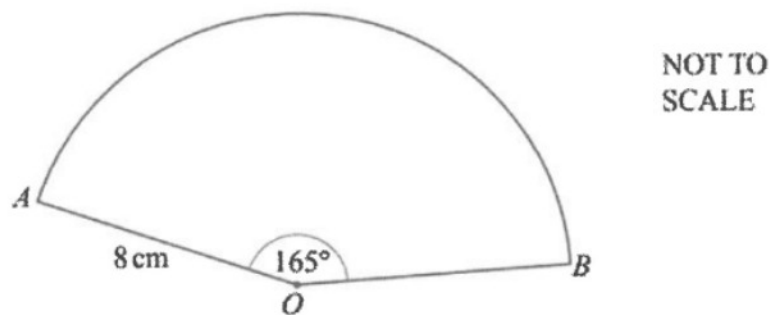
$$\frac{16}{3} \pi = \frac{2}{3} \pi r^3$$

$$\begin{aligned} 16\pi &= 2\pi r^3 \\ \div 2\pi &\quad \div 2\pi \end{aligned}$$

$$8 = r^3$$

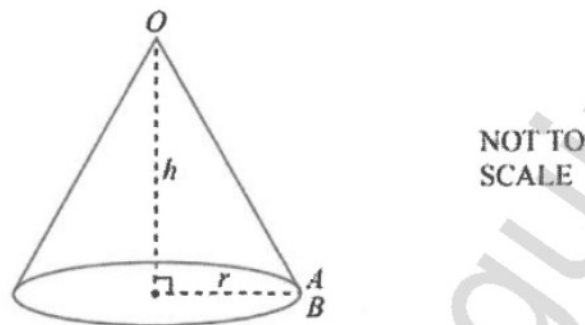
$$\begin{aligned} r &= \sqrt[3]{8} \\ &= \underline{2 \text{ cm}} \end{aligned}$$

$$r = \underline{2} \text{ cm} \quad [3]$$



The diagram shows a sector of a circle with centre O , radius 8 cm and sector angle 165° .

(c)



A cone is made from the sector by joining OA to OB .

(i) Calculate the radius, r , of the cone.

Arc length of sector:

$$\begin{aligned}
 l &= \frac{165}{360} \times 2\pi r \\
 &= \frac{11}{24} \times 2\pi \times 8 \\
 &= \frac{22}{3}\pi
 \end{aligned}$$

Radius of base:

$$\begin{aligned}
 C &= 2\pi r \\
 \frac{22}{3}\pi &= 2\pi r
 \end{aligned}$$

← becomes circumference of the base of cone.

$$r = \frac{11}{3} \text{ cm [2]}$$

(ii) Calculate the volume of the cone.

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Find height using pythagoras:



$$h^2 + \left(\frac{11}{3}\right)^2 = 8^2$$

$$h^2 + \frac{121}{9} = 64$$

$$h^2 = \frac{455}{9}$$

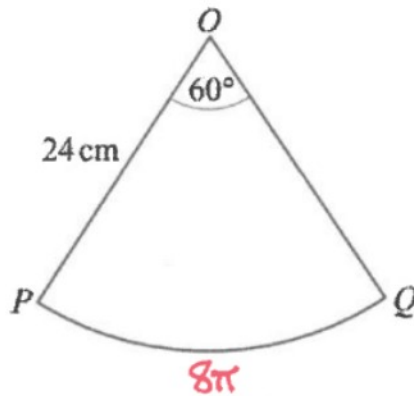
$$h = 7.11 \text{ cm}$$

Volume:

$$\begin{aligned}
 V &= \frac{1}{3} \times \pi \times \left(\frac{11}{3}\right)^2 \times 7.11 \\
 &= 100 \text{ cm}^3 \text{ (3sf)}
 \end{aligned}$$

$$\underline{100} \text{ cm}^3 \text{ [4]}$$

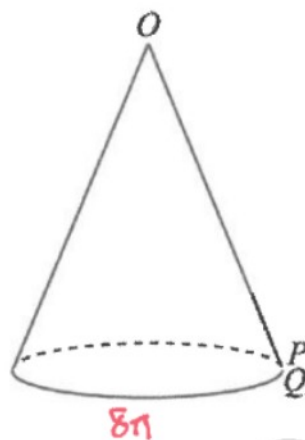
(b)



NOT TO
SCALE

The diagram shows a sector OPQ of a circle with centre O and radius 24 cm. The sector angle is 60° .

A cone is made from this sector by joining OP to OQ .



NOT TO
SCALE

Calculate the volume of the cone.

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Arc length of sector:

$$\begin{aligned} l &= \frac{60}{360} \times 2\pi \times 24 \\ &= \frac{1}{6} \times 48\pi \\ &= 8\pi \end{aligned}$$

↳ becomes circumference of the base of the cone.

Radius of base:

$$\begin{aligned} C &= 2\pi r \\ 8\pi &= 2\pi r \\ \div 2\pi &\quad \div 2\pi \\ \underline{r} &= 4\text{ cm} \end{aligned}$$

Height of cone:

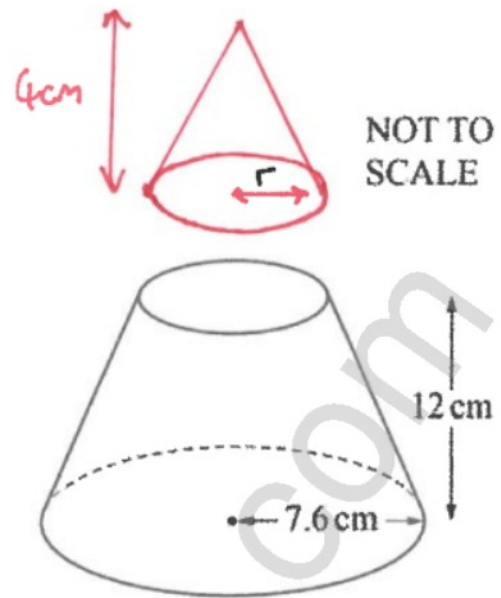
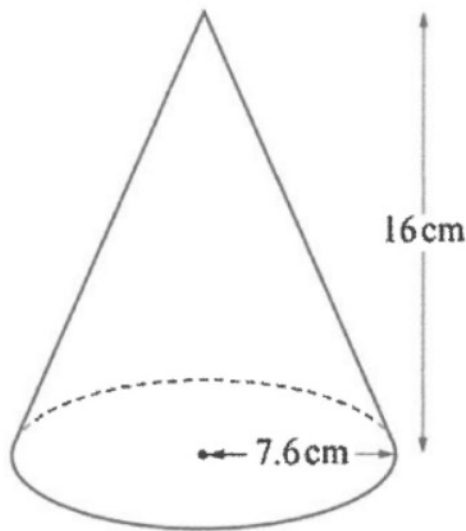


$$\begin{aligned} h^2 + 4^2 &= 24^2 \\ h^2 + 16 &= 576 \\ h^2 &= 560 \\ h &= \sqrt{560} \end{aligned}$$

Volume: $V = \frac{1}{3} \times \pi \times 4^2 \times \sqrt{560}$
 $= 396\text{ cm}^3$

396 cm³ [6]

(b)



The diagram shows a solid cone with radius 7.6 cm and height 16 cm.
A cut is made parallel to the base of the cone and the top section is removed.
The remaining solid has height 12 cm, as shown in the diagram.

Calculate the volume of the remaining solid.

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Think of the remaining shape as a large cone with a small cone removed from the top.

Volume of large cone:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 7.6^2 \times 16 \\ &= \underline{967.78 \text{ cm}^3} \end{aligned}$$

Radius of small cone:

The two cones are similar:

$$\begin{aligned} \text{Scale factor} &= \frac{16}{4} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{Radius} &= 7.6 \div 4 \\ &= \underline{1.9 \text{ cm}} \end{aligned}$$

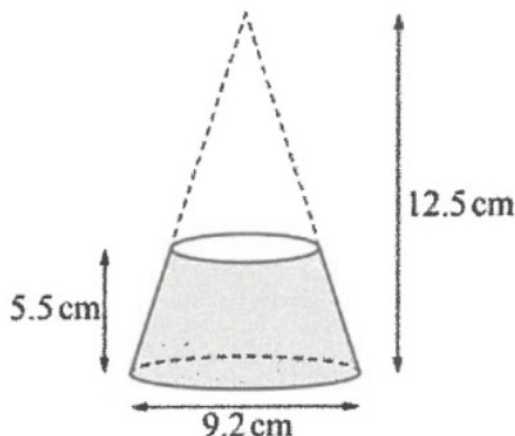
Volume of small cone:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 1.9^2 \times 4 \\ &= \underline{15.12 \text{ cm}^3} \end{aligned}$$

Volume left: $967.78 - 15.12$

$$= \underline{953} \text{ cm}^3 [4]$$

(3sf)



NOT TO SCALE

A solid is made by cutting a small cone from a larger cone, as shown in the diagram.

The height of the larger cone is 12.5 cm.

The height of the solid is 5.5 cm.

The diameter of the base of the larger cone is 9.2 cm.

Work out the volume of the solid.

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Volume of large cone:

$$V = \frac{1}{3} \times \pi \times 4.6^2 \times 12.5$$

$$= \underline{276.98 \text{ cm}^3}$$

Volume of small cone:

$$\text{height} = 12.5 - 5.5$$

$$= 7 \text{ cm}$$

$$\text{Scale factor} = \frac{12.5}{7}$$

$$= \frac{25}{14}$$

Because the cones are similar, we can either find the radius, then use $V = \frac{1}{3}\pi r^2 h$ OR use LSF, ASF, VSF

$$\rightarrow \text{LSF} = \frac{25}{14} \rightarrow \text{VSF} = \left(\frac{25}{14}\right)^3$$

$$\text{VSF} = \frac{15625}{2744}$$

Volume of small cone:

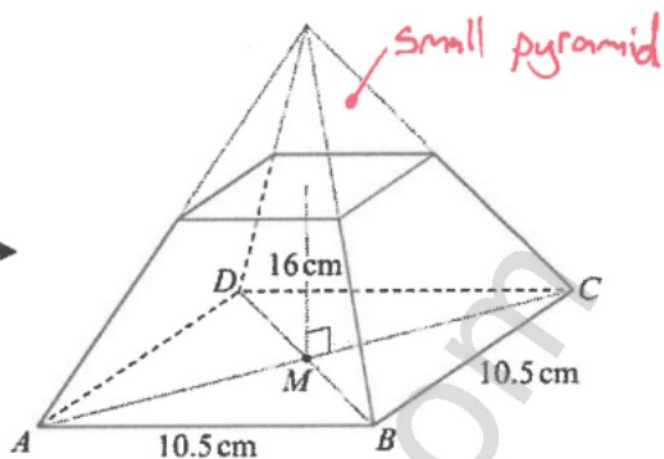
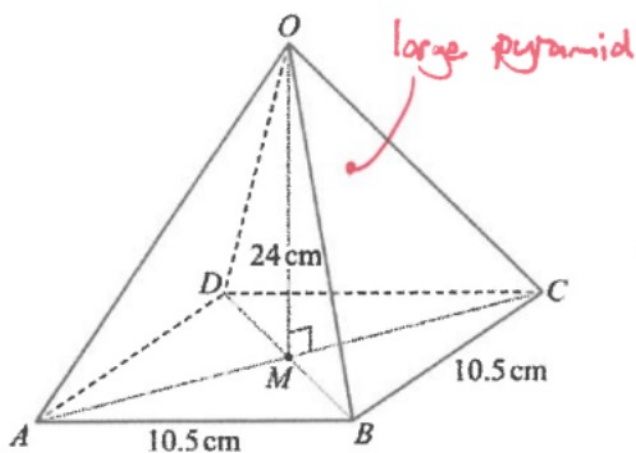
$$276.98 \div \frac{15625}{2744}$$

$$= \underline{48.64 \text{ cm}^3}$$

Volume of solid:

$$276.98 - 48.64$$

$$= \underline{228 \text{ cm}^3} \quad (3 \text{ sf})$$



The diagram shows a frustum of the pyramid $OABCD$.
The height of the frustum is 16 cm.

Calculate the volume of the frustum.

Volume of large pyramid:

$$V = \frac{1}{3} Ah$$

$$= \frac{1}{3} \times \underbrace{10.5 \times 10.5}_{\text{base}} \times 24$$

$$= \underline{882}$$

Height of small pyramid:

$$24 - 16 = 8 \text{ cm}$$

Scale factor:

$$\text{LSF} = \frac{24}{8} = \underline{3}$$

$$\text{VSF} = 3^3 = \underline{27}$$

Volume of small pyramid:

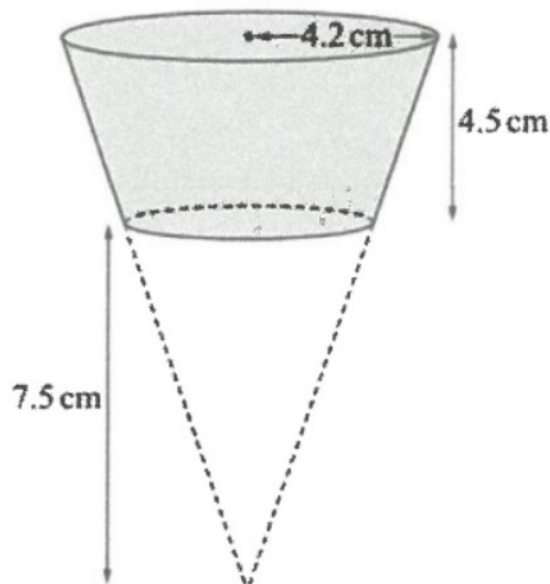
$$882 \div 27 = \frac{98}{3}$$

Volume of frustum:

$$882 - \frac{98}{3} = 849.3 \text{ cm}^3$$

849

..... cm^3 [5]



NOT TO
SCALE

The diagram shows a frustum made by removing a small cone from a large cone.
The height of the small cone is 7.5 cm.
The height of the frustum is 4.5 cm.
The radius of the large cone is 4.2 cm.

Work out the volume of the frustum.

Volume of large cone:

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 4.2^2 \times 12 \\ &= 221.67 \text{ cm}^3 \end{aligned}$$

Scale factor: height of large

$$\text{LSF} = \frac{12}{7.5} = \frac{8}{5}$$

height of small

$$\text{VSF} = \left(\frac{8}{5}\right)^3 = \frac{512}{125}$$

Volume of small cone:

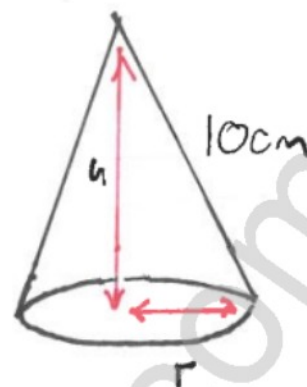
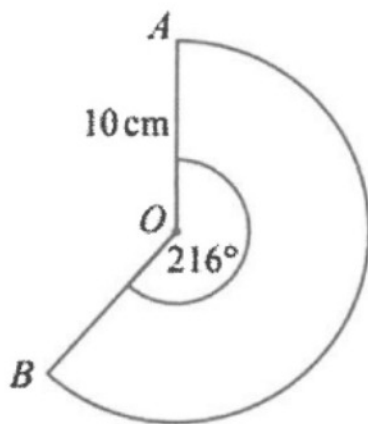
$$\begin{aligned} &221.67 \div \frac{512}{125} \\ &= 54.12 \end{aligned}$$

Volume of frustum:

$$221.67 - 54.12$$

$$= 168 \text{ (3sf)} \text{ cm}^3 \text{ [4]}$$

(b)



AOB is a sector of a circle, centre O .
 $AO = 10\text{ cm}$ and the sector angle is 216° .

(ii) A cone is made from this sector by joining OA to OB .

Calculate the volume of the cone.

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Arc length of sector:

$$\begin{aligned} l &= \frac{216}{360} \times 2\pi r \\ &= \frac{3}{5} \times 2 \times \pi \times 10 \\ &= 12\pi \end{aligned}$$

$\hookrightarrow$ circumference of base

Radius of base:

$$\begin{aligned} C &= 2\pi r \\ 12\pi &= 2\pi r \\ \div 2\pi &\quad \div 2\pi \\ \underline{r = 6\text{ cm}} \end{aligned}$$

Pythagoras to find height:

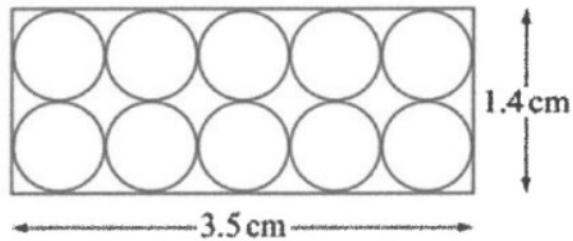


$$\begin{aligned} h^2 + 6^2 &= 10^2 \\ h^2 + 36 &= 100 \\ h^2 &= 64 \\ \underline{h = 8\text{ cm}} \end{aligned}$$

Volume:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 6^2 \times 8 \\ &= 96\pi \text{ cm}^3 \\ &= \underline{302\text{ cm}^3 \text{ to 3sf}} \end{aligned}$$

(iii)



NOT TO
SCALE

10 of the solids are placed in a box in the shape of a cuboid of length 17.5 cm.
The diagram shows one end of the box.

Calculate the volume of the empty space in the box.

Cuboid volume:

$$1.4 \times 3.5 \times 17.5 = 85.75$$

Volume of 10 solids:

$$6.35 \times 10 = 63.5$$

Empty space:

$$85.75 - 63.5 = 22.25$$

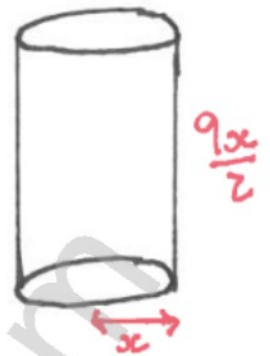
22.3

..... cm³ [3]

- (b) A solid metal cylinder with radius x and height $\frac{9x}{2}$ is melted.
All the metal is used to make a sphere with radius r .

Find r in terms of x .

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]



Cylinder:

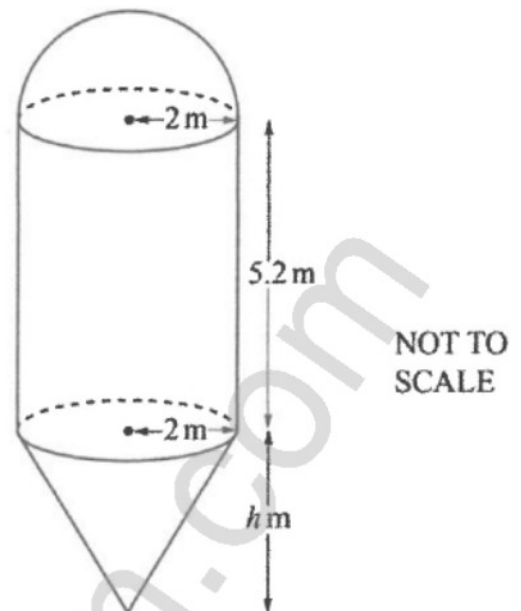
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times x^2 \times \frac{9x}{2} \\ &= \frac{9}{2} \pi x^3 \end{aligned}$$

Sphere:

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \\ \frac{9}{2} \pi x^3 &= \frac{4}{3} \pi r^3 \\ \div \pi &\quad \div \pi \\ \frac{9}{2} x^3 &= \frac{4}{3} r^3 \\ 27x^3 &= 8r^3 \\ \div 8 &\quad \div 8 \\ r^3 &= \frac{27x^3}{8} \\ r &= \sqrt[3]{\frac{27x^3}{8}} \\ r &= \frac{3x}{2} \end{aligned}$$

- 7 (a) The diagram shows a container for storing grain.

The container is made from a hemisphere, a cylinder and a cone, each with radius 2 m. The height of the cylinder is 5.2 m and the height of the cone is h m.



- (i) Calculate the volume of the hemisphere. Give your answer as a multiple of π .

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$$\begin{aligned} V &= \frac{1}{2} \times \frac{4}{3} \pi r^3 \\ &= \frac{2}{3} \times \pi \times 2^3 \\ &= \frac{16}{3} \pi \end{aligned}$$

$$\frac{16}{3} \pi \dots \text{m}^3 \quad [2]$$

- (ii) The total volume of the container is $\frac{88\pi}{3} \text{m}^3$.

Calculate the value of h .

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Cylinder:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 2^2 \times 5.2 \\ &= \frac{104}{5} \pi \end{aligned}$$

Volume of cone:

$$\frac{88\pi}{3} - \frac{104\pi}{5} - \frac{16\pi}{3} = \frac{16\pi}{5}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$\frac{16\pi}{5} = \frac{1}{3} \times \pi \times 2^2 \times h$$

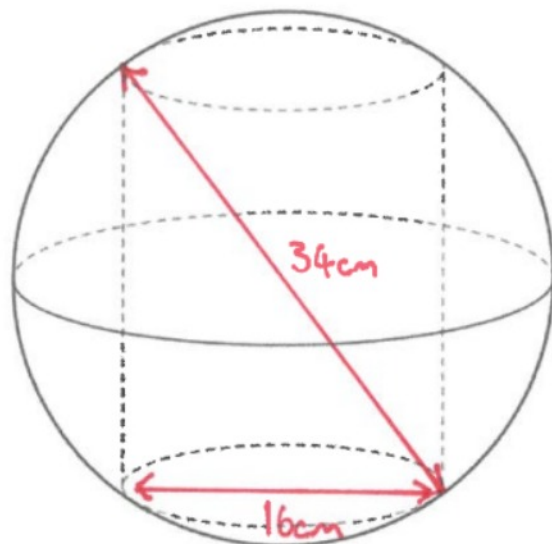
$$\frac{16}{5} \pi = \frac{4}{3} \pi h$$

$$\frac{16}{5} = \frac{4}{3} h$$

$$48 = 20h$$

$$h = 2.4 \text{ m}$$

(b)



NOT TO
SCALE

The diagram shows a cylinder with radius 8 cm inside a sphere with radius 17 cm. Both ends of the cylinder touch the curved surface of the sphere.

(i) Show that the height of the cylinder is 30 cm.

Pythagoras:



$$h^2 + 16^2 = 34^2$$

$$h^2 + 256 = 1156$$

$$h^2 = 900$$

$$h = 30 \text{ cm}$$

[2]

(ii) Calculate the volume of the cylinder as a percentage of the volume of the sphere.

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Sphere:

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \times 17^3$$

$$= 20579.53$$

Cylinder:

$$V = \pi r^2 h$$

$$= \pi \times 8^2 \times 30$$

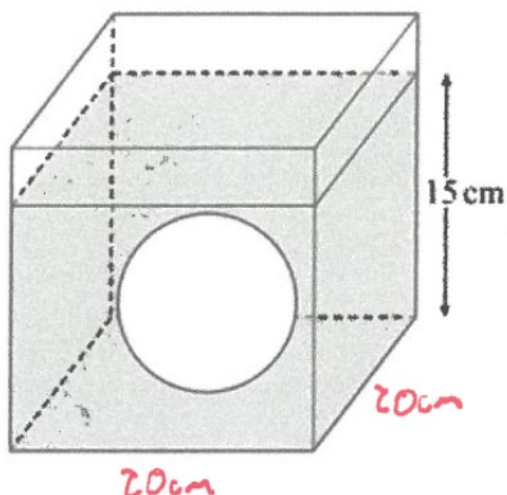
$$= 6031.86$$

% Volume:

$$\frac{6031.86}{20579.53} \times 100$$

$$= 29.3\%$$

(c)



NOT TO
SCALE

The diagram shows a solid sphere with radius 6 cm inside a cube with side length 20 cm. The cube contains water to a depth of 15 cm. The sphere is removed.

Calculate the new depth of water in the cube.

[The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

ie. shaded bit

Volume of sphere:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 6^3 \\ &= 288\pi \end{aligned}$$

Volume of water = Volume of cuboid - sphere

$$\begin{aligned} &= 20 \times 20 \times 15 - 288\pi \\ &= 6000 - 288\pi \end{aligned}$$

New height:

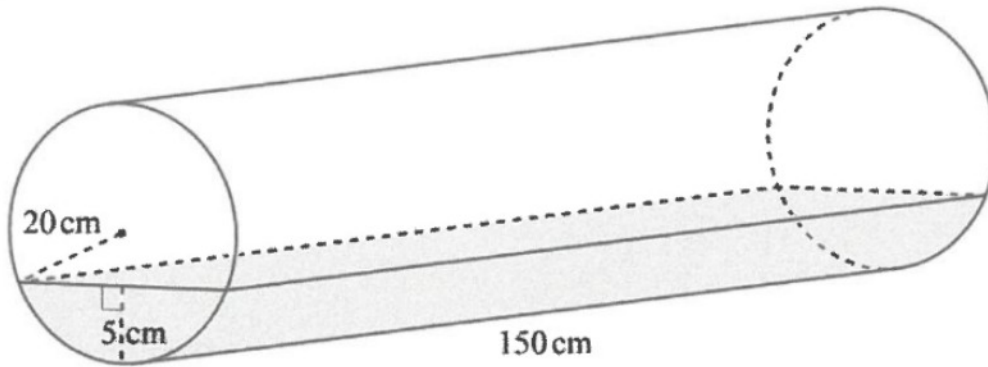
$$V = 20 \times 20 \times h$$

$$6000 - 288\pi = 400h$$

$$h = \frac{6000 - 288\pi}{400}$$

$$= \underline{12.7} \dots \text{cm [3]}$$

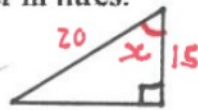
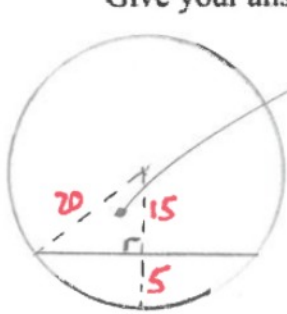
(c)



NOT TO
SCALE

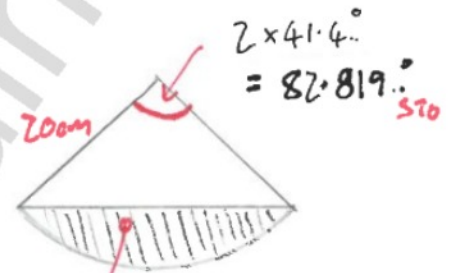
The diagram shows a cylinder of length 150 cm on horizontal ground.
The cylinder has radius 20 cm.
The cylinder contains water to a depth of 5 cm, as shown in the diagram.

Calculate the volume of water in the cylinder.
Give your answer in litres.



$$\cos x = \frac{15}{20}$$

$$x = \cos^{-1}\left(\frac{15}{20}\right) \\ = 41.4^\circ \text{ STO}$$



Shaded Area: Sector - Triangle

Area of sector:

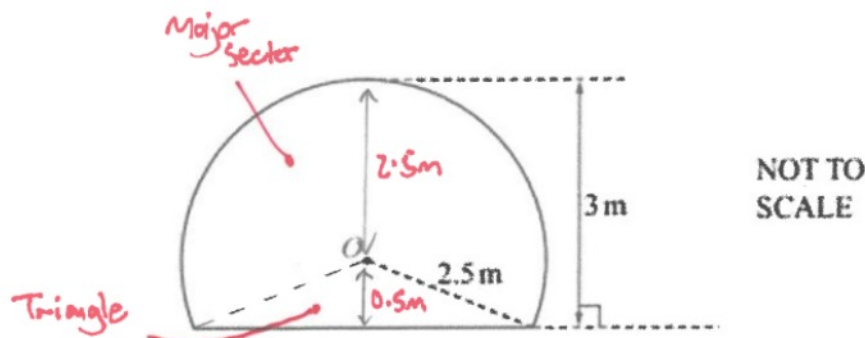
$$A = \frac{82.819}{360} \times \pi r^2 \\ = \frac{82.819}{360} \times \pi \times 20^2 \\ = \underline{289.0937}$$

Area of triangle:

$$A = \frac{1}{2} ab \sin C \\ = \frac{1}{2} \times 20 \times 20 \times \sin(82.819^\circ) \\ = \underline{198.4313}$$

$$\text{Shaded Area} = 289.0937 - 198.4313 \\ = \underline{90.6624}$$

$$\text{Volume} = 90.6624 \times 150 \\ = 13599.36 \text{ cm}^3 \\ = 13599.36 \text{ ml} \\ = \underline{\underline{13.6 \text{ litres}}}$$



The diagram shows the major segment of a circle, centre O , radius 2.5 m.

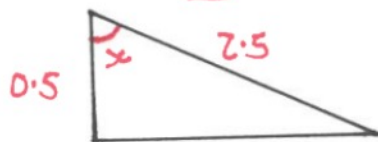
The segment is the cross section of a tunnel with height 3 m.

The length of the tunnel is 800 m and it has the same cross section throughout its length.

Calculate the volume of the tunnel.

Area = Area of major sector + Area of triangle

Use triangle to find angle:

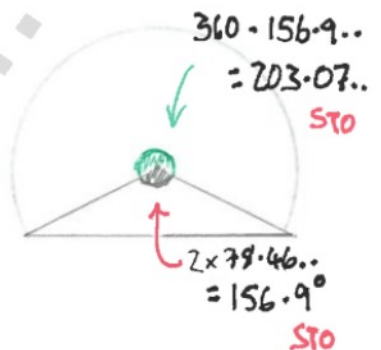


$\sin^{-1} \frac{0.5}{2.5}$

$$\cos x = \frac{0.5}{2.5}$$

$$x = \cos^{-1} \left(\frac{0.5}{2.5} \right)$$

$$= 78.46^\circ \text{ STO}$$



Area of major sector:

$$A = \frac{203.07}{360} \times \pi r^2$$

$$= \frac{203.07}{360} \times \pi \times 2.5^2$$

$$= 11.075 \text{ STO}$$

Area of triangle:

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 2.5 \times 2.5 \times \sin(156.9)$$

$$= 1.224 \text{ STO}$$

$$\text{Total Area: } 11.075 + 1.224$$

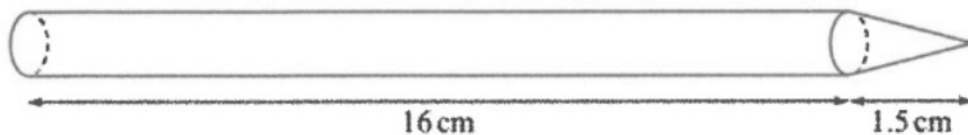
$$= 12.300$$

$$\text{Volume} = 12.300 \times 800$$

$$= 9840 \text{ m}^3 [7]$$

(3sf)

8 (a)

NOT TO
SCALE

The diagram shows a solid made from a cylinder and a cone.
The height of the cylinder is 16 cm and the height of the cone is 1.5 cm.
The radius of the cylinder and the base radius of the cone are each 0.35 cm.

(ii) Calculate the volume of the solid.

[The volume, V , of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Cylinder:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 0.35^2 \times 16 \\ &= \underline{6.1575} \end{aligned}$$

Cone:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 0.35^2 \times 1.5 \\ &= \underline{0.1924} \end{aligned}$$

Total Volume: $6.1575 + 0.1924 = 6.3499$

$$\underline{6.35} \text{ cm}^3 \text{ [3]}$$

(3sf)