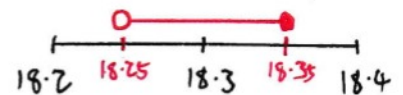


A Upper and Lower Bounds

- 8 The length, l cm, of a line is 18.3 cm, correct to the nearest millimetre.



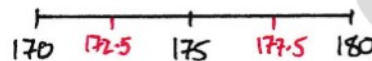
Complete this statement about the value of l .

$$18.3 \begin{cases} 18.35 \\ 18.25 \end{cases}$$

$$18.25 \leq l < 18.35 \quad [2]$$

- 15 Ella's height is 175 cm, correct to the nearest 5 cm.

Write down the upper bound of Ella's height.



$$175 \begin{cases} 177.5 \\ 172.5 \end{cases}$$

$$177.5 \text{ cm} \quad [1]$$

- 22 (a) A bag of rice has a mass of 25 kg, correct to the nearest kilogram.

Calculate the lower bound of the total mass of 10 of these bags.

$$25 \begin{cases} 25.5 \\ 24.5 \end{cases}$$

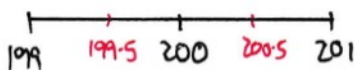
$$24.5 \times 10 = 245$$

$$245 \text{ kg} \quad [1]$$

- (b) Virat has 200 metres of wire, correct to the nearest metre.

He cuts the wire into n pieces of length 3 metres, correct to the nearest 20 centimetres.

Calculate the largest possible value of n .

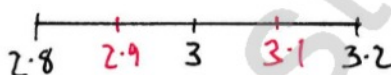


$$200 \begin{cases} 200.5 \\ 199.5 \end{cases}$$

For the most pieces, we want the wire to be as long as possible and the pieces to be as short as possible: $n =$

$$69$$

[3]



$$3 \begin{cases} 3.1 \\ 2.9 \end{cases}$$

$$n = \frac{200.5}{2.9} = 69.14$$

- 10 (a) The lengths of the sides of a triangle are 11.4 cm, 14.8 cm and 15.7 cm, all correct to 1 decimal place.

Calculate the upper bound of the perimeter of the triangle.

$$11.4 < \frac{11.45}{11.35}$$

$$11.45 + 14.85 + 15.75 = 42.05$$

$$14.8 < \frac{14.85}{14.75}$$

$$15.7 < \frac{15.75}{15.65}$$

$$\underline{42.05} \text{ cm [2]}$$

- 16 The sides of a regular hexagon are 80 mm, correct to the nearest millimetre.

Calculate the lower bound of the perimeter of the hexagon.

$$80 < \frac{80.5}{79.5}$$

$$6 \times 79.5 = 477$$

$$\underline{477} \text{ mm [2]}$$

- 18 The sides of an isosceles triangle are measured correct to the nearest millimetre. One side has a length of 8.2 cm and another has a length of 9.4 cm.

Find the largest possible value of the perimeter of this triangle.

For largest perimeter, triangle must have two 9.4 cm sides (rather than two 8.2 cm sides)

$$8.2 < \frac{8.25}{8.15}$$

$$8.25 + 2 \times 9.45$$

$$9.4 < \frac{9.45}{9.35}$$

$$= 8.25 + 18.9$$

$$\underline{27.15} \text{ cm [3]}$$

$w = 11$ and $h = 9.5$, both correct to 2 significant figures.

Find the lower bound and the upper bound for P .

$$w \begin{cases} 11.5 \\ 10.5 \end{cases}$$

$$h \begin{cases} 9.55 \\ 9.45 \end{cases}$$

$$\text{UB: } 2(11.5) + 2(9.55) \\ = 42.1$$

$$\text{LB: } 2(10.5) + 2(9.45) \\ = 39.9$$

$$\text{Lower bound} = 39.9$$

$$\text{Upper bound} = 42.1 \quad [3]$$

- (b) A cuboid measures 10 cm by 4 cm by 6 cm.
Each side is measured correct to the nearest centimetre.

Complete the inequality for the volume, V , of this cuboid.

$$10 \begin{cases} 10.5 \\ 9.5 \end{cases}$$

$$\text{LB: } V = 9.5 \times 3.5 \times 5.5 \\ = 182.875$$

$$4 \begin{cases} 4.5 \\ 3.5 \end{cases}$$

$$\text{UB: } V = 10.5 \times 4.5 \times 6.5 \\ = 307.125$$

$$6 \begin{cases} 6.5 \\ 5.5 \end{cases}$$

$$182.875 \text{ cm}^3 \leq V < 307.125 \text{ cm}^3 \quad [3]$$

18 $P = 2(w + h)$

$w = 12$ correct to the nearest whole number.
 $h = 4$ correct to the nearest whole number.

Work out the upper bound for the value of P .

$$12 < \frac{12.5}{11.5}$$

$$4 < \frac{4.5}{3.5}$$

$$\begin{aligned} P &= 2(12.5 + 4.5) \\ &= 2 \times 17 \\ &= 34 \end{aligned}$$

34

[2]

- 24 Violet and Wilfred recorded their times to run 200 m, correct to the nearest second.
 Violet took 36 seconds and Wilfred took 39 seconds.

Work out the upper bound of the difference between their times.

$$V: 36 < \frac{36.5}{35.5}$$

$$\begin{aligned} UB &= W_{UB} - V_{LB} \\ &= 39.5 - 35.5 \\ &= 4 \end{aligned}$$

$$W: 39 < \frac{39.5}{38.5}$$

4

s [2]

- 21 Neha has a piece of ribbon of length 23 cm, correct to the nearest cm.
From this ribbon she cuts off a piece with length 87 mm, correct to the nearest mm.

Work out the lower bound and the upper bound for the length of the remaining ribbon.
Give your answer in centimetres.

$$23 \begin{cases} 23.5 \\ 22.5 \end{cases}$$

$$87 \begin{cases} 87.5 & (8.75 \text{ cm}) \\ 86.5 & (8.65 \text{ cm}) \end{cases}$$

$$\begin{aligned} LB &= 23_{LB} - 8.7_{UB} \\ &= 22.5 - 8.75 \\ &= \underline{13.75} \end{aligned}$$

$$\begin{aligned} UB &= 23_{UB} - 8.7_{LB} \\ &= 23.5 - 8.65 \\ &= \underline{14.85} \end{aligned}$$

$$\text{Lower bound} = \underline{13.75} \text{ cm}$$

$$\text{Upper bound} = \underline{14.85} \text{ cm [3]}$$

22

$$I = \frac{V}{R}$$

V is 50, correct to the nearest 10.

R is 13, correct to the nearest integer.

Calculate the upper bound of I .

$$V \begin{cases} \underline{55} \\ 45 \end{cases}$$

$$R \begin{cases} 13.5 \\ \underline{12.5} \end{cases}$$

$$\begin{aligned} I_{UB} &= \frac{V_{UB}}{R_{LB}} \\ &= \frac{55}{12.5} \\ &= 4.4 \end{aligned}$$

$$\underline{4.4} \text{ [3]}$$

- 18 A car travels at a constant speed.
It travels a distance of 146.2 m, correct to 1 decimal place.
This takes 7 seconds, correct to the nearest second.

Calculate the upper bound for the speed of the car.

$$\text{distance } 146.2 < \frac{146.25}{146.15}$$

$$\text{time } 7 < \frac{7.5}{6.5}$$

$$\text{Speed}_{\text{UB}} = \frac{\text{distance}_{\text{UB}}}{\text{time}_{\text{LB}}}$$

$$= \frac{146.25}{6.5}$$

$$= 22.5 \dots\dots\dots 22.5 \dots\dots\dots \text{ m/s [3]}$$

- 21 A car travels 14 km, correct to the nearest kilometre.
This takes 12 minutes, correct to the nearest minute.

Calculate the lower bound of the speed of the car.
Give your answer in kilometres per minute.

$$\text{distance } 14 < \frac{14.5}{13.5}$$

$$\text{time } 12 < \frac{12.5}{11.5}$$

$$\text{Speed}_{\text{LB}} = \frac{\text{distance}_{\text{LB}}}{\text{time}_{\text{UB}}}$$

$$= \frac{13.5}{12.5}$$

$$= 1.08$$

$$\dots\dots\dots 1.08 \dots\dots\dots \text{ km/min [3]}$$

- 20 The area of a rectangle is 55.2 cm², correct to 1 decimal place.
The length of the rectangle is 9 cm, correct to the nearest cm.

Calculate the upper bound of the width of the rectangle.

$$\text{Area } 55.2 < \frac{55.25}{55.15}$$

$$\text{length } 9 < \frac{9.5}{8.5}$$

$$\text{Width}_{\text{UB}} = \frac{\text{area}_{\text{UB}}}{\text{length}_{\text{LB}}}$$

$$= \frac{55.25}{8.5}$$

$$= 6.5$$

$$\dots\dots\dots 6.5 \dots\dots\dots \text{ cm [3]}$$

- 20 A piece of metal has volume 1240 cm^3 , correct to the nearest 20 cm^3 .
The mass of the piece of metal is 7800 g , correct to the nearest 100 g .

Calculate the lower bound of the density of the metal.

[Density = mass $\div$ volume.]

volume $1240 < \begin{matrix} \underline{1250} \\ 1230 \end{matrix}$

mass $7800 < \begin{matrix} 7850 \\ \underline{7750} \end{matrix}$

$$\begin{aligned} \text{density}_{\text{LB}} &= \frac{\text{mass}_{\text{LB}}}{\text{Volume}_{\text{UB}}} \\ &= \frac{7750}{1250} \\ &= 6.2 \end{aligned}$$

$\dots\dots\dots 6.2 \dots\dots\dots \text{g/cm}^3$ [3]

- (iii) Greta flies home to Spain.

The plane flies a distance of 2200 km , correct to the nearest 100 km .

The average speed of the plane is 740 km/h , correct to the nearest 20 km/h .

Calculate the lower bound of the time taken, in hours and minutes, for this flight.

distance: $2200 < \begin{matrix} 2250 \\ \underline{2150} \end{matrix}$

speed: $740 < \begin{matrix} \underline{750} \\ 730 \end{matrix}$

$$\begin{aligned} \text{time}_{\text{LB}} &= \frac{\text{distance}_{\text{LB}}}{\text{Speed}_{\text{UB}}} \\ &= \frac{2150}{750} \\ &= 2.8666\dots \end{aligned}$$

Convert to hrs and mins:

$2.8666\dots \times 60 = 172 \text{ mins} \dots\dots 2 \text{ h} \dots\dots 52 \text{ min}$ [3]

$172 \text{ mins} = \underline{2 \text{ hrs } 52 \text{ mins}}$

- 23 A train travels between two stations.
The distance between the stations is 220 km, correct to the nearest kilometre.
The speed of the train is 125 km/h, correct to the nearest 5 km/h.

Calculate the upper bound for the time the journey takes.
Give your answer in hours and minutes.

$$\text{distance: } 220 \begin{cases} 220.5 \\ 219.5 \end{cases}$$

$$\text{speed: } 125 \begin{cases} 127.5 \\ 122.5 \end{cases}$$

$$\begin{aligned} \text{time}_{\text{UB}} &= \frac{\text{distance}_{\text{UB}}}{\text{speed}_{\text{LB}}} \\ &= \frac{220.5}{122.5} \\ &= 1.8 \end{aligned}$$

Convert to hrs and mins:

$$1.8 \times 60 = 108 \text{ mins} \quad \dots 1 \text{ h } 48 \text{ min [4]}$$

$$108 \text{ mins} = \underline{1 \text{ hr } 48 \text{ mins}}$$

- (c) When running, Chao has a stride length of 70 cm, correct to the nearest 5 cm.
Chao runs a distance of 11.2 km, correct to the nearest 0.1 km.

Work out the minimum number of strides that Chao could take to complete this distance.

$$\text{stride length: } 70 \begin{cases} 72.5 \\ 67.5 \end{cases}$$

$$\text{distance: } 11.2 \begin{cases} 11.25 \\ 11.15 \end{cases}$$

$$\text{Strides}_{\text{LB}} = \frac{\text{distance}_{\text{UB}}}{\text{stride length}_{\text{UB}}}$$

First convert to same units:

$$\begin{aligned} 11.15 \text{ km} &= 11150 \text{ m} \\ &= 1115000 \text{ cm} \end{aligned}$$

$$\text{Strides}_{\text{LB}} = \frac{1115000}{72.5}$$

$$= 15379.31\dots \quad \leftarrow \text{not quite there with } 15379 \text{ strides..}$$

$$= \underline{15380} \dots [4]$$

22 A metal cube has mass 14.2 g, correct to 1 decimal place.

(a) Find the lower bound of the mass of the cube.

$$14.2 < \begin{cases} 14.25 \\ 14.15 \end{cases}$$

$$\dots\dots\dots 14.15 \dots\dots\dots \text{g [1]}$$

(b) The side length of the cube is 1.3 cm, correct to 1 decimal place.

Calculate the upper bound of the density of the metal.
[Density = mass $\div$ volume]

side: $1.3 < \begin{cases} 1.35 \\ 1.25 \end{cases}$

$$\text{density}_{\text{UB}} = \frac{\text{mass}_{\text{UB}}}{\text{volume}_{\text{LB}}}$$

$$\begin{aligned} \text{volume}_{\text{LB}} &= 1.25^3 \\ &= 1.953125 \end{aligned}$$

$$\text{density}_{\text{UB}} = \frac{14.25}{1.953125}$$

$$\dots\dots\dots 7.296 \dots\dots\dots \text{g/cm}^3 \text{ [3]}$$

(c) On a farm, there are 500 hens, correct to the nearest 10.

(i) In one year, the mean number of eggs laid per hen was 320 eggs, correct to the nearest 20.

Calculate the upper bound for the total number of eggs all the hens lay in that year.

hens: $500 < \begin{matrix} 505^* \\ 495 \end{matrix}$

mean: $320 < \begin{matrix} 330 \\ 310 \end{matrix}$

$$\begin{aligned} \text{total eggs}_{\text{UB}} &= \text{mean}_{\text{UB}} \times \text{number of hens}_{\text{UB}} \\ &= 330 \times 505 \\ &= \underline{166\,650} \end{aligned}$$

$$\text{mean} = \frac{\text{total eggs}}{\text{number of hens}}$$

$\underline{166\,650}$ (or $165\,816$) [3]

* could also use 504 here because you can't have 504.999... hens

(ii) Another farm has 800 hens, correct to the nearest 20.

Calculate the lower bound for the difference between the number of hens on the two farms.

hens: $800 < \begin{matrix} 810^* \\ 790 \end{matrix}$

$$\begin{aligned} \text{difference}_{\text{LB}} &= 800_{\text{LB}} - 500_{\text{UB}} \\ &= 790 - 505 \\ &= \underline{285} \end{aligned}$$

* or 809

$\underline{285}$ (or 286) [2]

(d) At a football match, there are 29 800 people, correct to the nearest 100.

(i) At the end of the football match, the people leave at a rate of 400 people per minute, correct to the nearest 50 people.

Calculate the lower bound for the number of minutes it takes for all the people to leave.

People: 29 800 $\begin{cases} 29\,850 \text{ (or } 29\,849) \\ 29\,750 \end{cases}$ time_{LB} = $\frac{\text{Number of people}_{LB}}{\text{Rate}_{UB}}$

Rate: 400 $\begin{cases} 425 \text{ (or } 424) \\ 375 \end{cases}$ = $\frac{29\,750}{425}$ $\leftarrow \text{or } 424$

= $\frac{29\,750}{425}$

= 70

Rate = $\frac{\text{Number of people}}{\text{time}}$ 70 (or 70.2) min [3]

(ii) At a cricket match there are 27 500 people, correct to the nearest 100.

Calculate the upper bound for the difference between the number of people at the football match and at the cricket match.

Cricket: 27 500 $\begin{cases} 27\,550 \text{ (or } 27\,549) \\ 27\,450 \end{cases}$ difference_{UB} = football_{UB} - cricket_{LB}

= 29 850 - 27 450

(29 849)

= 2400

2400 (or 2399) [2]