

A Sequences: Other

- 8 The n th term of a sequence is $n^2 + 12$.

Find the first three terms of this sequence.

$$n^2: 1, 4, 9$$

$$n^2 + 12: 13, 16, 21$$

$$\dots 13, \dots 16, \dots 21 \dots [2]$$

- 5 The n th term of a sequence is $n^2 - 1$.

Find the first three terms of this sequence.

$$n^2: 1, 4, 9$$

$$n^2 - 1: 0, 3, 8$$

$$\dots 0, \dots 3, \dots 8 \dots [2]$$

- 6 (a) The n th term of a sequence is $n^2 + 3n$.

Find the first three terms of this sequence.

$$n^2: 1, 4, 9$$

$$3n: 3, 6, 9$$

$$n^2 + 3n: 4, 10, 18$$

$$\dots 4, \dots 10, \dots 18 \dots [2]$$

- 8 (a) The n th term of a sequence is $10 - n^2$.

Write down the first three terms of this sequence.

$$n^2: 1, 4, 9$$

$$10 - n^2: 9, 6, 1$$

$$\dots 9, \dots 6, \dots 1 \dots [2]$$

10 (a) The n th term of a sequence is $n^2 + 7$.

Find the first three terms of this sequence.

$$n^2: \quad 1 \quad , \quad 4 \quad , \quad 9$$

$$n^2 + 7: \quad 8 \quad , \quad 11 \quad , \quad 16$$

$$\dots 8 \quad , \quad 11 \quad , \quad 16 \dots [2]$$

(b) These are the first five terms of another sequence.

$$4 \quad 7 \quad 12 \quad 19 \quad 28$$

$$\begin{array}{ccccccc} & \nearrow +3 & & \nearrow +5 & & \nearrow +7 & & \nearrow +9 \\ & \searrow +2 & & \searrow +2 & & \searrow +2 & & \end{array}$$

Find the n th term.

Half second difference: $2 \div 2 = 1$

Use this as coefficient of n^2 : $1n^2$

$$n^2: \quad 1 \quad 4 \quad 9 \quad 16 \quad 25$$

$$4 \quad 7 \quad 12 \quad 19 \quad 28 \quad \searrow +3$$

$$\underline{n^2 + 3}$$

$$\dots [2]$$

8 Find the next term and the n th term for this sequence.

$$1, \quad 7, \quad 17, \quad 31, \quad 49, \quad 71$$

$$\begin{array}{ccccccc} & \nearrow +6 & & \nearrow +10 & & \nearrow +14 & & \nearrow +18 & & \nearrow +22 \\ & \searrow +4 & & \searrow +4 & & \searrow +4 & & \end{array}$$

Half second difference: $4 \div 2 = 2$

Coeff of n^2 : $2n^2$

$$2n^2: \quad 2 \quad 8 \quad 18 \quad 32 \quad 50$$

$$1 \quad 7 \quad 17 \quad 31 \quad 49 \quad \searrow -1$$

$$\underline{2n^2 - 1}$$

next term $\dots 71 \dots$

n th term $\dots 2n^2 - 1 \dots [3]$

14 Find the n th term of this sequence.



Half second difference: $6 \div 2 = 3$

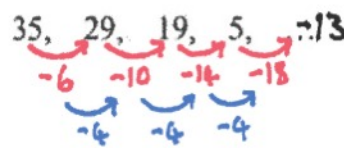
Coeff of n^2 : $3n^2$



$$\underline{3n^2 + 5}$$

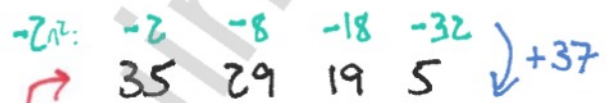
..... [2]

11 Find the next term and an expression for the n th term of this sequence.



Half second difference: $-4 \div 2 = -2$

Coeff of n^2 : $-2n^2$



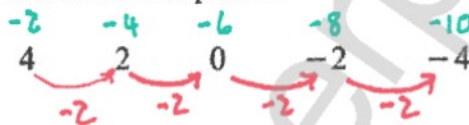
$$\underline{-2n^2 + 37}$$

next term = -13

n th term = $-2n^2 + 37$ [3]

(c) Find the n th term of each sequence.

(i)



$$\underline{-2n + 6}$$

..... [2]

(ii)



Half second difference: $4 \div 2 = 2$

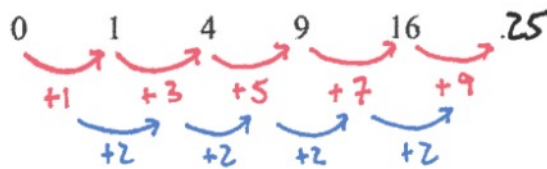
Coeff of n^2 : $2n^2$



$$\underline{2n^2 - 1}$$

..... [2]

- 6 Find the next term and the n th term of this sequence.

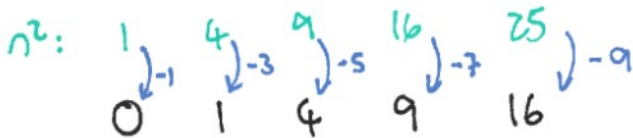


Half second difference: $2 \div 2 = 1$

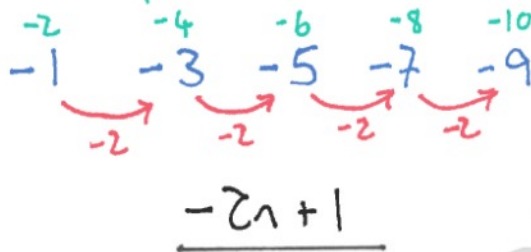
Coeff of n^2 : $1n^2$

next term = 25

n th term = $n^2 - 2n + 1$ [3]
or $(n-1)^2$



Find n th term of differences (blue numbers):



$$n^2 - 2n + 1$$

- 8 (a) 3, 9, 27, 81, ... 243

Write down the next term in this sequence.

243 [1]

- 8 (a) The n th term of a sequence is $n^2 - 3$.

Find the first three terms of this sequence.

$$n^2: \quad 1 \quad , \quad 4 \quad , \quad 9$$

$$n^2 - 3: \quad -2 \quad , \quad 1 \quad , \quad 6$$

$$\underline{\quad -2 \quad , \quad 1 \quad , \quad 6 \quad} \quad [2]$$

- (b) These are the first five terms of a different sequence.

$$1 \xrightarrow{\times 3} 3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \xrightarrow{\times 3} 81$$

Find the n th term of this sequence.

$$\text{First term: } 1 \quad (a)$$

$$\text{Ratio: } 3 \quad (r)$$

$$\text{use formula: } ar^{n-1} = 1 \times 3^{n-1}$$

$$\underline{\quad 3^{n-1} \quad} \quad [2]$$

- 11 Here are the first five terms of a sequence.

$$\frac{1}{4} \xrightarrow{\times 4} 1 \xrightarrow{\times 4} 4 \xrightarrow{\times 4} 16 \xrightarrow{\times 4} 64 \xrightarrow{\times 4} 256$$

- (a) Find the next term.

$$\underline{\quad 256 \quad} \quad [1]$$

- (b) Find the n th term.

$$\text{First term: } \frac{1}{4}$$

$$\text{Ratio: } 4$$

$$ar^{n-1}: \frac{1}{4} \times 4^{n-1}$$

$$\underline{\quad \frac{1}{4} \times 4^{n-1} \quad} \quad [2]$$

20 Find the n th term of each sequence.

(a) $-1, 0, 7, 26, 63, \dots$

$+1$
 $+6$
 $+7$
 $+12$
 $+19$
 $+26$
 $+37$

Same third difference means it's a cubic sequence:

$n^3: 1, 8, 27, 64, 125$

(b) $24, 12, 6, 3, 1.5, \dots$

$\times \frac{1}{2}$
 $\times \frac{1}{2}$
 $\times \frac{1}{2}$
 $\times \frac{1}{2}$

First term: 24

Ratio: $\frac{1}{2}$

$ar^{n-1}: 24 \times \left(\frac{1}{2}\right)^{n-1}$

These are close to the sequence, but shifted along one:

$n^3: 1 \rightarrow 8 \rightarrow 27 \rightarrow 64 \rightarrow 125$

So instead of n^3 , do $(n-1)^3$:

$(n-1)^3: 0 \quad 1 \quad 8 \quad 27 \quad 64$
 $-1 \quad 0 \quad 7 \quad 26 \quad 63$
 $= \underline{(n-1)^3 - 1}$

(b) Find the n th term of each sequence.

(i) $6, 13, 32, 69, 130, \dots$

$+7$
 $+12$
 $+19$
 $+24$
 $+37$
 $+61$

→ Cubic sequence

coeff of $n^3 = 6 \div 6 = 1$

(ii) $100, 50, 25, 12.5, 6.25, \dots$

$\times \frac{1}{2}$
 $\times \frac{1}{2}$
 $\times \frac{1}{2}$
 $\times \frac{1}{2}$

$a: 100$

$r: \frac{1}{2}$

$ar^{n-1}: 100 \times \left(\frac{1}{2}\right)^{n-1}$

$n^3: 1 \quad 8 \quad 27 \quad 64 \quad 125$
 $6 \quad 13 \quad 32 \quad 69 \quad 130 \quad \downarrow +5$

$n^3 + 5$

..... [2]

$100 \times \left(\frac{1}{2}\right)^{n-1}$ [2]

16 Find the n th term of each sequence.

(a) 8, 15, 34, 71, 132, ...

$+7$
 $+12$
 $+19$
 $+24$
 $+37$
 $+61$
 $+18$
 $+6$
 $+6$

→ Cubic sequence

coeff of n^3 : $6 \div 6 = 1$

(b) $\frac{2}{1}, \frac{3}{4}, \frac{4}{16}, \frac{5}{64}, \frac{6}{256}, \dots$

Numerator: 2, 3, 4, 5, 6

$+1$
 $+1$
 $+1$
 $+1$
 $n+1$

Denominator:

These are the square numbers:

n^2

n^3 : 1, 8, 27, 64, 125

8, 15, 34, 71, 132

$+7$

$n^3 + 7$

..... [2]

$\frac{n+1}{n^2}$

..... [3]

(c) Find the n th term for each of these sequences.

(i) 13, 16, 19, 22, 25, ...

$+3$
 $+3$
 $+3$
 $+3$

$3n + 10$

$3n + 10$

..... [2]

(ii) 3, 17, 55, 129, 251, ...

$+14$
 $+24$
 $+38$
 $+74$
 $+122$
 $+36$
 $+48$
 $+12$
 $+12$

$2n^3$: 2, 16, 54, 128, 250

3, 17, 55, 129, 251

$+1$

→ Cubic sequence

coeff of n^3 : $12 \div 6 = 2$

$2n^3 + 1$

..... [2]

8 Find an expression for the n th term of each sequence.

(b) 1, -2, 3, -4, 5, ...

Without negatives:

1, 2, 3, 4, 5

→ $\frac{n}{2}$

(b) 1, 5, 25, 125, 625, ...

a: 1

r: 5

$ar^{n-1}: 1 \times 5^{n-1}$

→ 5^{n-1}

To alternate between positive and negative:

$(-1)^n$

problem: this is negative when $n=1$, positive when $n=2$, etc. We want it to be positive for $n=1$, then negative.

→ $(-1)^{n+1}$

$= n \times (-1)^{n+1}$

(c) The sum of the first n terms of another sequence is $\frac{n}{2}(5n-1)$.

(i) Use $n=2$ to find the sum of the first two terms in this sequence.

$$\frac{2}{2}(5(2)-1) = 1 \times 9 = 9$$

9

[1]

(ii) Find the 3rd term of this sequence.

Find sum of 3 terms:

Sum of 3 terms - Sum of 2 terms

$$\frac{3}{2}(5(3)-1) = \frac{3}{2} \times 14 = 21$$

$$21 - 9 = 12 \leftarrow \text{third term}$$

12

[2]

- 6 (a) These are the first five terms of a sequence.

27 26 23 18 11 2 -9

Find the next two terms in the sequence.

2 -9 [2]

- (b) The table shows information about two different sequences.

	First five terms of sequence	n th term
Sequence A	3 10 17 24 31	$7n - 4$
Sequence B	2 11 26 47 74	$3n^2 - 1$

Complete the table.

A: 3 10 17 24 31

$\rightarrow 7n - 4$

[4]

B: 2 11 26 47 74

Half 2nd diff is coeff of n^2 :

$$3n^2$$

$3n^2$: 3 12 27 48 75

2 11 26 47 74

$$\rightarrow 3n^2 - 1$$

12 The table shows the first five terms of sequences A , B and C .

	1st term	2nd term	3rd term	4th term	5th term	n th term
Sequence A	8	3	-2	-7	-12	$-5n + 13$
Sequence B	2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{n+1}{n}$
Sequence C	$\frac{1}{2}$	1	2	4	8	$\frac{1}{2} \times 2^{n-1}$

Complete the table to show the n th term of each sequence.

A:

$$\begin{array}{ccccccccc}
 & -5 & & -10 & & -15 & & -20 & & -25 & & \downarrow +13 \\
 & 8 & \xrightarrow{-5} & 3 & \xrightarrow{-5} & -2 & \xrightarrow{-5} & -7 & \xrightarrow{-5} & -12 & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \rightarrow -5n + 13
 \end{array}$$

[5]

B:

$$\begin{array}{ccccccccc}
 & 2 & & \frac{3}{2} & & \frac{4}{3} & & \frac{5}{4} & & \frac{6}{5} \\
 \text{Numerator:} & 2 & \xrightarrow{+1} & 3 & \xrightarrow{+1} & 4 & \xrightarrow{+1} & 5 & \xrightarrow{+1} & 6 & & \downarrow +1 \\
 & & & & & & & & & & & \rightarrow n+1 \\
 \text{Denominator:} & 1 & & 2 & & 3 & & 4 & & 5 & & \\
 & & & & & & & & & & & \rightarrow n
 \end{array}$$

$$\left. \begin{array}{c} \rightarrow n+1 \\ \rightarrow n \end{array} \right\} \frac{n+1}{n}$$

C:

$$\begin{array}{ccccccc}
 & \frac{1}{2} & \xrightarrow{\times 2} & 1 & \xrightarrow{\times 2} & 2 & \xrightarrow{\times 2} & 4 & \xrightarrow{\times 2} & 8 \\
 a: & \frac{1}{2} & & & & & & & & \\
 r: & 2 & & & & & & & & \\
 ar^{n-1}: & \frac{1}{2} \times 2^{n-1} & & & & & & & &
 \end{array}$$

6 (a)

Sequence	1st term	2nd term	3rd term	4th term	5th term	n th term
A	-7	-3	1	5	9	$4n-11$
B	7	13	23	37	55	$2n^2+5$
C	$\frac{2}{27}$	$\frac{3}{81}$	$\frac{4}{243}$	$\frac{5}{729}$	$\frac{6}{2187}$	$27 \times 3^{n-1}$

Complete the table for the three sequences.

A: $-7 \xrightarrow{+4} -3 \xrightarrow{+4} 1 \xrightarrow{+4} 5 \xrightarrow{+4} 9$

$\rightarrow \underline{4n-11}$

B: $7 \xrightarrow{+6} 13 \xrightarrow{+10} 23 \xrightarrow{+14} 37 \xrightarrow{+18} 55$

Half 2nd diff as coeff of n^2 : $2n^2$

$\rightarrow \underline{2n^2+5}$

C: $\frac{2}{27} \quad \frac{3}{81} \quad \frac{4}{243} \quad \frac{5}{729}$

Numerator: $2 \xrightarrow{+1} 3 \xrightarrow{+1} 4 \xrightarrow{+1} 5 \xrightarrow{+1} 6$

$\rightarrow n+1$

Denominator: $27 \xrightarrow{\times 3} 81 \xrightarrow{\times 3} 243 \xrightarrow{\times 3} 729 \xrightarrow{\times 3} 2187$

$a: 27$

$r: 3$

$ar^{n-1}: \underline{27 \times 3^{n-1}}$

[10]

13 The table shows the first 5 terms of sequences A , B and C .

	1st term	2nd term	3rd term	4th term	5th term	n th term
Sequence A	5	12	31	68	129	$n^3 + 4$
Sequence B	$\frac{10}{3}$	$\frac{9}{4}$	$\frac{8}{5}$	$\frac{7}{6}$	$\frac{6}{7}$	$\frac{-n+11}{n+2}$
Sequence C	4	8	16	32	64	$4 \times 2^{n-1}$

Complete the table to show the n th term of each sequence.

A: 5 12 31 68 129

$+7$ $+19$ $+37$ $+61$
 $+12$ $+18$ $+24$
 $+6$ $+6$

n^3 : 1 8 27 64 125
 5 12 31 68 129 $\downarrow +4$

$\rightarrow n^3 + 4$

$\rightarrow$ Cubic sequence.
 coeff of $n^3 = 6 \div 6 = 1$

B: $\frac{10}{3}$ $\frac{9}{4}$ $\frac{8}{5}$ $\frac{7}{6}$ $\frac{6}{7}$

[6]

Numerator: -1 -2 -3 -4 -5 $\downarrow +11$

10 9 8 7 6

-1 -1 -1 -1 -1

$\frac{-n+11}{n+2}$

Denominator: 1 2 3 4 5 $\downarrow +2$

3 4 5 6 7

$+1$ $+1$ $+1$ $+1$ $+1$

$n+2$

$\frac{-n+11}{n+2}$

C: 4 8 16 32 64

$\times 2$ $\times 2$ $\times 2$ $\times 2$

a : 4
 r : 2
 ar^{n-1} : $4 \times 2^{n-1}$

11 The table shows three sequences.

	1st term	2nd term	3rd term	4th term	5th term	6th term	n th term
Sequence A	28	22	16	10	4	-2	$-6n + 34$
Sequence B	$\frac{1}{6}$	$\frac{2}{7}$	$\frac{3}{8}$	$\frac{4}{9}$	$\frac{1}{2}$	$\frac{6}{11}$	$\frac{n}{n+5}$
Sequence C	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	$\frac{1}{4} \times 2^{n-1}$

Complete the table.

A: $28 \xrightarrow{-6} 22 \xrightarrow{-6} 16 \xrightarrow{-6} 10 \xrightarrow{-6} 4 \xrightarrow{-6} -2$

$-6n + 34$

B: $\frac{1}{6} \quad \frac{2}{7} \quad \frac{3}{8} \quad \frac{4}{9} \quad \frac{5}{10}$

Numerator: 1 2 3 4 5

Denominator: $6 \xrightarrow{+1} 7 \xrightarrow{+1} 8 \xrightarrow{+1} 9 \xrightarrow{+1} 10 \xrightarrow{+1} 11$

$\frac{n}{n+5}$

C: $\frac{1}{4} \xrightarrow{\times 2} \frac{1}{2} \xrightarrow{\times 2} 1 \xrightarrow{\times 2} 2 \xrightarrow{\times 2} 4 \xrightarrow{\times 2} 8$

$a: \frac{1}{4}$

$r: 2$

$ar^{n-1}: \frac{1}{4} \times 2^{n-1}$

[8]

Sequence	1st term	2nd term	3rd term	4th term	5th term	n th term
A	13	9	5	1	-3	$-4n + 17$
B	0	7	26	63	124	$n^3 - 1$
C	$\frac{7}{8}$	$\frac{8}{16}$	$\frac{9}{32}$	$\frac{10}{64}$	$\frac{11}{128}$	$\frac{n+6}{8 \times 2^{n-1}}$

(a) Complete the table for the three sequences.

A: $13 \xrightarrow{-4} 9 \xrightarrow{-4} 5 \xrightarrow{-4} 1 \xrightarrow{-4} -3$
 $-4n + 17$

B: $0 \xrightarrow{+7} 7 \xrightarrow{+19} 26 \xrightarrow{+37} 63 \xrightarrow{+61} 124$
 $n^3: 1 \xrightarrow{+7} 8 \xrightarrow{+27} 27 \xrightarrow{+64} 64$
 $0 \xrightarrow{+7} 7 \xrightarrow{+26} 26 \xrightarrow{+63} 63$
 $n^3 - 1$
 Cubic Sequence
 coeff of n^3 is $6 \div 6 = 1$

C: $\frac{7}{8} \xrightarrow{+1} \frac{8}{16} \xrightarrow{+1} \frac{9}{32} \xrightarrow{+1} \frac{10}{64}$
 Numerator: $7 \xrightarrow{+1} 8 \xrightarrow{+1} 9 \xrightarrow{+1} 10$
 $n+6$
 Denominator: $8 \xrightarrow{\times 2} 16 \xrightarrow{\times 2} 32 \xrightarrow{\times 2} 64$
 $a: 8$
 $r: 2$
 $ar^{n-1}: 8 \times 2^{n-1}$
 $\rightarrow \frac{n+6}{8 \times 2^{n-1}}$

(b) One term in Sequence C is $\frac{p}{q}$.

Write down the next term in Sequence C in terms of p and q .

$\frac{7}{8} \xrightarrow{+1} \frac{8}{16} \xrightarrow{+1} \frac{9}{32} \xrightarrow{+1} \frac{10}{64}$
 $\frac{p}{q} \xrightarrow{+1} \frac{p+1}{2q}$
 $\frac{p+1}{2q}$ [2]