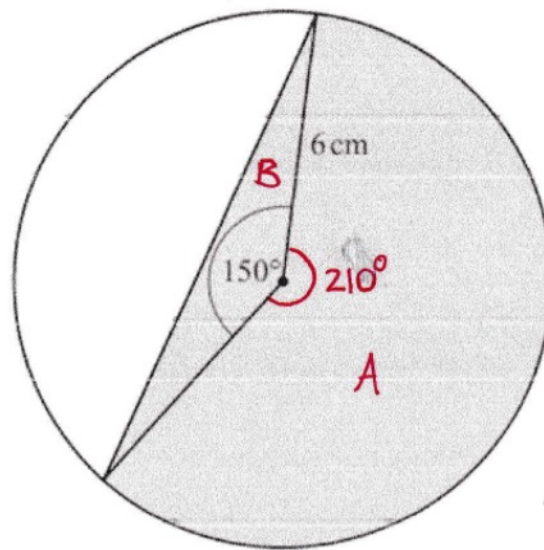


NOT TO
SCALE

A sector of a circle with radius 6 cm has a sector angle of 150° .

Find the exact value of the area of the shaded region.
Give your answer in its simplest form.

Area of sector A:

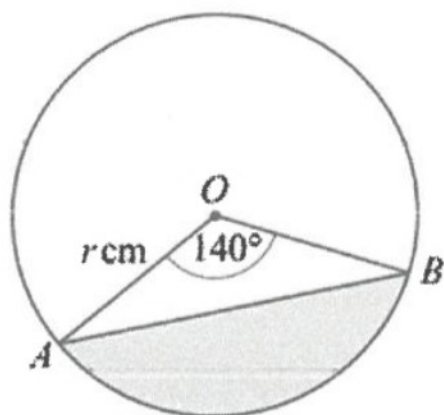
$$\begin{aligned}
 \text{Area} &= \frac{210}{360} \times \pi r^2 \\
 &= \frac{7}{12} \times \pi \times 6^2 \\
 &= \frac{7}{12} \times 36\pi \\
 &= \underline{21\pi}
 \end{aligned}$$

Area of triangle B:

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} ab \sin C \\
 &= \frac{1}{2} \times 6 \times 6 \times \sin 150 \\
 &= \frac{1}{2} \times 36 \times \frac{1}{2} \\
 &= \underline{9}
 \end{aligned}$$

$$\underline{9 + 21\pi} \text{ cm}^2 [4]$$

(b)



A and B are points on a circle, centre O , radius r cm.
The area of the shaded segment is 65 cm^2 .

Calculate the value of r .

Area of sector:

$$\begin{aligned}\text{Area} &= \frac{140}{360} \times \pi r^2 \\ &= \frac{7}{18} \pi r^2\end{aligned}$$

Area of triangle:

$$\begin{aligned}\text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times r \times r \times \sin 140 \\ &= 0.32139 r^2\end{aligned}$$

Shaded Area = Sector - triangle:

$$\text{Area} = \frac{7}{18} \pi r^2 - 0.32139 r^2$$

$$65 = \left(\frac{7}{18} \pi - 0.32139 \right) r^2$$

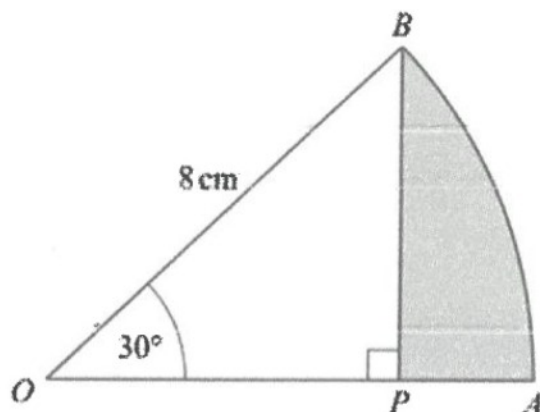
$$65 = 0.9003 r^2$$

$$r^2 = \frac{65}{0.9003}$$

$$r = \sqrt{\frac{65}{0.9003}}$$

$$r = 8.50 \text{ cm} \quad [4]$$

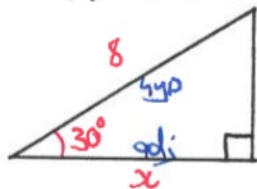
(34)



NOT TO
SCALE

OAB is the sector of a circle, centre O .
 $OB = 8\text{ cm}$ and angle $AOB = 30^\circ$.
 BP is perpendicular to OA .

(a) Calculate AP .



S^OH C^AH T^A

$$\cos 30 = \frac{x}{8}$$

$$\begin{aligned} x &= 8 \cos 30 \\ &= 8 \times \frac{\sqrt{3}}{2} \\ &= 4\sqrt{3} \end{aligned}$$

$$\underline{AP = OA - x:}$$

$$\begin{aligned} AP &= 8 - 4\sqrt{3} \\ &= 1.07 \end{aligned}$$

$$AP = \underline{1.07} \text{ cm [3]}$$

(3sf)

(b) Work out the area of the shaded region APB .

Area of Sector:

$$\begin{aligned} \text{Area} &= \frac{30}{360} \times \pi r^2 \\ &= \frac{1}{12} \times \pi \times 8^2 \\ &= \frac{1}{12} \times \pi \times 64 \\ &= \frac{16}{3} \pi \end{aligned}$$

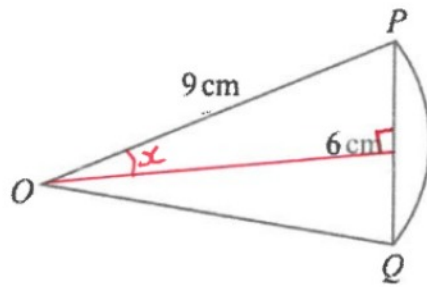
Area of Triangle:

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 4\sqrt{3} \times 8 \times \sin 30 \\ &= \frac{1}{2} \times 32\sqrt{3} \times \frac{1}{2} \\ &= 8\sqrt{3} \end{aligned}$$

$$\underline{2.90} \text{ cm}^2 \text{ [3]}$$

Shaded Area = Sector - Triangle:

$$\begin{aligned} \text{Area} &= \frac{16}{3} \pi - 8\sqrt{3} \\ &= \underline{2.90 \text{ cm}^2} \text{ (3sf)} \end{aligned}$$

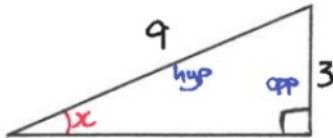


NOT TO
SCALE

The diagram shows a sector of a circle with centre O and radius 9 cm.
The length of the chord PQ is 6 cm.

Calculate the length of the arc PQ .

S°H C°H T°A



$$\sin x = \frac{3}{9}$$

$$\sin x = \frac{1}{3}$$

$$x = \sin^{-1}\left(\frac{1}{3}\right)$$

$$= 19.471^\circ$$

Angle of sector = $2x$:

$$2 \times 19.471 = 38.942^\circ$$

Arc length:

$$\text{length} = \frac{38.942}{360} \times 2\pi r$$

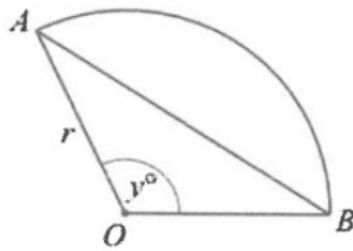
$$= \frac{38.942}{360} \times 2 \times \pi \times 9$$

$$= 6.12 \text{ cm}$$

6.12

..... cm [3]

(b)



NOT TO
SCALE

The diagram shows a sector, OAB , of a circle with centre O and radius r .
The area of triangle OAB is half the area of the sector.
Angle $AOB = y^\circ$ and is obtuse.

(i) Show that $360 \sin y = \pi y$.

Area of Triangle:

$$\begin{aligned}\text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times r \times r \times \sin y \\ &= \frac{1}{2} r^2 \sin y\end{aligned}$$

Area of Sector:

$$\text{Area} = \frac{y}{360} \times \pi r^2$$

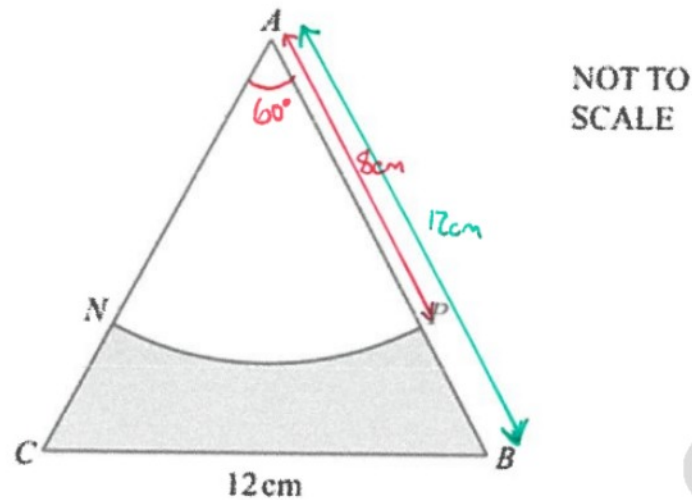
Area of sector = 2 x triangle:

$$\frac{y}{360} \times \pi r^2 = 2 \times \frac{1}{2} r^2 \sin y$$

$$\frac{y}{360} \times \pi r^2 = r^2 \sin y$$

$$y \pi r^2 = 360 r^2 \sin y$$

$$\pi y = 360 \sin y$$



NOT TO
SCALE

The diagram shows an equilateral triangle ABC with all sides of length 12 cm.

ANP is a sector of a circle, centre A .

N lies on AC such that $AN : NC = 2 : 1$.

P lies on AB such that $AP : PB = 2 : 1$.

Calculate the area of the shaded region.

Area of Sector:

$$\begin{aligned} \text{Area} &= \frac{60}{360} \times \pi r^2 \\ &= \frac{1}{6} \times \pi \times 8^2 \\ &= \frac{32}{3} \pi \end{aligned}$$

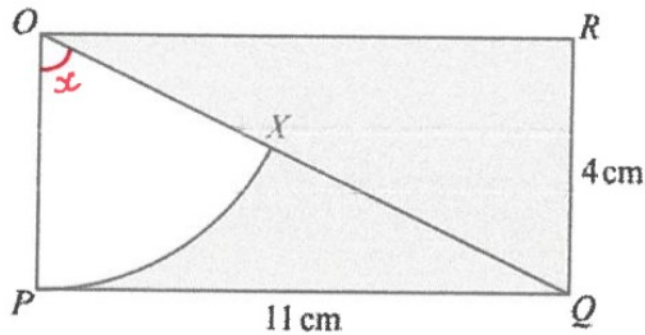
Area of Triangle:

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 12 \times 12 \times \sin 60 \\ &= \frac{1}{2} \times 144 \times \frac{\sqrt{3}}{2} \\ &= 36\sqrt{3} \end{aligned}$$

Shaded Area = Triangle - Sector:

$$\begin{aligned} \text{Area} &= 36\sqrt{3} - \frac{32}{3}\pi \\ &= \underline{28.8 \text{ cm}^2} \end{aligned}$$

..... 28.8 cm^2 [4]



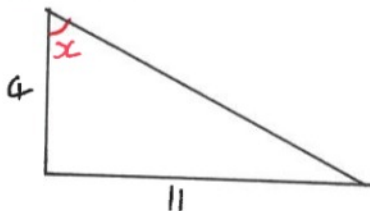
NOT TO
SCALE

The diagram shows a rectangle $OPQR$ with length 11 cm and width 4 cm. OQ is a diagonal and OPX is a sector of a circle, centre O .

Calculate the percentage of the rectangle that is shaded.

Angle x :

$S^{\circ}H C^{\circ}H T^{\circ}A$



$$\begin{aligned}\tan x &= \frac{11}{4} \\ x &= \tan^{-1}\left(\frac{11}{4}\right) \\ &= \underline{70.017^{\circ}}\end{aligned}$$

Shaded Area = Rectangle - Sector:

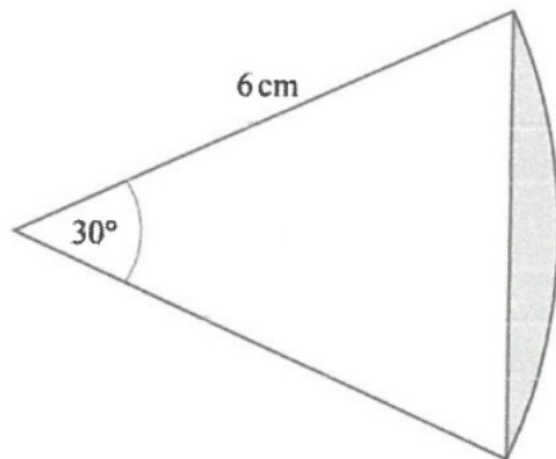
$$\begin{aligned}\text{Area} &= 11 \times 4 - 9.7762 \\ &= 44 - 9.7762 \\ &= \underline{34.2238}\end{aligned}$$

Percentage Shaded:

$$\frac{34.2238}{44} \times 100 = \underline{77.8\%}$$

Area of sector:

$$\begin{aligned}\text{Area} &= \frac{70.017}{360} \times \pi \times r^2 \\ &= \frac{70.017}{360} \times \pi \times 4^2 \\ &= \underline{9.7762}\end{aligned}$$



NOT TO
SCALE

The area of the shaded segment is $(a\pi + b) \text{ cm}^2$.

Find the value of a and the value of b .

Area of Sector:

$$\begin{aligned} \text{Area} &= \frac{30}{360} \times \pi r^2 \\ &= \frac{1}{12} \times \pi \times 6^2 \\ &= \frac{1}{12} \times 36\pi \\ &= \underline{3\pi} \end{aligned}$$

Area of Triangle:

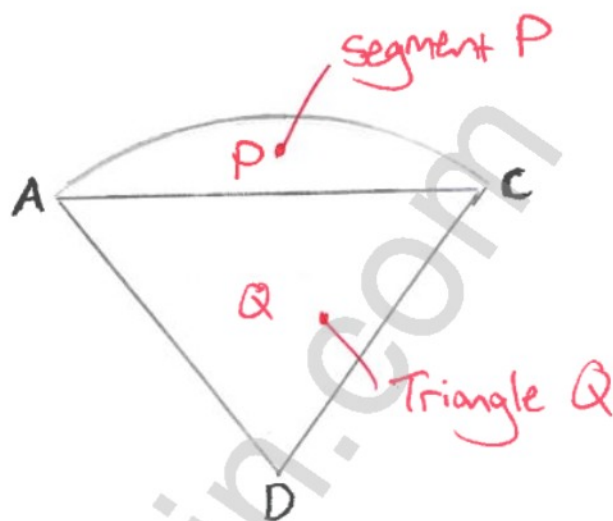
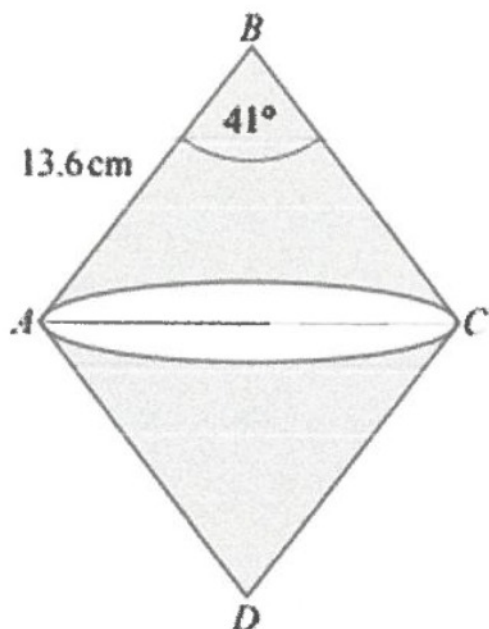
$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 6 \times 6 \times \sin 30 \\ &= \frac{1}{2} \times 36 \times \frac{1}{2} \\ &= \underline{9} \end{aligned}$$

Shaded segment:

$$\text{Area} = 3\pi - 9$$

$$a = \underline{3}$$

$$b = \underline{-9} \quad [4]$$



$ABCD$ is a rhombus with side length 13.6 cm.
 Angle $ABC = 41^\circ$.
 BAC is a sector of a circle with centre B .
 DAC is a sector of a circle with centre D .

Calculate the shaded area.

$$\text{Shaded Area} = 2 \times \text{Triangle Q} - 2 \times \text{Segment P}.$$

Triangle:

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 13.6 \times 13.6 \times \sin 41^\circ \\ &= 60.672 \end{aligned}$$

Sector:

$$\begin{aligned} \text{Area} &= \frac{41}{360} \times \pi r^2 \\ &= \frac{41}{360} \times \pi \times 13.6^2 \\ &= 66.177 \end{aligned}$$

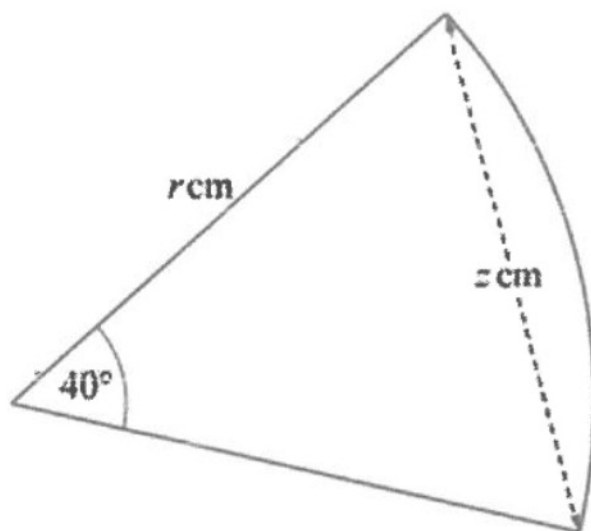
Segment = Sector - Triangle:

$$\begin{aligned} \text{Area} &= 66.177 - 60.672 \\ &= \underline{5.505} \end{aligned}$$

Shaded Area = 2x Triangle - 2x Segment:

$$\begin{aligned} \text{Area} &= 2 \times 60.672 - 2 \times 5.505 \\ &= 121.344 - 11.01 \\ &= 110.334 \\ &= \underline{110 \text{ cm}^2} \quad (3\text{sf}) \end{aligned}$$

(c)



NOT TO
SCALE

This sector of a circle has radius r and perimeter 20 cm .

Find the value of z .

Arc length:

$$\begin{aligned} l &= \frac{40}{360} \times 2\pi r \\ &= \frac{1}{9} \times 2\pi r \\ &= \frac{2}{9} \pi r \end{aligned}$$

Perimeter = 20 cm :

$$\begin{aligned} \text{Perimeter} &= r + r + \frac{2}{9} \pi r \\ &= 2r + \frac{2}{9} \pi r \end{aligned}$$

$$2r + \frac{2}{9} \pi r = 20$$

$$\begin{aligned} r(2 + \frac{2}{9} \pi) &= 20 \\ r &= \frac{20}{2 + \frac{2}{9} \pi} \end{aligned}$$

$$= \underline{7.4125... \text{ cm}}$$

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

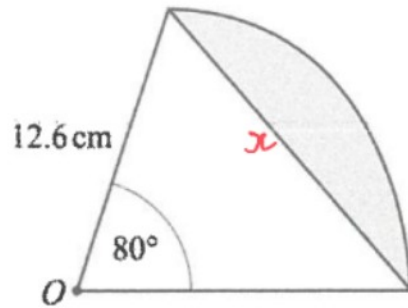
$$z^2 = r^2 + r^2 - 2 \times r \times r \times \cos 40$$

$$z^2 = (7.4125...)^2 + (7.4125...)^2 - 2 \times 7.4125...^2 \cos 40$$

$$z = \sqrt{2(7.4125...)^2 - 2(7.4125...)^2 \cos 40}$$

$$= \underline{5.07 \text{ cm}}$$

$$z = \underline{5.07 \text{ cm}} \quad [6]$$



NOT TO
SCALE

The diagram shows a sector of a circle, centre O , radius 12.6 cm.

Calculate the perimeter of the shaded segment.

Arc Length:

$$\begin{aligned}
 l &= \frac{80}{360} \times 2\pi r \\
 &= \frac{2}{9} \times 2 \times \pi \times 12.6 \\
 &= \frac{28}{5} \pi
 \end{aligned}$$

Side x (Cosine Rule):

$$\begin{aligned}
 a^2 &= b^2 + c^2 - 2bc \cos A \\
 x^2 &= 12.6^2 + 12.6^2 - 2 \times 12.6 \times 12.6 \times \cos 80 \\
 x &= \sqrt{12.6^2 + 12.6^2 - 2 \times 12.6^2 \cos 80} \\
 &= 16.198
 \end{aligned}$$

Perimeter = Arc + x :

$$\frac{28}{5} \pi + 16.198 = \underline{\underline{33.8 \text{ cm}}}$$