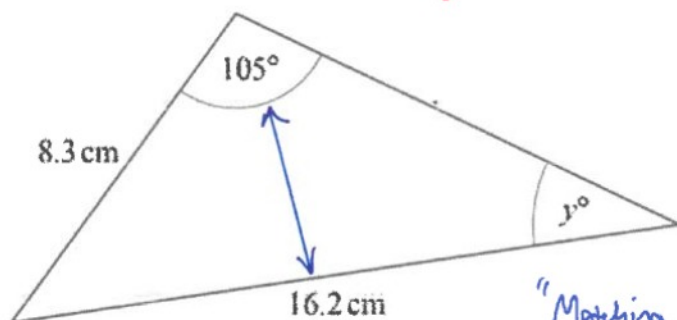


A Sine and Cosine Rules

18

Sine Rule for Angles: $\frac{\sin A}{a} = \frac{\sin B}{b}$

Sine Rule for Sides: $\frac{a}{\sin A} = \frac{b}{\sin B}$



NOT TO SCALE

Calculate the value of y .

"Matching pair" of angle and side opposite means we can use the Sine rule.

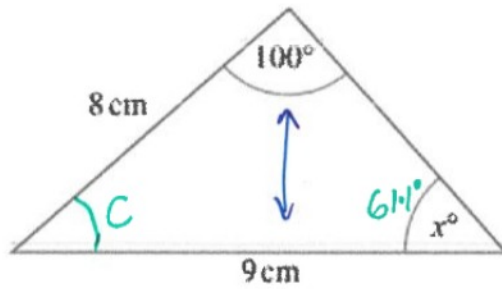
$$\frac{\sin y}{8.3} = \frac{\sin 105}{16.2} \times 8.3$$

$$\sin y = \frac{8.3 \sin 105}{16.2}$$

$$y = \sin^{-1} \left(\frac{8.3 \sin 105}{16.2} \right)$$

$$= 29.7^\circ$$

$$y = 29.7^\circ \quad [3]$$



NOT TO
SCALE

(a) Calculate the value of x .

$$\frac{\sin x}{8} = \frac{\sin 100}{9}$$

$$\sin x = \frac{8 \sin 100}{9}$$

$$x = \sin^{-1}\left(\frac{8 \sin 100}{9}\right)$$

$$x = 61.1^\circ$$

$$x = 61.1^\circ \quad [3]$$

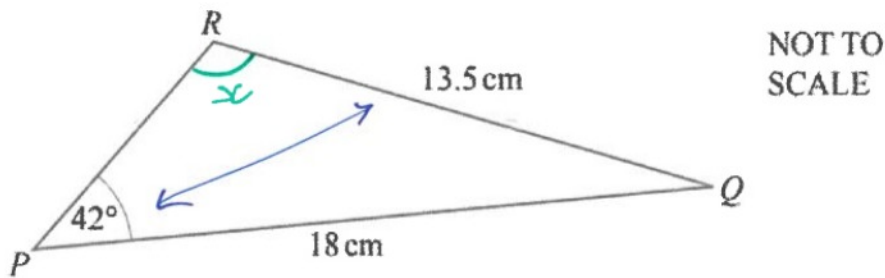
(b) Calculate the area of the triangle.

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 8 \times 9 \times \sin 18.9^\circ$$

$$= 11.7 \text{ cm}^2$$

$$11.7 \text{ cm}^2 \quad [3]$$



Calculate the obtuse angle PRQ .

$$\frac{\sin x}{18} = \frac{\sin 42}{13.5}$$

$$\sin x = \frac{18 \sin 42}{13.5}$$

$$x = \sin^{-1}\left(\frac{18 \sin 42}{13.5}\right)$$

$$= 63.1^\circ$$

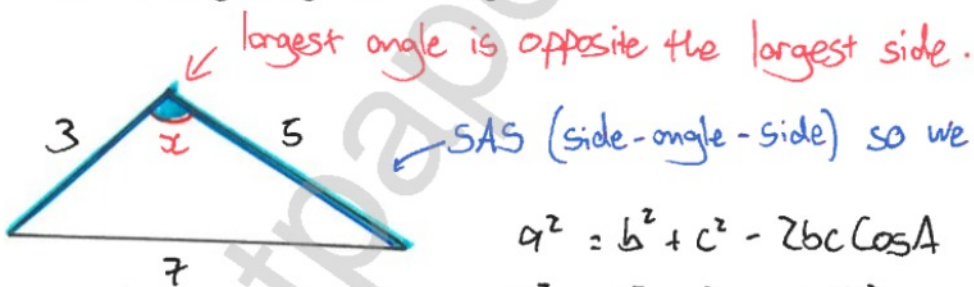
→ Obtuse so:

$$180 - 63.1 = \underline{116.9^\circ}$$

Angle $PRQ = \underline{116.9^\circ}$ [4]

- 15 A triangle has sides of length 3 cm, 5 cm and 7 cm.

Work out the largest angle in the triangle.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$7^2 = 3^2 + 5^2 - 2(3)(5) \cos x$$

$$49 = 9 + 25 - 30 \cos x$$

$$49 = 34 - 30 \cos x$$

$$\begin{array}{r} -34 \\ 15 \end{array} = \begin{array}{r} -34 \\ -30 \cos x \end{array}$$

$$\begin{array}{r} \div -30 \\ -\frac{1}{2} \end{array} = \cos x$$

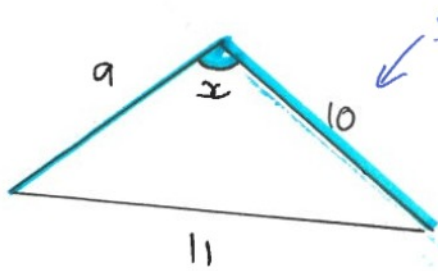
$$x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$= 120^\circ$$

120^\circ [3]

- 23 A triangle has sides of length 11 cm, 10 cm and 9 cm.

Calculate the largest angle in the triangle.



SAS → Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$11^2 = 9^2 + 10^2 - 2 \times 9 \times 10 \cos x$$

$$121 = 181 - 180 \cos x$$

$$\begin{array}{r} -181 \\ -181 \end{array} \quad \begin{array}{r} -181 \\ -181 \end{array}$$

$$-60 = -180 \cos x$$

$$\div -180 \quad \div -180$$

$$\cos x = \frac{1}{3}$$

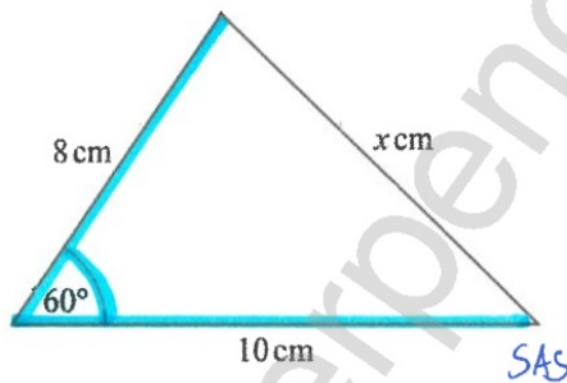
$$x = \cos^{-1}\left(\frac{1}{3}\right)$$

$$= 70.5$$

$$\dots\dots\dots 70.5^\circ \quad [4]$$

11

Non-Calculator



NOT TO
SCALE

Find the value of x^2 .

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 60$$

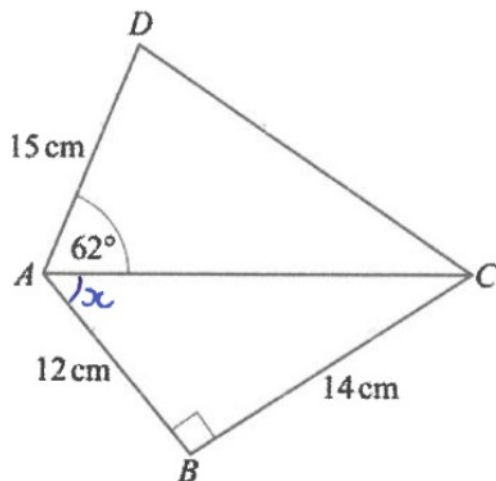
$$x^2 = 64 + 100 - 160 \cos 60$$

$$= 164 - 160 \times \frac{1}{2}$$

$$= 164 - 80$$

$$= 84$$

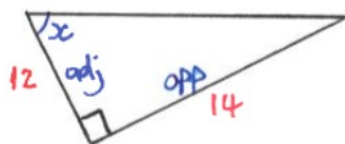
$$x^2 = \dots\dots\dots 84 \quad [3]$$



NOT TO
SCALE

The diagram shows a quadrilateral, $ABCD$, formed from two triangles, ABC and ACD . ABC is a right-angled triangle.

(a) Calculate angle BAC .

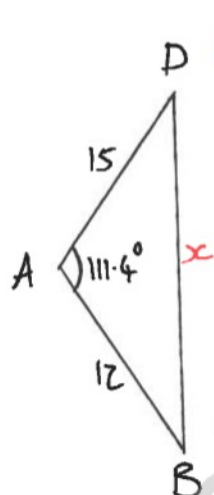


S^oHC[^]HT^oA

$$\tan x = \frac{14}{12}$$

$$x = \tan^{-1}\left(\frac{14}{12}\right) \text{ Angle } BAC = \underline{49.4^\circ} \quad [2]$$

(b) Calculate BD .



SAS

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \times \cos(111.4)$$

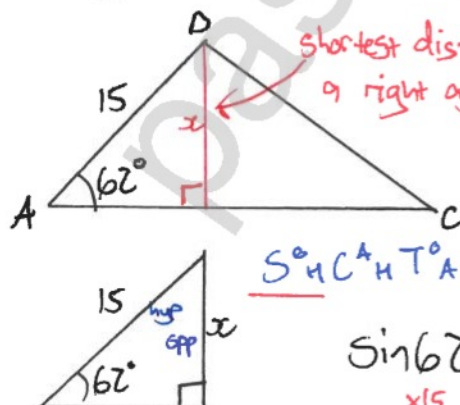
$$x^2 = 369 - 360 \cos(111.4)$$

$$x = \sqrt{369 - 360 \cos(111.4)}$$

$$= \underline{22.4 \text{ cm}}$$

$$BD = \underline{22.4} \text{ cm} \quad [4]$$

(c) Calculate the shortest distance from D to AC .



shortest distance always makes
a right angle with AC

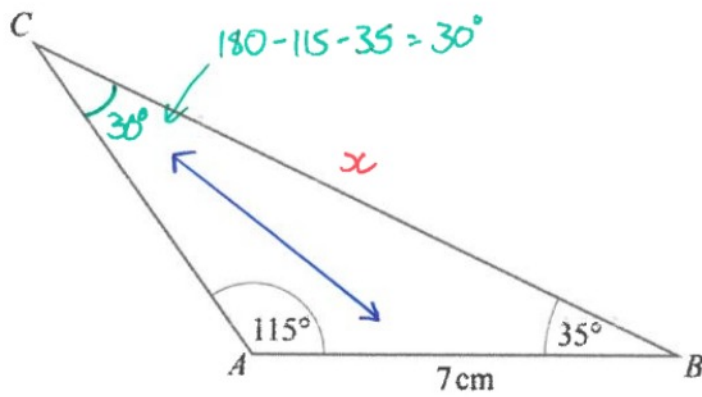
S^oHC[^]HT^oA

$$\sin 62^\circ = \frac{x}{15}$$

$$x = 15 \sin 62^\circ$$

$$= \underline{13.2 \text{ cm}}$$

$$\underline{13.2} \text{ cm} \quad [3]$$



NOT TO
SCALE

Calculate the length BC.

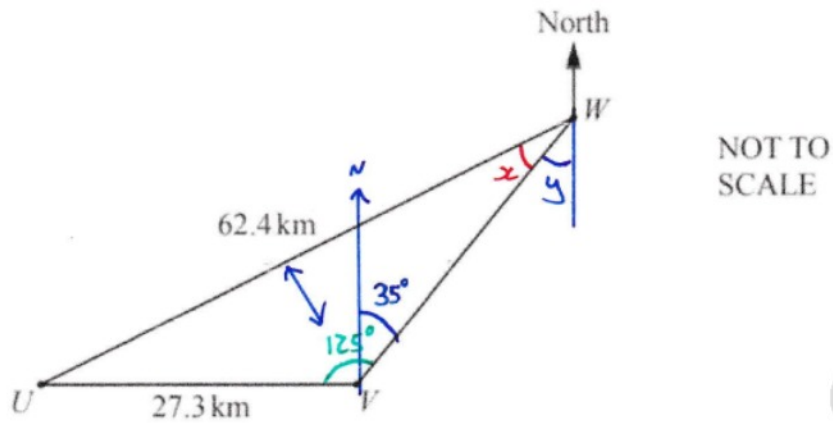
$$\frac{x}{\sin 115} = \frac{7}{\sin 30}$$

$\times \sin 115$ $\times \sin 115$

$$x = \frac{7 \sin 115}{\sin 30}$$

$$x = 12.7\text{ cm}$$

$$BC = 12.7 \text{ cm [4]}$$



The diagram shows the position of three towns, U , V and W .
 U is due west of V and angle $UVW = 125^\circ$.

Calculate the bearing of U from W .

$$\frac{\sin x}{27.3} = \frac{\sin 125}{62.4}$$

$\times 27.3$ $\times 27.3$

$$\sin x = \frac{27.3 \sin 125}{62.4}$$

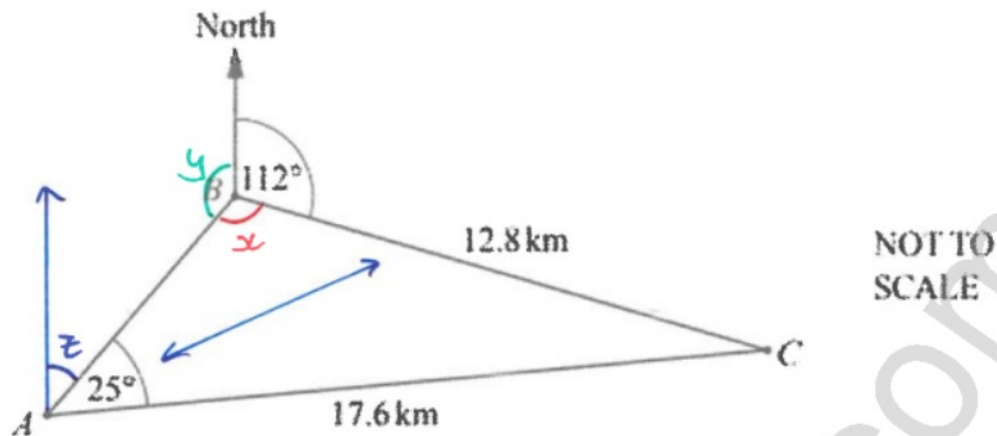
$$x = \sin^{-1}\left(\frac{27.3 \sin 125}{62.4}\right)$$

$$\underline{x = 21.0^\circ}$$

$$\text{Angle } y = 35^\circ \text{ (alternate angles)}$$

$$\text{Bearing of } U \text{ from } W = 180 + 35 + 21 = \underline{236^\circ}$$

$$\underline{236^\circ} \quad [4]$$



The diagram shows the positions of three ships A , B and C .
 $AC = 17.6$ km, $BC = 12.8$ km and angle $BAC = 25^\circ$.
 The bearing of C from B is 112° and angle ABC is obtuse.

Calculate the bearing of B from A .

x :

$$\frac{\sin x}{17.6} = \frac{\sin 25}{12.8}$$

$$\sin x = \frac{17.6 \sin 25}{12.8}$$

$$x = \sin^{-1}\left(\frac{17.6 \sin 25}{12.8}\right)$$

$$= 35.5^\circ$$

but x is obtuse, so:

$$x = 180 - 35.5$$

$$= \underline{144.5^\circ}$$

z :

Co-interior angle sum to 180° , so

$$z = 180 - 103.5$$

$$= \underline{76.5^\circ}$$

y :

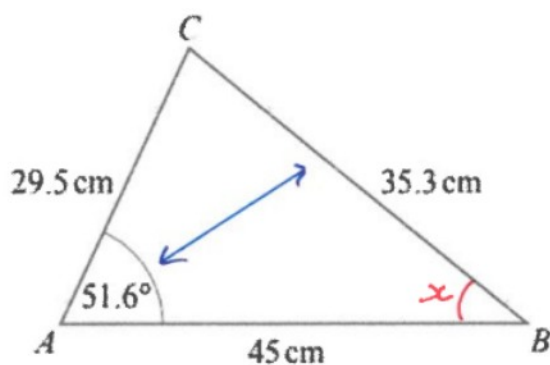
$$y = 360 - (112 + 144.5)$$

$$= \underline{103.5^\circ}$$

$$\underline{076.5^\circ}$$

[5]

4 (a)



NOT TO
SCALE

In triangle ABC , $AB = 45$ cm, $AC = 29.5$ cm, $BC = 35.3$ cm and angle $CAB = 51.6^\circ$.

(i) Calculate angle ABC .

$$\frac{\sin x}{29.5} = \frac{\sin 51.6}{35.3}$$

$\times 29.5$ $\times 29.5$

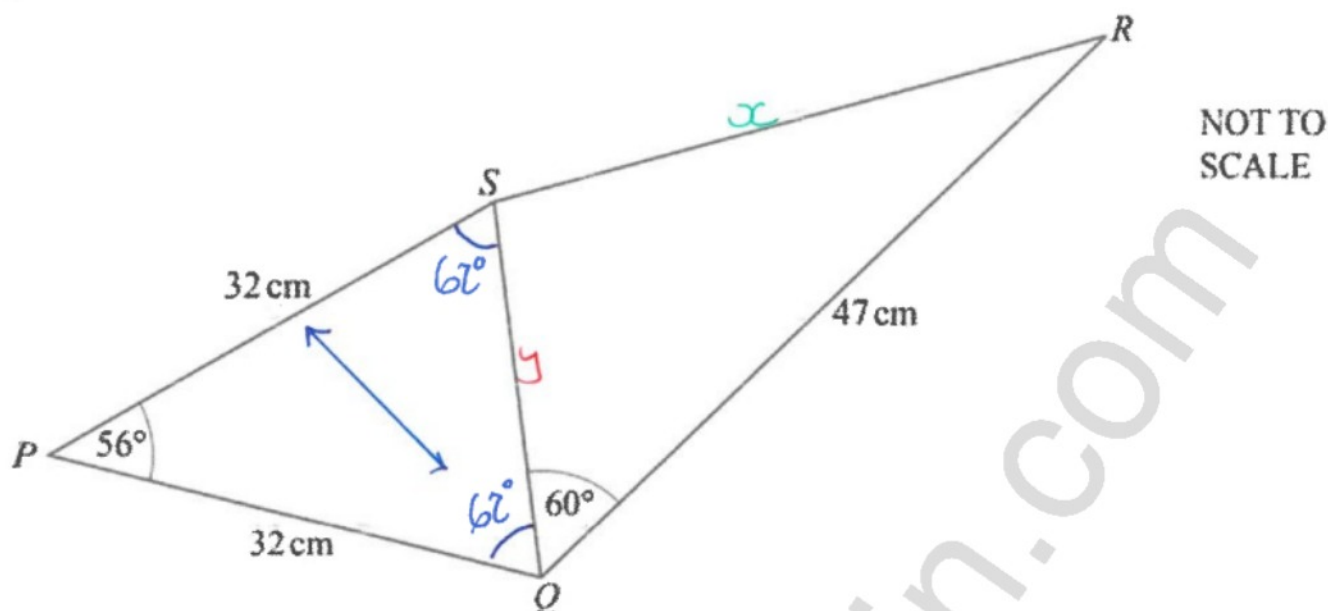
$$\sin x = \frac{29.5 \sin 51.6}{35.3}$$

$$x = \sin^{-1} \left(\frac{29.5 \sin 51.6}{35.3} \right)$$

$$= 40.9^\circ$$

Angle $ABC = 40.9^\circ$ [3]

(b)



The diagram shows a quadrilateral $PQRS$ formed from two triangles, PQS and QRS . Triangle PQS is isosceles, with $PQ = PS = 32$ cm and angle $SPQ = 56^\circ$. $QR = 47$ cm and angle $SQR = 60^\circ$.

(i) Calculate SR .

PQS is isosceles, so

$$\angle PSQ = \angle PQS:$$

$$180 - 56 = 124$$

$$124 \div 2 = 62^\circ$$

$$\frac{y}{\sin 56} = \frac{32}{\sin 62}$$

$\times \sin 56$ $\times \sin 62$

$$y = \frac{32 \sin 56}{\sin 62}$$
$$= 30.046...$$

STO

x : Cosine rule:

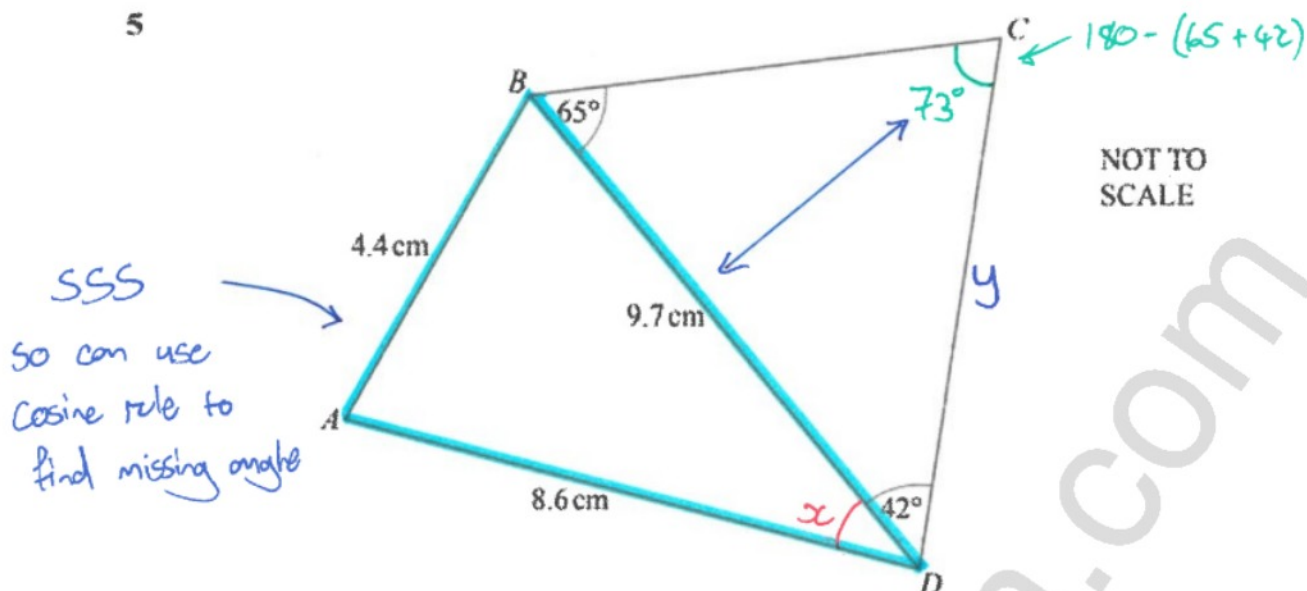
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 47^2 + (30.046)^2 - 2 \times 47 \times 30.046 \times \cos(60)$$

$$x = \sqrt{47^2 + (30.046)^2 - 2 \times 47 \times 30.046 \times \cos(60)}$$

$$= 41.2 \text{ cm}$$

$$SR = 41.2 \text{ cm [4]}$$



(a) Calculate angle ADB.

Cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$4.4^2 = 8.6^2 + 9.7^2 - 2 \times 8.6 \times 9.7 \cos x$$

$$19.36 = 168.05 - 166.84 \cos x$$

$$-168.05 \quad -168.05$$

$$\begin{aligned} -148.69 &= -166.84 \cos x \\ \div -166.84 &\quad \div -166.84 \end{aligned}$$

$$\cos x = \frac{148.69}{166.84}$$

$$x = 26.97^\circ$$

$$\text{Angle ADB} = 27.0^\circ \quad [3]$$

(b) Calculate DC.

$$\frac{y}{\sin 65} = \frac{9.7}{\sin 73}$$

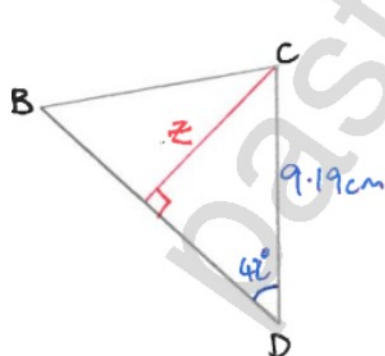
$\times \sin 65 \quad \times \sin 65$

$$y = \frac{9.7 \sin 65}{\sin 73}$$

$$y = \frac{9.19 \text{ cm}}{\text{STO}}$$

$$DC = 9.19 \text{ cm} \quad [4]$$

(c) Calculate the shortest distance from C to BD.

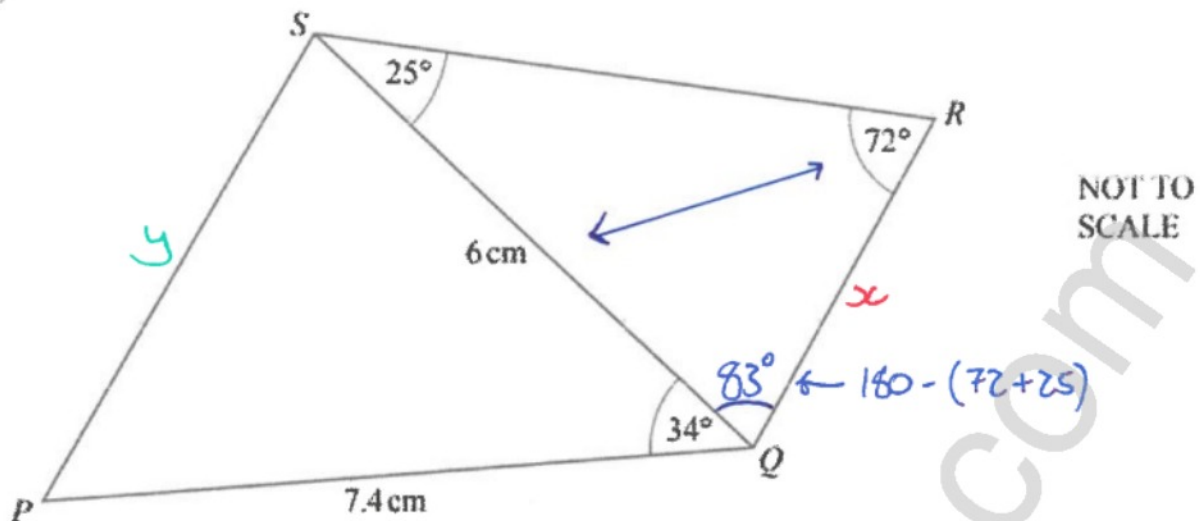


$$\sin 42 = \frac{z}{9.19 \dots}$$

$\times 9.19 \dots \quad \times 9.19 \dots$

$$\begin{aligned} z &= (9.19 \dots) \times \sin 42 \\ &= 6.15 \text{ cm} \end{aligned}$$

$$6.15 \text{ cm} \quad [3]$$



The diagram shows a quadrilateral $PQRS$ formed from two triangles, PQS and QRS .

Calculate

(i) QR ,

$$\frac{x}{\sin 25} = \frac{6}{\sin 72}$$

$x \sin 25$ $x \sin 25$

$$x = \frac{6 \sin 25}{\sin 72}$$

$$x = \frac{2.67 \text{ cm}}{\text{STO}}$$

$$QR = 2.67 \text{ cm [3]}$$

(ii) PS ,
cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$y^2 = 6^2 + 7.4^2 - 2 \times 6 \times 7.4 \times \cos 34$$

$$y = \sqrt{90.76 - 88.8 \cos 34}$$

$$= 4.14 \text{ cm}$$

$$PS = 4.14 \text{ cm [3]}$$

(iii) the area of quadrilateral $PQRS$.

ΔPQS : Area = $\frac{1}{2} ab \sin C$

$$= \frac{1}{2} \times 6 \times 7.4 \times \sin 34$$

$$= 12.414 \text{ cm}^2$$

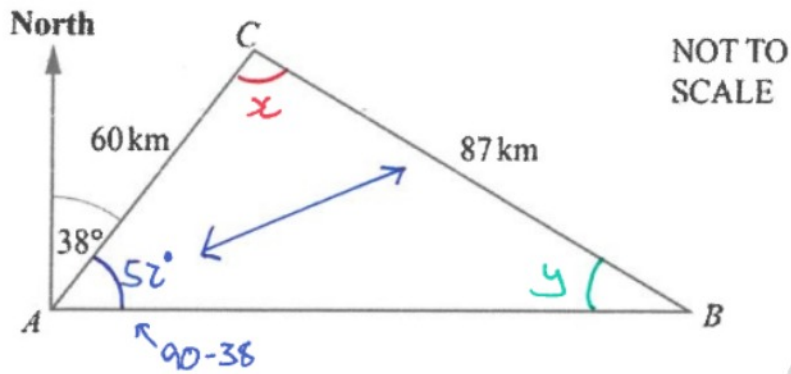
ΔQRS : Area = $\frac{1}{2} ab \sin C$

$$= \frac{1}{2} \times 6 \times (2.66...) \times \sin 83$$

$$= 7.939 \text{ cm}^2$$

Total: $12.414 + 7.939 = 20.4 \text{ cm}^2$ [4]

(3sf)



The diagram shows the straight roads between town A , town B and town C .
 $AC = 60$ km, $CB = 87$ km and B is due east of A .
 The bearing of C from A is 038° .

(a) Show that angle $ACB = 95.1^\circ$, correct to 1 decimal place.

y:

$$\frac{\sin y}{60} = \frac{\sin 52}{87}$$

$$\sin y = \frac{60 \sin 52}{87}$$

$$y = \sin^{-1} \left(\frac{60 \sin 52}{87} \right)$$

$$= 32.919^\circ$$

x:

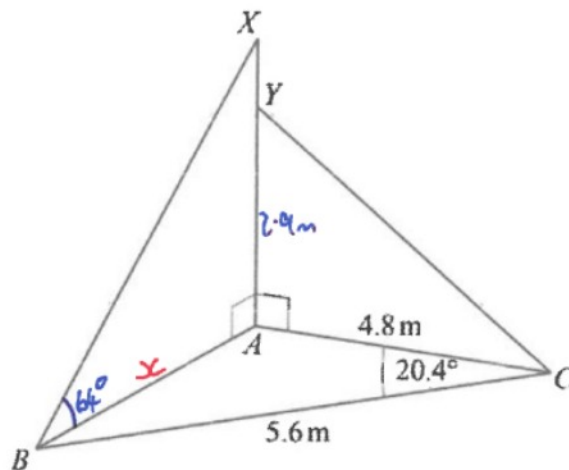
$$x = 180 - (52 + 32.919\dots)$$

$$= 95.08\dots$$

$$= 95.1^\circ \text{ to 1dp}$$

[5]

5 (a)



ABC is a scalene triangle on horizontal ground.

AYX is a straight vertical post, held in place by two straight wires XB and YC .

$AC = 4.8$ m, $BC = 5.6$ m and angle $ACB = 20.4^\circ$.

(i) Calculate AB .

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$$x^2 = 4.8^2 + 5.6^2 - 2 \times 4.8 \times 5.6 \cos 20.4$$

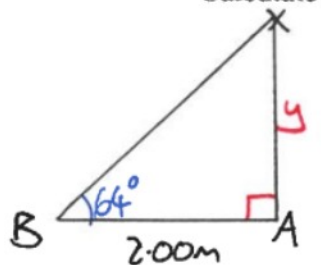
$$x = \sqrt{54.4 - 53.76 \cos 20.4}$$

$$= 2.00 \text{ m} \quad \text{STO}$$

$$AB = 2.00 \text{ m [3]}$$

(ii) Angle $XBA = 64^\circ$.

Calculate AX .



(unrounded)

$\sin^{-1} \frac{y}{2.00} = 64^\circ$

$$\tan 64 = \frac{y}{2.00}$$

$$y = 2.00 \tan 64$$

$$= 4.1066..$$

$$= 4.11 \text{ m}$$

$$AX = 4.11 \text{ m [2]}$$

(iii) $AY = 2.9$ m.

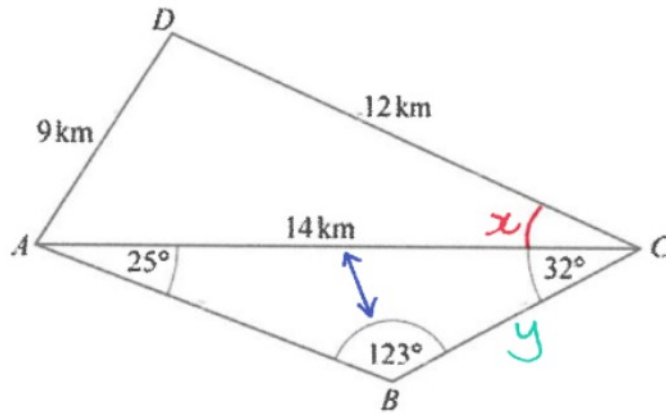
Calculate the area of triangle YAC .

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 4.8 \times 2.9$$

$$= 6.96 \text{ m}^2$$

$$6.96 \text{ m}^2 [2]$$



NOT TO SCALE

(a) Calculate angle ACD. SSS $\rightarrow$ cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$9^2 = 14^2 + 12^2 - 2 \times 14 \times 12 \times \cos x$$

$$81 = 340 - 336 \cos x$$

$$\begin{matrix} -340 & -340 \end{matrix}$$

$$\begin{matrix} -259 & \div -336 \end{matrix} = \begin{matrix} -336 \cos x & \div -336 \end{matrix}$$

$$\cos x = \frac{259}{336}$$

$$x = \cos^{-1}\left(\frac{259}{336}\right)$$

$$= 39.6^\circ$$

$$\text{Angle ACD} = 39.6^\circ \quad [4]$$

(b) Show that $BC = 7.05$ km, correct to 2 decimal places.

$$\frac{y}{\sin 25} = \frac{14}{\sin 123}$$

$$\begin{matrix} \times \sin 25 & \times \sin 25 \end{matrix}$$

$$y = \frac{14 \sin 25}{\sin 123}$$

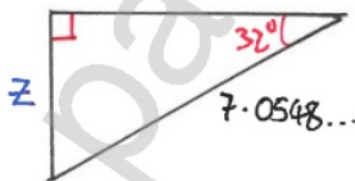
$$= 7.0548...$$

$$= \underline{7.05 \text{ (to 2dp)}}$$

$$\text{STO}$$

[3]

(c) Calculate the shortest distance from B to AC.

S^o H C[^] H T^o A

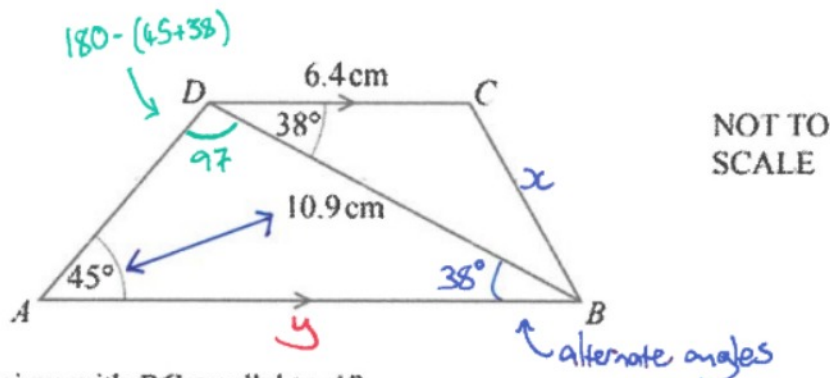
$$\sin 32 = \frac{z}{7.0548...}$$

$$\begin{matrix} \times 7.0548... & \times 7.0548... \end{matrix}$$

$$z = 7.0548... \sin 32$$

$$= 3.74 \text{ km}$$

$$= 3.74 \text{ km} \quad [3]$$



$ABCD$ is a trapezium with DC parallel to AB .
 $DC = 6.4$ cm, $DB = 10.9$ cm, angle $CDB = 38^\circ$ and angle $DAB = 45^\circ$.

(a) Find CB .

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 6.4^2 + 10.9^2 - 2 \times 6.4 \times 10.9 \times \cos 38^\circ$$

$$x = \sqrt{159.77 - 139.52 \cos 38^\circ}$$

$$= 7.06 \text{ cm}$$

$$CB = 7.06 \text{ cm [3]}$$

(b) (i) Find angle ADB .

alternate angles are equal, so

$$\angle ABD = 38^\circ$$

$$180 - (45 + 38) = 97^\circ$$

$$\text{Angle } ADB = 97^\circ \text{ [1]}$$

(ii) Find AB .

$$\frac{y}{\sin 97^\circ} = \frac{10.9}{\sin 45^\circ}$$

$$y = \frac{10.9 \sin 97^\circ}{\sin 45^\circ}$$

$$y = 15.3 \text{ cm}$$

$$AB = 15.3 \text{ cm [3]}$$

(c) Calculate the area of the trapezium.

$$\underline{\triangle ABD}: \text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 10.9 \times 15.3 \sin 38^\circ$$

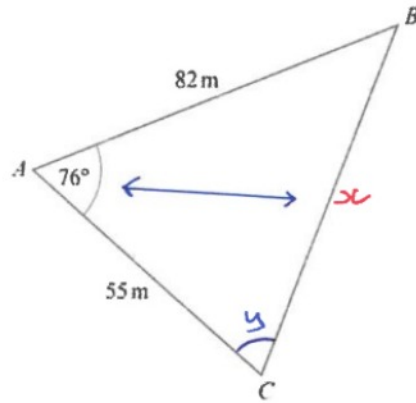
$$= 51.337$$

$$\underline{\triangle BDC}: \text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 6.4 \times 10.9 \sin 38^\circ$$

$$= 21.474$$

$$\text{Total: } 51.337 + 21.474 = 72.8 \text{ cm}^2 \text{ [3]}$$



NOT TO SCALE

x:

The diagram shows a field ABC.

Cosine rule:

(a) Calculate BC.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 55^2 + 82^2 - 2 \times 55 \times 82 \times \cos 76$$

$$x = \sqrt{9749 - 9020 \cos 76}$$

$$x = 87.0 \text{ m} \quad \text{STO}$$

$$BC = 87.0 \text{ m} \quad [3]$$

(b) Calculate angle ACB.

y:

Sine rule:

$$\frac{\sin y}{82} = \frac{\sin 76}{86.987..} \quad \leftarrow \text{from (a)}$$

$$\sin y = \frac{82 \sin 76}{86.987..}$$

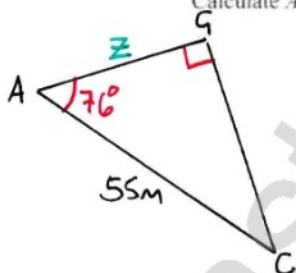
$$y = \sin^{-1} \left(\frac{82 \sin 76}{86.987..} \right) = 66.2^\circ$$

$$\text{Angle ACB} = 66.2^\circ \quad [3]$$

(c) A gate, G, lies on AB at the shortest distance from C.

Calculate AG.

SOHCAHTOA

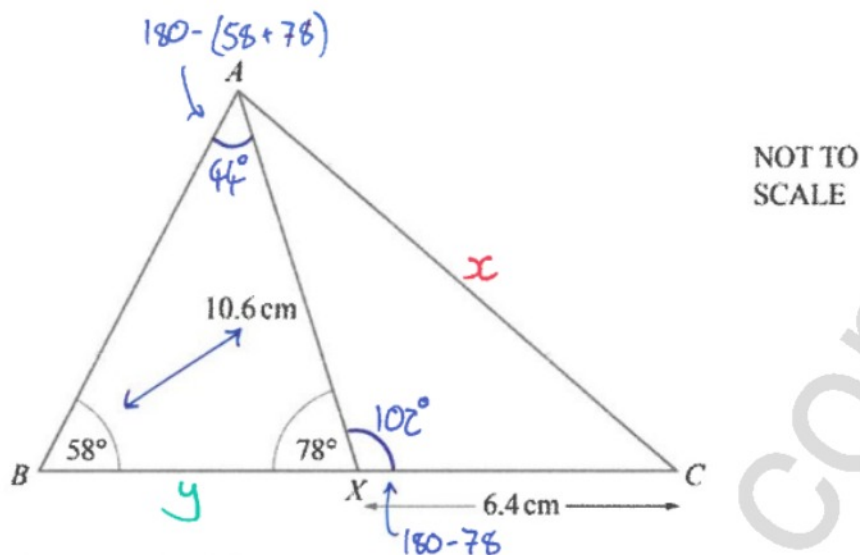


$$\cos 76 = \frac{z}{55}$$

$$z = 55 \cos 76$$

$$= 13.3 \text{ m}$$

$$AG = 13.3 \text{ m} \quad [3]$$



NOT TO
SCALE

The diagram shows triangle ABC.

X is a point on BC.

AX = 10.6 cm, XC = 6.4 cm, angle ABC = 58° and angle AXB = 78°.

x:

(a) Calculate AC.

Cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 6.4^2 + 10.6^2 - 2 \times 6.4 \times 10.6 \cos 102^\circ$$

$$x = \sqrt{153.32 - 135.68 \cos 102^\circ}$$

$$= 13.5 \text{ cm}$$

$$AC = 13.5 \text{ cm [4]}$$

(b) Calculate BX.

y:

$$\frac{y}{\sin 44^\circ} = \frac{10.6}{\sin 58^\circ}$$

$\times \sin 44^\circ$ $\times \sin 44^\circ$

$$y = \frac{10.6 \sin 44^\circ}{\sin 58^\circ}$$

$$= 8.68 \text{ cm}$$

STO

$$BX = 8.68 \text{ cm [4]}$$

(c) Calculate the area of triangle ABC.

ΔABX : Area = $\frac{1}{2} ab \sin C$

$$= \frac{1}{2} \times 10.6 \times 8.68 \times \sin 78^\circ$$

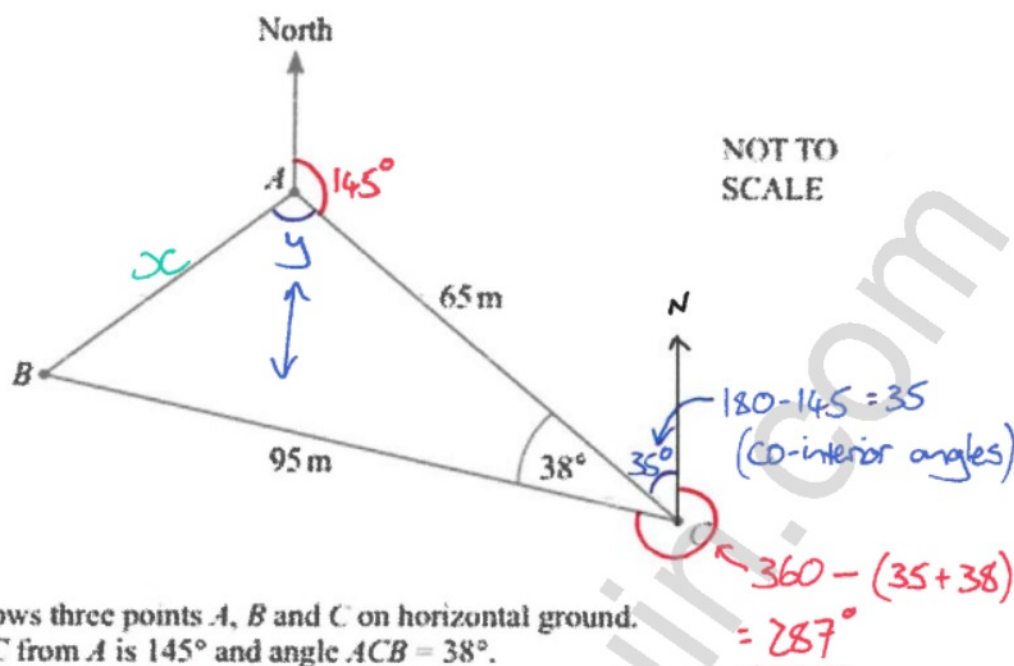
$$= 45.013$$

ΔACX : Area = $\frac{1}{2} ab \sin C$

$$= \frac{1}{2} \times 10.6 \times 6.4 \times \sin 102^\circ$$

$$= 33.179$$

Total: $45.013 + 33.179 = 78.2 \text{ cm}^2$ [3]



The diagram shows three points A , B and C on horizontal ground. The bearing of C from A is 145° and angle $ACB = 38^\circ$. $AC = 65$ m and $BC = 95$ m.

(a) Find the bearing of B from C .

See diagram for working.

287° [2]

(b) Show that $AB = 59.3$ m, correct to 1 decimal place.

x:
Cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 65^2 + 95^2 - 2 \times 65 \times 95 \times \cos 38$$

$$x = \sqrt{13250 - 12350 \cos 38}$$

$$x = 59.313$$

$$= \underline{59.3 \text{ to 1dp}}$$

[3]

(c) Angle BAC is obtuse.

Work out the bearing of B from A .

y:

$$\frac{\sin y}{95} = \frac{\sin 38}{59.3...}$$

$\times 95$ $\times 95$

$$\sin y = \frac{95 \sin 38}{59.3...}$$

$$y = \sin^{-1} \left(\frac{95 \sin 38}{59.3...} \right)$$

$$= 80.4^\circ$$

$\rightarrow$ BAC is obtuse, so:

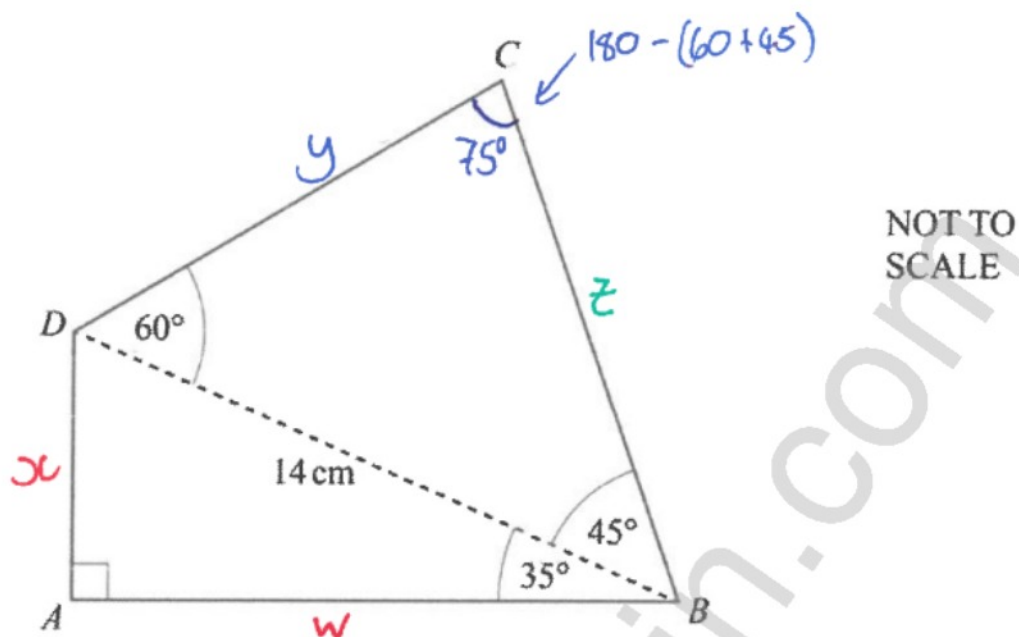
$$180 - 80.4 = \underline{99.6^\circ}$$

Bearing of B from A :

$$145 + 99.6 = \underline{244.6^\circ}$$

244.6° [4]

9 (a)



Calculate the perimeter of the quadrilateral ABCD.

x: $\text{S}^{\circ}\text{H}^{\text{C}}\text{A}^{\text{T}}\text{O}^{\text{A}}$

$$\sin 35 = \frac{x}{14}$$

$\times 14$ $\times 14$

$$x = 14 \sin 35$$

$$= \underline{8.0301 \text{ cm}}$$

w: $\text{S}^{\circ}\text{H}^{\text{C}}\text{A}^{\text{T}}\text{O}^{\text{A}}$

$$\cos 35 = \frac{w}{14}$$

$\times 14$ $\times 14$

$$w = 14 \cos 35$$

$$= \underline{11.468 \text{ cm}}$$

y:

$$\frac{y}{\sin 45} = \frac{14}{\sin 75}$$

$\times \sin 45$ $\times \sin 45$

$$y = \frac{14 \sin 45}{\sin 75}$$

$$= \underline{10.249 \text{ cm}}$$

z:

$$\frac{z}{\sin 60} = \frac{14}{\sin 75}$$

$\times \sin 60$ $\times \sin 60$

$$z = \frac{14 \sin 60}{\sin 75}$$

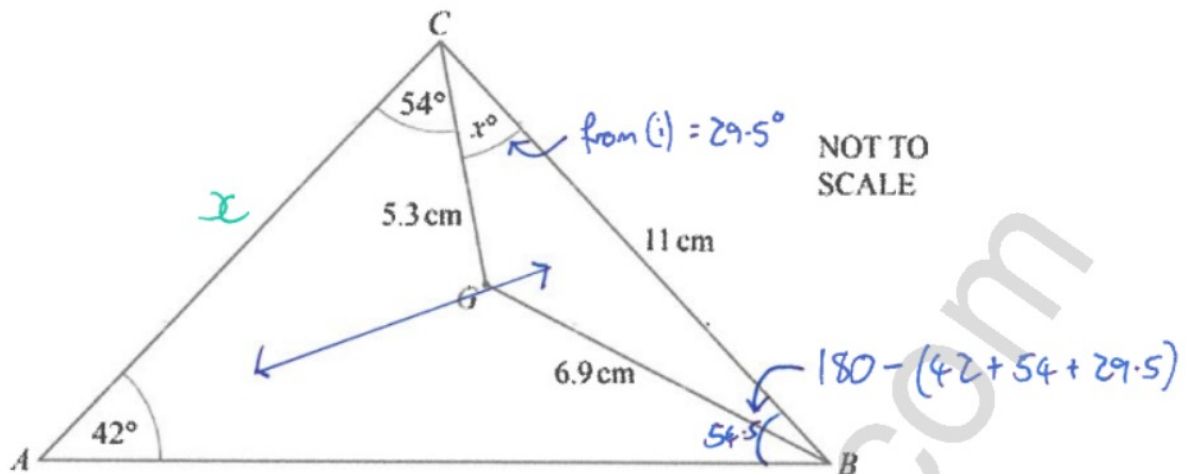
$$= \underline{12.552 \text{ cm}}$$

$$\text{Perimeter} = 8.0301 + 11.468 + 10.249 + 12.552$$

$$= \underline{42.3 \text{ cm}} \text{ (3sf)}$$

$$\underline{42.3} \text{ cm [7]}$$

6 (a)



The diagram shows triangle ABC with point G inside.
 $CB = 11$ cm, $CG = 5.3$ cm and $BG = 6.9$ cm.
 Angle $CAB = 42^\circ$ and angle $ACG = 54^\circ$.

(i) Calculate the value of x .

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$$6.9^2 = 5.3^2 + 11^2 - 2 \times 5.3 \times 11 \cos x$$

$$47.61 = 28.09 + 121 - 116.6 \cos x$$

$$47.61 = 149.09 - 116.6 \cos x$$

$$-149.09 \quad -149.09$$

$$-101.48 = -116.6 \cos x$$

$$\div -116.6 \quad \div -116.6$$

$$\cos x = \frac{101.48}{116.6}$$

$$x = \cos^{-1} \left(\frac{101.48}{116.6} \right)$$

$$= 29.5^\circ$$

$$x = 29.5^\circ \quad [4]$$

(ii) Calculate AC .

x :

$$\frac{x}{\sin 54.5} = \frac{11}{\sin 42}$$

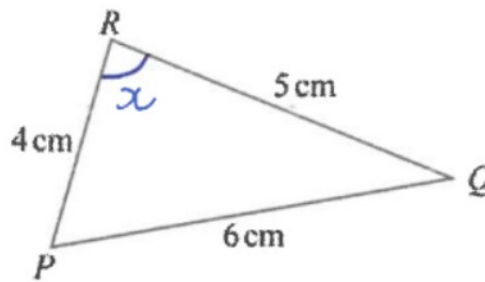
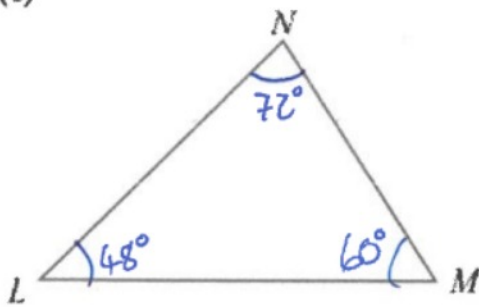
$\times \sin 54.5 \quad \times \sin 54.5$

$$x = \frac{11 \sin 54.5}{\sin 42}$$

$$= 13.4 \text{ cm}$$

$$AC = 13.4 \text{ cm} \quad [4]$$

(c)



NOT TO
SCALE

In triangle LMN, the ratio angle L : angle M : angle N = 4 : 5 : 6.
In triangle PQR, $PQ = 6 \text{ cm}$, $PR = 4 \text{ cm}$ and $QR = 5 \text{ cm}$.

Calculate the difference between the largest angle in triangle PQR and the largest angle in triangle LMN.

LMN: $4 + 5 + 6 = 15$

$$180 \div 15 = 12$$

$$4 : 5 : 6$$

$$\begin{array}{c} \times 12 \quad \times 12 \\ \hline 48 : 60 : 72 \end{array}$$

PQR: Largest angle is opposite longest side (x)

cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$6^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos x$$

$$36 = 41 - 40 \cos x$$

$$\begin{array}{cc} -41 & -41 \\ \hline \end{array}$$

$$-5 = -40 \cos x$$

$$\begin{array}{cc} \div -40 & \div -40 \\ \hline \end{array}$$

$$\cos x = \frac{1}{8}$$

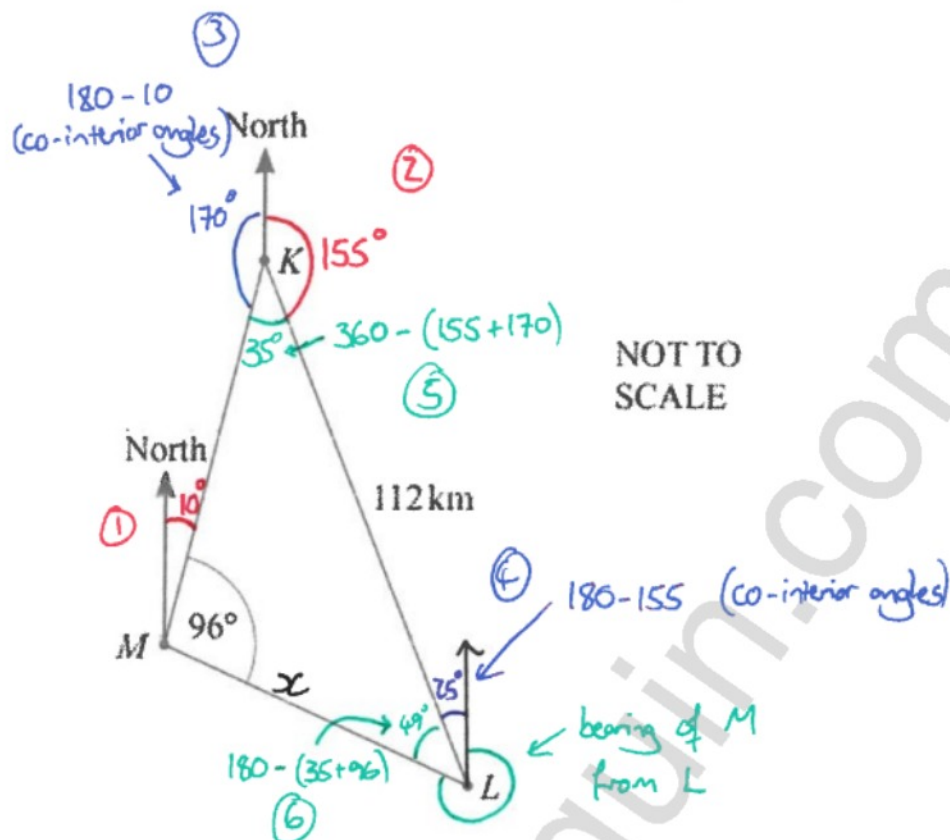
$$x = \cos^{-1}\left(\frac{1}{8}\right)$$

$$= 82.8^\circ$$

Difference:

$$82.8 - 72 = 10.8^\circ \quad \dots \quad 10.8^\circ \quad \dots \quad [7]$$

(c)



The diagram shows the positions of a lighthouse, L , and two ships, K and M . The bearing of L from K is 155° and $KL = 112$ km. The bearing of K from M is 010° and angle $KML = 96^\circ$.

Find the bearing and distance of ship M from the lighthouse, L .

Bearing:

See working on diagram above (order: ①, ②, ③, ④, ⑤, ⑥)

$$\begin{aligned}\text{Bearing} &= 360 - (49 + 25) \\ &= \underline{286^\circ}\end{aligned}$$

Distance:

$$\frac{x}{\sin 35} = \frac{112}{\sin 96}$$

$\times \sin 35$ $\times \sin 35$

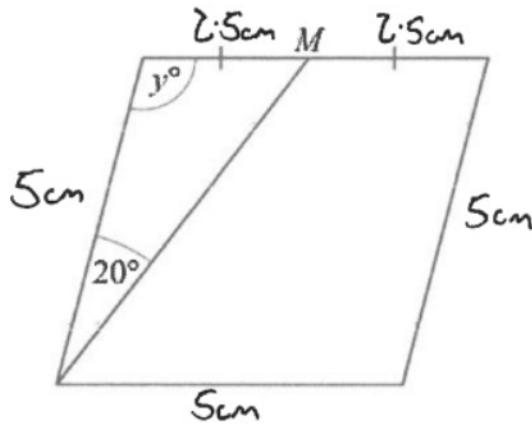
$$x = \frac{112 \sin 35}{\sin 96}$$

$$= \underline{64.6 \text{ km}}$$

Bearing $\underline{286^\circ}$

Distance $\underline{64.6}$ km [5]

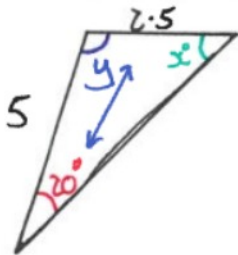
(b)



NOT TO
SCALE

This rhombus has perimeter 20 cm and angle y is obtuse.
 M is the midpoint of one of the sides.

Find the value of y .



$$\frac{\sin x}{5} = \frac{\sin 20}{2.5}$$

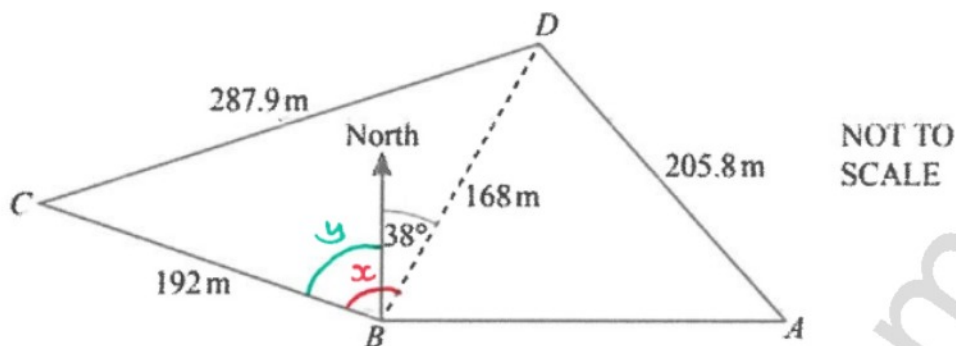
$$\sin x = \frac{5 \sin 20}{2.5}$$

$$x = \sin^{-1}\left(\frac{5 \sin 20}{2.5}\right)$$
$$= 43.2^\circ$$

each side: $\frac{20}{4} = 5 \text{ cm}$

$$y = 180 - (20 + 43.2)$$
$$= 116.8^\circ$$

$$y = 116.8^\circ \quad [5]$$



The diagram shows a field, $ABCD$, on horizontal ground.
 $BC = 192$ m, $CD = 287.9$ m, $BD = 168$ m and $AD = 205.8$ m.

- (a) (i) Calculate angle CBD and show that it rounds to 106.0° , correct to 1 decimal place.

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$287.9^2 = 168^2 + 192^2 - 2 \times 168 \times 192 \times \cos x$$

$$82886.41 = 65088 - 64512 \cos x$$

$$-65088 \quad -65088$$

$$17798.41 = -64512 \cos x$$

$$\div -64512 \quad \div -64512$$

$$\cos x = \frac{-17798.41}{64512}$$

$$x = \cos^{-1} \left(\frac{-17798.41}{64512} \right)$$

$$x = 106.015...$$

$$\approx \underline{106.0 \text{ to 1 dp}}$$

[4]

- (ii) The bearing of D from B is 038° .

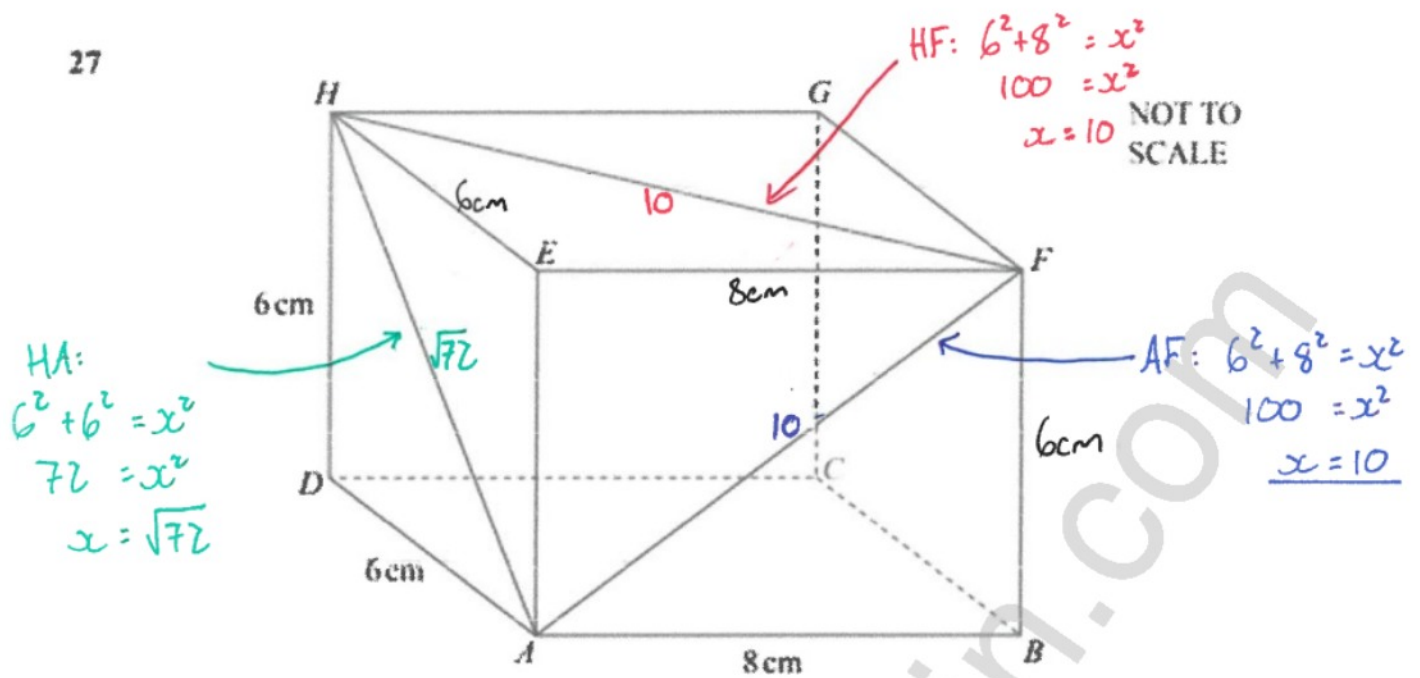
Find the bearing of C from B .

y: $y = 106.0 - 38$
 $= 68.0^\circ$

$$292.0^\circ$$

[1]

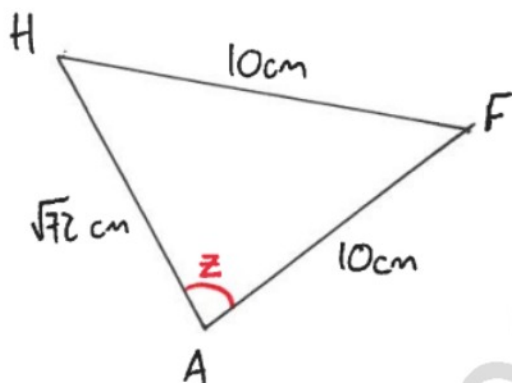
Bearing: $360 - 68.0 = 292.0^\circ$



The diagram shows a cuboid.

$AB = 8\text{ cm}$, $AD = 6\text{ cm}$ and $DH = 6\text{ cm}$.

Calculate angle HAF .



Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$10^2 = 10^2 + (\sqrt{72})^2 - 2 \times 10 \times \sqrt{72} \cos x$$

$$100 = 172 - 120\sqrt{2} \cos x$$

$$-72 = -120\sqrt{2} \cos x$$

$$\div 120\sqrt{2} \quad \div -120\sqrt{2}$$

$$\frac{72}{120\sqrt{2}} = \cos x$$

$$x = \cos^{-1} \left(\frac{72}{120\sqrt{2}} \right)$$

$$= 64.9^\circ$$

Angle $HAF = 64.9^\circ$ [6]