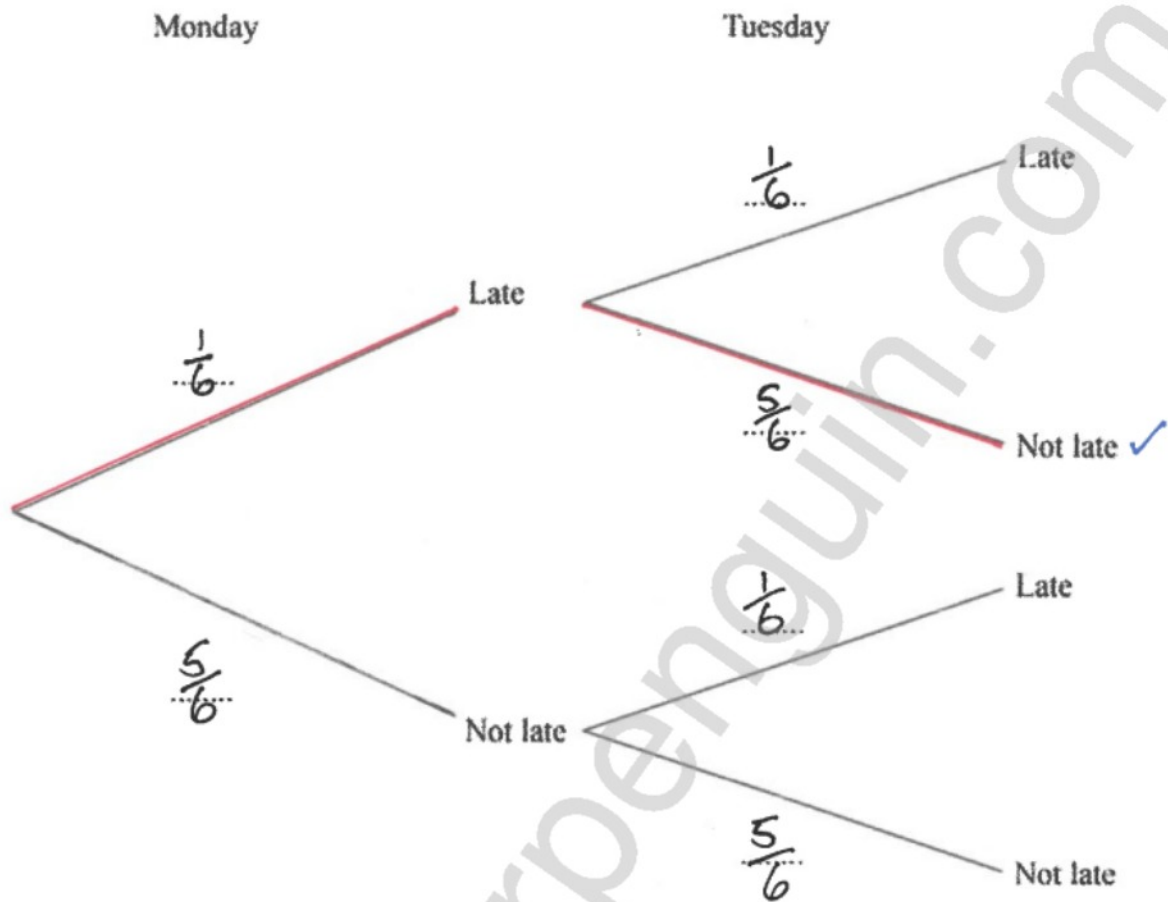


A Probability (Trees etc.)

11 (a) The probability that Shalini is late for school on any day is $\frac{1}{6}$.

(i) Complete the tree diagram for Monday and Tuesday.



[2]

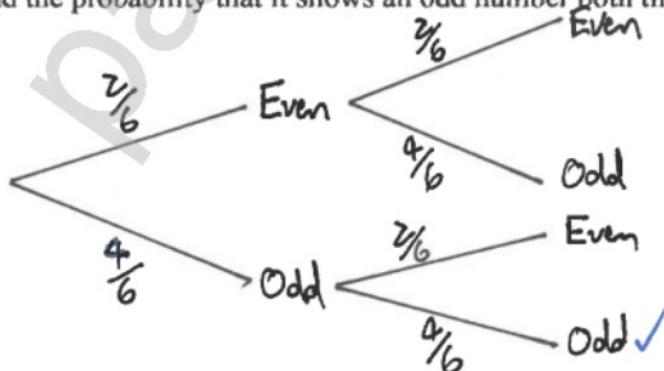
(ii) Calculate the probability that Shalini is late on Monday but is not late on Tuesday.

$$\frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$$

$$\frac{5}{36} \dots\dots\dots [2]$$

10 The faces of a die are numbered 1, 1, 2, 3, 3 and 4.
When it is rolled it is equally likely to show any face.
The die is rolled twice.

Find the probability that it shows an odd number both times.

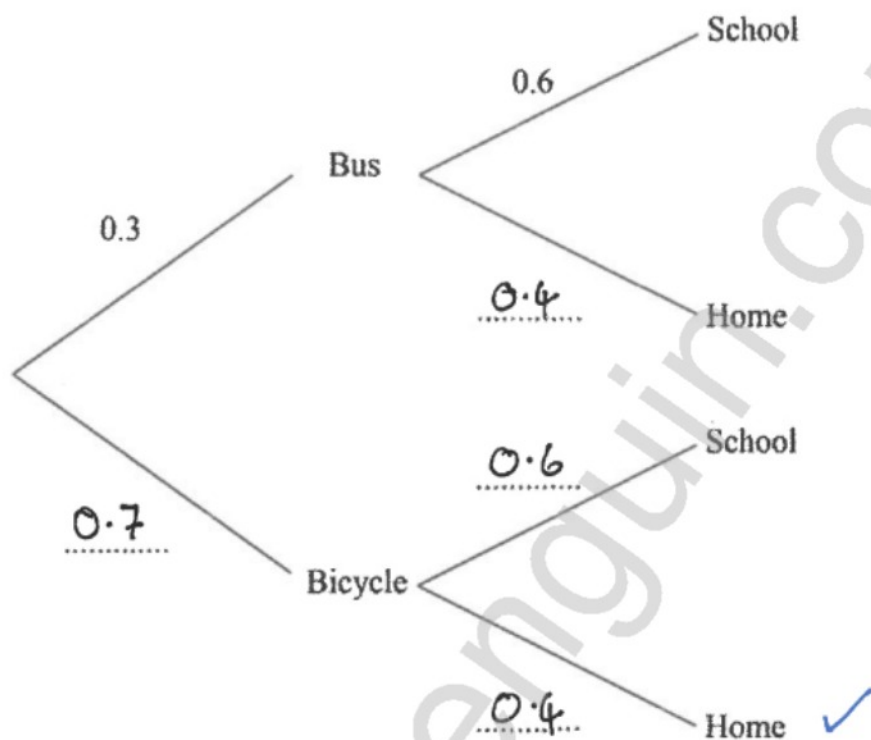


$$\frac{4}{6} \times \frac{4}{6} = \frac{16}{36}$$

$$= \frac{4}{9} \dots\dots\dots [2]$$

- 7 Iraj travels to school either by bus or on a bicycle.
 The probability that he goes by bus is 0.3 .
 He can have lunch at home or at school.
 The probability that he has lunch at school is 0.6 .

(a) Complete the tree diagram.



[2]

- (b) Find the probability that Iraj travels on a bicycle to school and goes home for lunch.

$$0.7 \times 0.4 = 0.28$$

0.28 [2]

- 7 Karen has 3 blue hats, 5 red hats and 2 white hats.
She also has 4 blue scarves, 3 red scarves and 1 white scarf.

(a) Karen takes a hat at random and replaces it.

Find the probability that it is white.

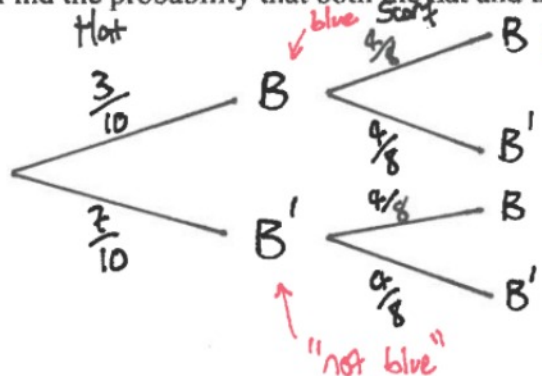
$$\frac{2}{10} = \frac{1}{5}$$

$$\frac{1}{5}$$

[1]

(b) Karen takes a hat and a scarf at random.

Find the probability that both the hat and the scarf are blue.



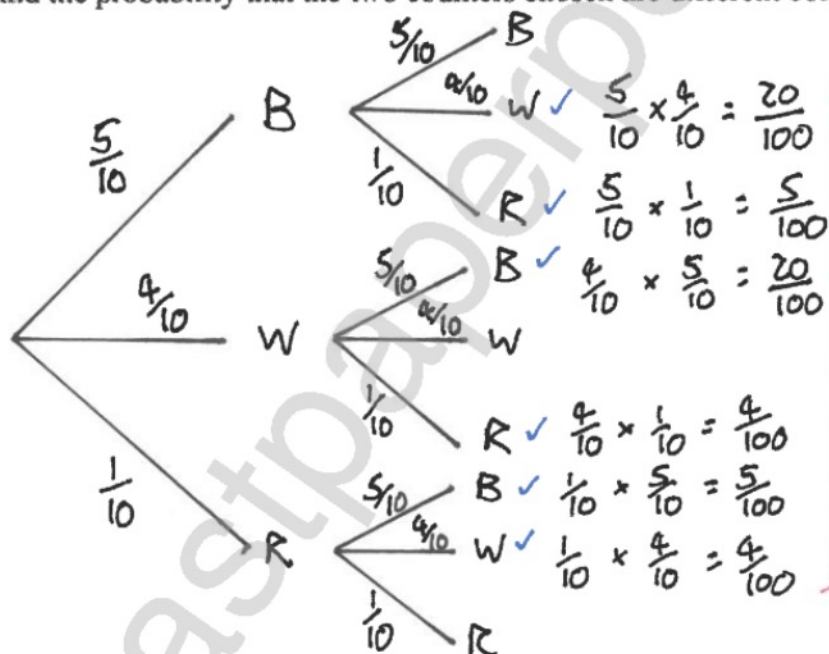
$$\frac{3}{10} \times \frac{4}{8} = \frac{12}{80}$$

$$= \frac{3}{20}$$

[2]

- 15 A bag has 5 black counters, 4 white counters and 1 red counter.
One counter is chosen at random and is replaced.
A second counter is then chosen at random.

Find the probability that the two counters chosen are different colours.



$$\frac{58}{100}$$

$$\frac{29}{50}$$

[4]

(can also sum BB, WW and RR and subtract from 1)

- (b) Scott takes an examination.
The examination is in two parts, a theory test and a practical test.
Both parts must be passed to pass the examination.

The probability that Scott passes the theory test is 0.9 .
The probability that Scott passes the practical test is 0.8 .

Find the probability that

- (i) Scott passes the examination,

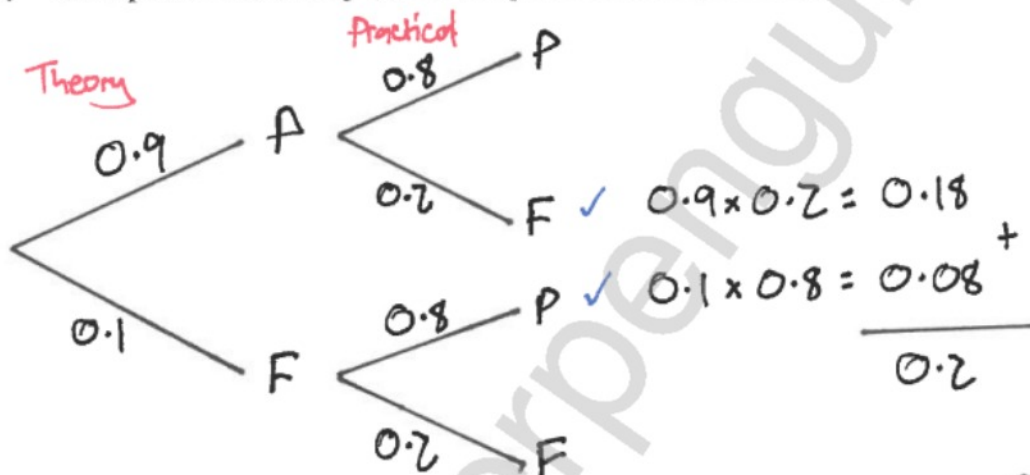
$$0.9 \times 0.8 = 0.72$$

(see tree diagram below)

0.72

[2]

- (ii) Scott passes the theory test or the practical test but not both.



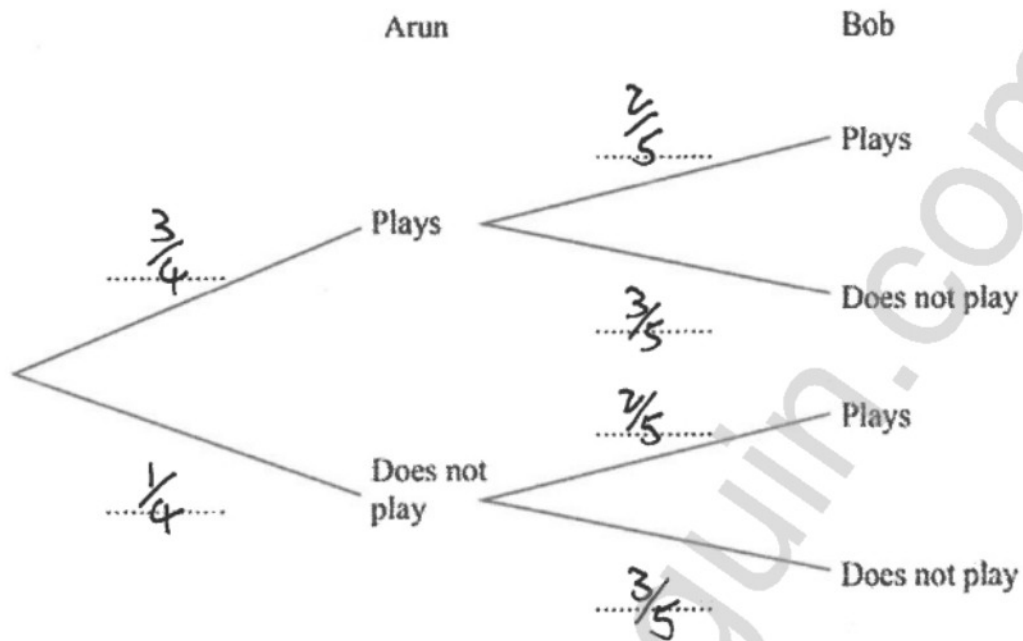
0.2

[3]

7 On any Saturday, the probability that Arun plays football is $\frac{3}{4}$.

On any Saturday, the probability that Bob plays football is $\frac{2}{5}$.

(a) (i) Complete the tree diagram.



[2]

(ii) Calculate the probability that, one Saturday, Arun and Bob both play football.

$$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$$

$\frac{3}{10}$ [2]

(iii) Calculate the probability that, one Saturday, either Arun plays football or Bob plays football, but not both.

only Arun: $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$

$$\frac{9}{20} + \frac{2}{20} = \frac{11}{20}$$

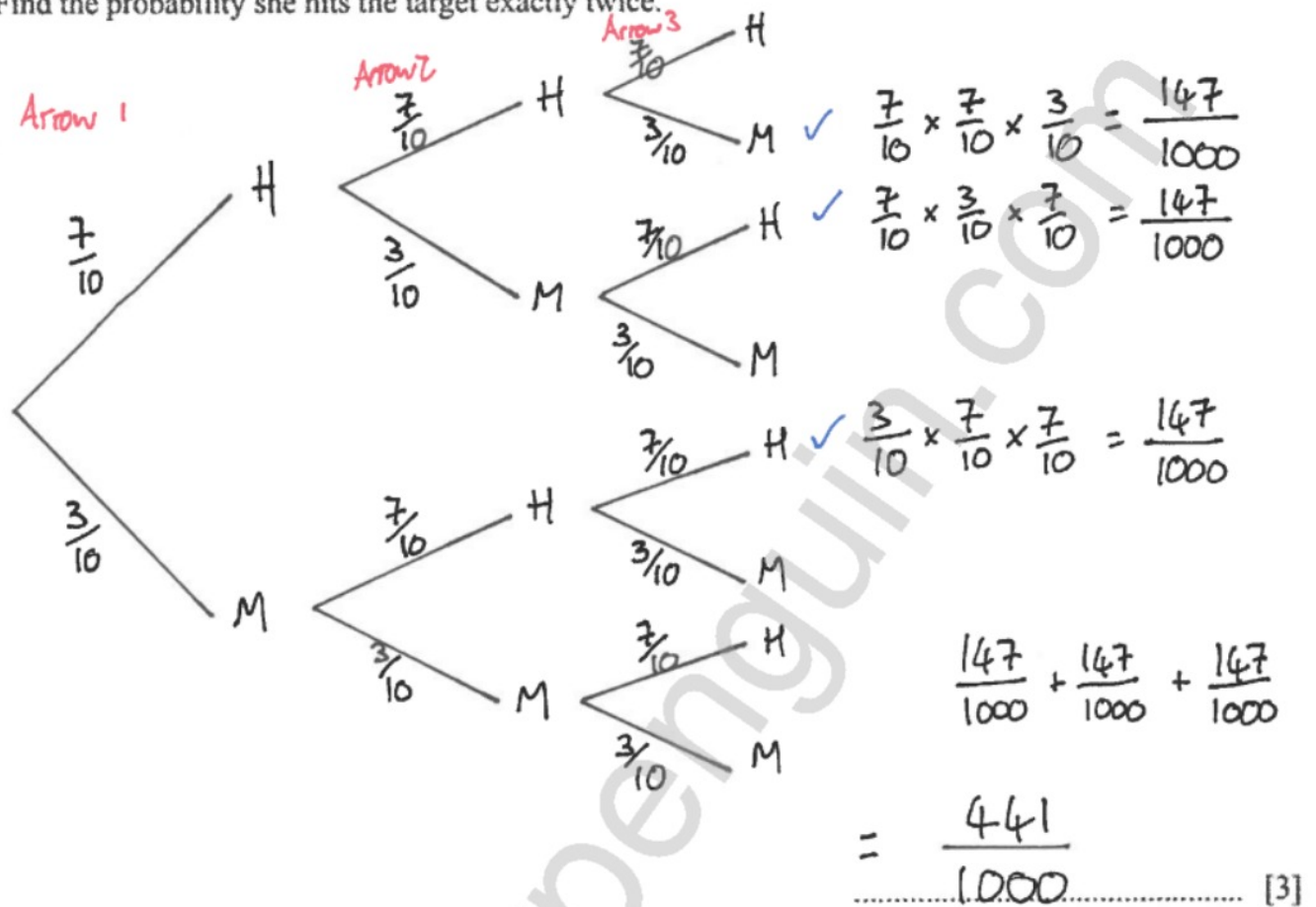
only Bob: $\frac{1}{4} \times \frac{2}{5} = \frac{2}{20}$

$\frac{11}{20}$ [3]

14 An archer shoots three arrows at a target.

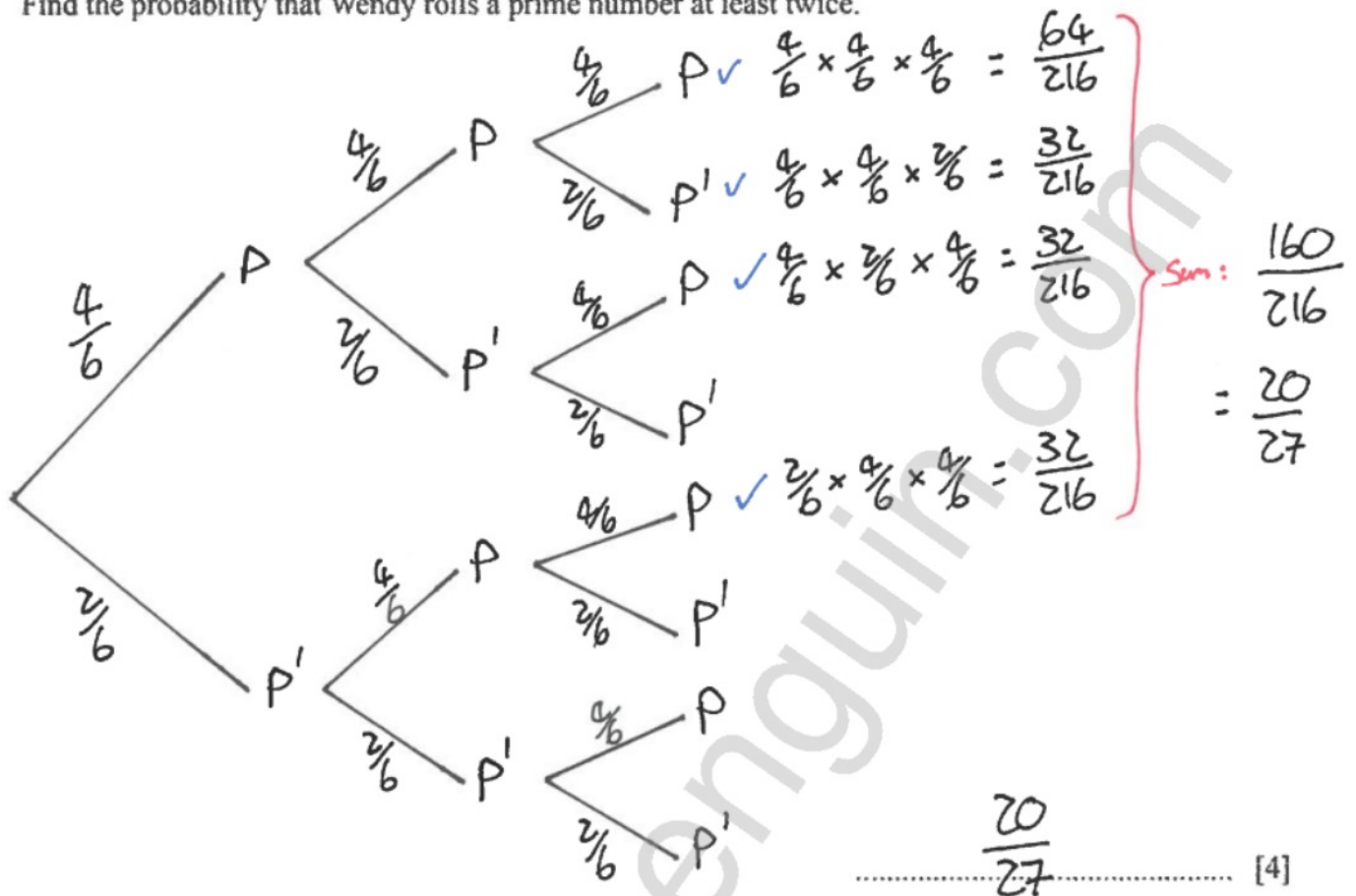
The probability that she hits the target with each arrow is $\frac{7}{10}$.

Find the probability she hits the target exactly twice.



- 17 An unbiased die is numbered 2, 3, 3, 4, 5, 6.
Wendy rolls the die three times.

Find the probability that Wendy rolls a prime number at least twice.



- 19 Katy picks a number at random from the numbers 2, 3 and 5.
She then picks a number at random from the numbers 5, 6, 7 and 9.
When she adds the two numbers the answer is even.

Find the probability that **exactly one** of the numbers picked is a 5.

	5	6	7	9
2	7	8 ✓	9	11
3	8 ✓	9	10 ✓	12 ✓
5	10 ✓	11	12 ✓	14 ✓

✓ = even

○ = only one is a 5

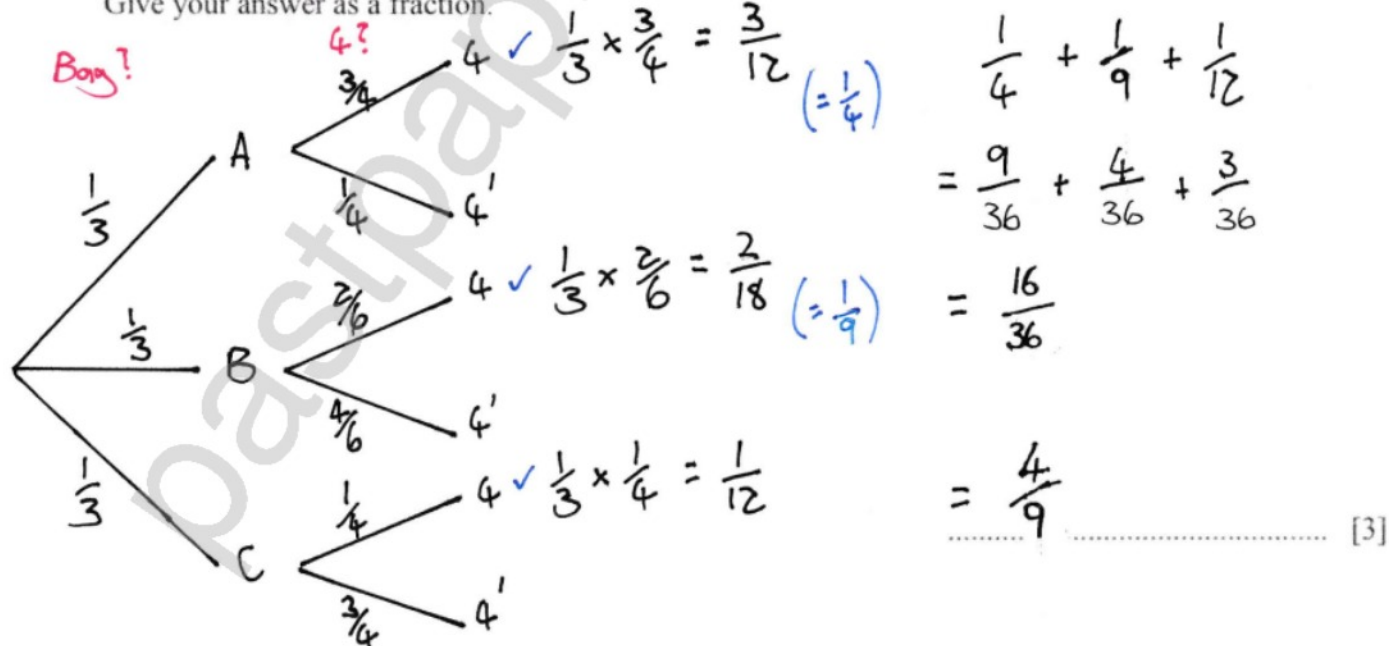
$$\frac{3}{7}$$

[3]

- 12 Bag A contains balls numbered 2, 4, 4, 4.
Bag B contains balls numbered 1, 1, 2, 3, 4, 4.
Bag C contains balls numbered 1, 2, 3, 4.

One of these three bags is chosen at random.
A ball is chosen at random from this bag.

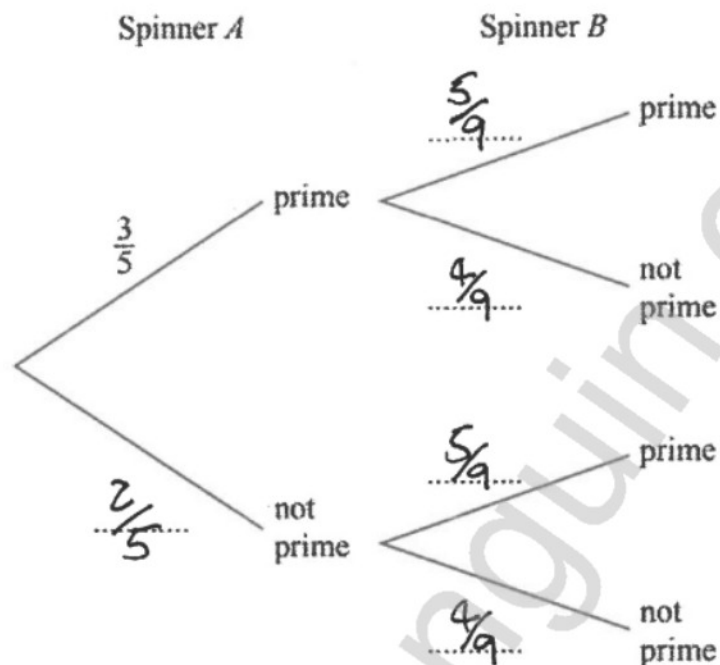
Find the probability that the ball chosen is numbered 4.
Give your answer as a fraction.



- 4 (a) Lucia has two fair spinners.
 Spinner A is five-sided and is numbered 1, 2, 3, 4, 5.
 Spinner B is nine-sided and is numbered 3, 3, 3, 4, 4, 4, 4, 5, 5.

Lucia spins the two spinners and records whether they land on a prime number.

- (i) Complete the tree diagram.



[2]

- (ii) Find the probability that

- (a) the two numbers are both prime

$$\frac{3}{5} \times \frac{5}{9} = \frac{15}{45} = \frac{1}{3}$$

$$\frac{1}{3}$$

[2]

- (b) the two numbers are **not** both prime.

$$1 - \frac{1}{3} = \frac{2}{3}$$

$$\frac{2}{3}$$

[1]

- (b) Lucia spins Spinner A 120 times.

Find the expected number of times the spinner lands on a prime number.

$$\frac{3}{5} \times 120 = 72$$

..... 72 [1]

- (d) Lucia keeps spinning Spinner B until it lands on a 4.

Find an expression, in terms of n , for the probability that this happens on the n th spin.

$$P(4) = \frac{4}{9}$$

e.g. first 4 on third spin:

$$\frac{5}{9} \times \frac{5}{9} \times \frac{4}{9} = \frac{5^2 \times 4}{9^3}$$

$\uparrow \quad \uparrow \quad \uparrow$
 $\text{not 4} \quad \text{not 4} \quad 4$

n th spin:

$$\frac{5^{n-1} \times 4}{9^n} \dots\dots\dots [2]$$

e.g. first 4 on fifth spin:

$$\frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times \frac{4}{9} = \frac{5^4 \times 4}{9^5}$$

7 On any day the probability that it rains is $\frac{1}{3}$.

When it rains the probability that Amira goes fishing is $\frac{3}{5}$.

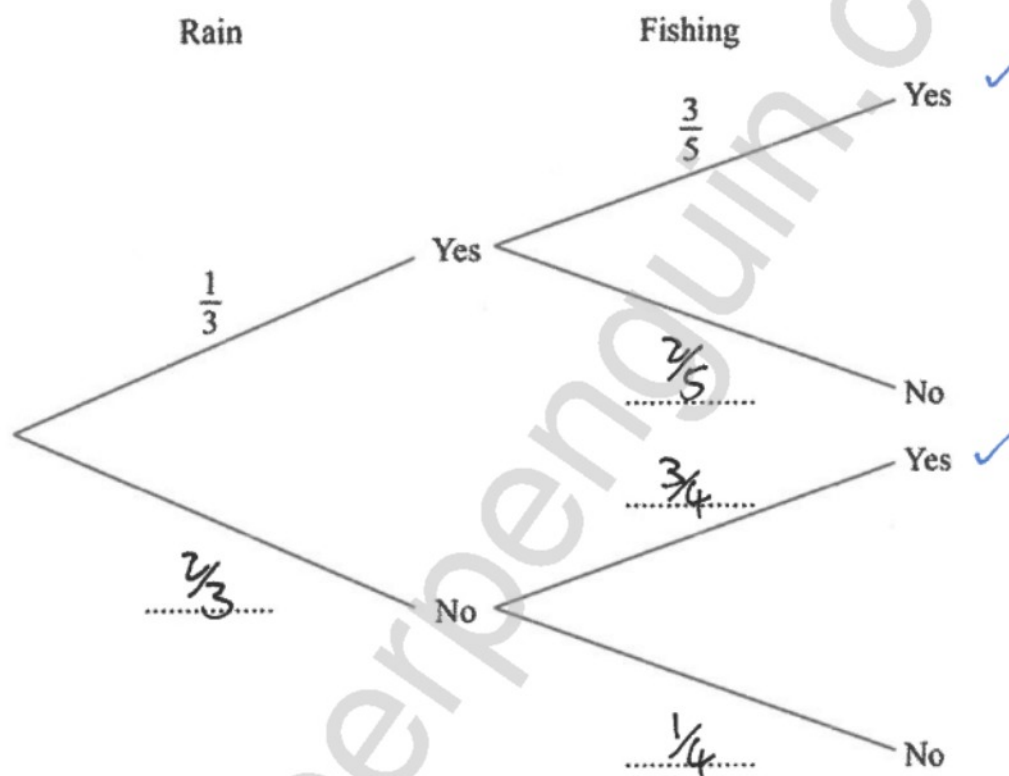
When it does not rain the probability that Amira goes fishing is $\frac{3}{4}$.

(a) In a period of 60 days on how many days is it expected to rain?

$$\frac{1}{3} \times 60 = 20$$

..... 20 [1]

(b) Complete the tree diagram.



[2]

(c) Find the probability that on any day Amira goes fishing.

$$\frac{1}{3} \times \frac{3}{5} = \frac{3}{15} = \frac{1}{5}$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

$$\begin{aligned} & \frac{1}{5} + \frac{1}{2} \\ &= \frac{2}{10} + \frac{5}{10} = \frac{7}{10} \end{aligned}$$

..... $\frac{7}{10}$ [3]

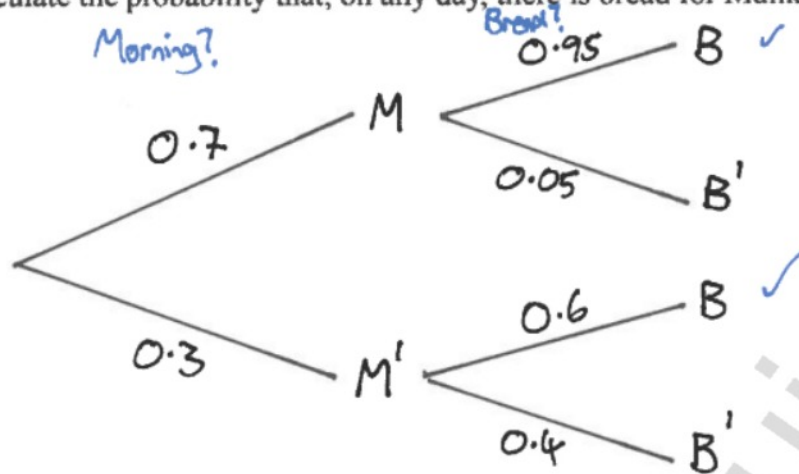
26 Malik goes to a shop every day to buy bread.

On any day, the probability that Malik goes to the shop in the morning is 0.7 .

If he goes in the morning, the probability that there is bread for Malik to buy is 0.95 .

If he goes later, the probability that there is bread for Malik to buy is 0.6 .

Calculate the probability that, on any day, there is bread for Malik to buy.

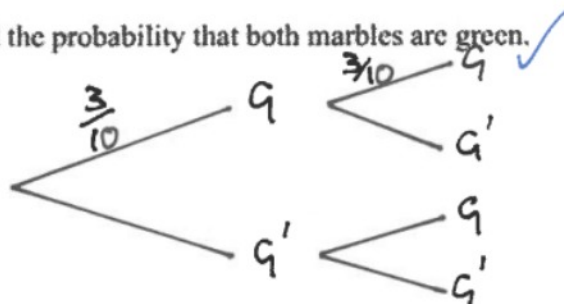


$$\begin{array}{r} 0.7 \times 0.95 = 0.665 \\ 0.3 \times 0.6 = 0.18 \quad + \\ \hline 0.845 \end{array}$$

..... 0.845 [3]

- 17 (a) A bag contains 6 red marbles, 3 green marbles and 1 blue marble.
Two marbles are picked at random from the bag **with replacement**.

Find the probability that both marbles are green. ✓

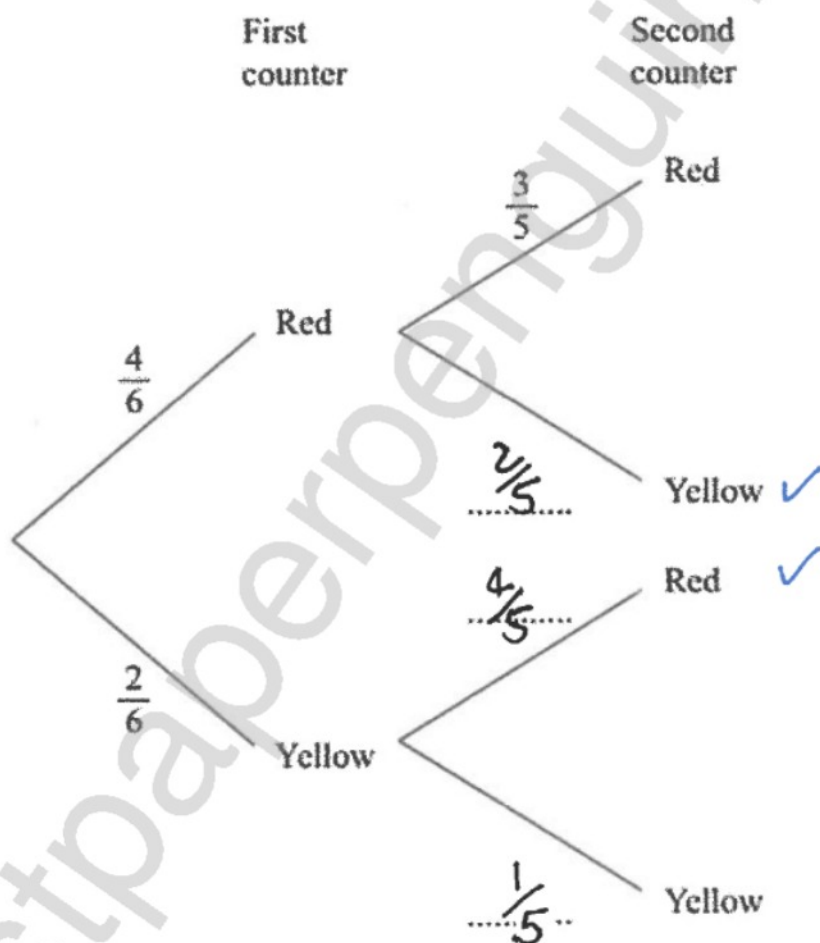


$$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$$

$$\frac{9}{100} \dots \dots \dots [2]$$

- (b) Another bag contains 4 red counters and 2 yellow counters.
Two counters are picked at random from this bag **without replacement**.

- (i) Complete the tree diagram.



- (ii) Find the probability that one of the two counters is yellow.

$$\frac{4}{6} \times \frac{2}{5} = \frac{8}{30}$$

$$\frac{2}{6} \times \frac{4}{5} = \frac{8}{30}$$

$$\frac{8}{30} + \frac{8}{30} = \frac{16}{30}$$

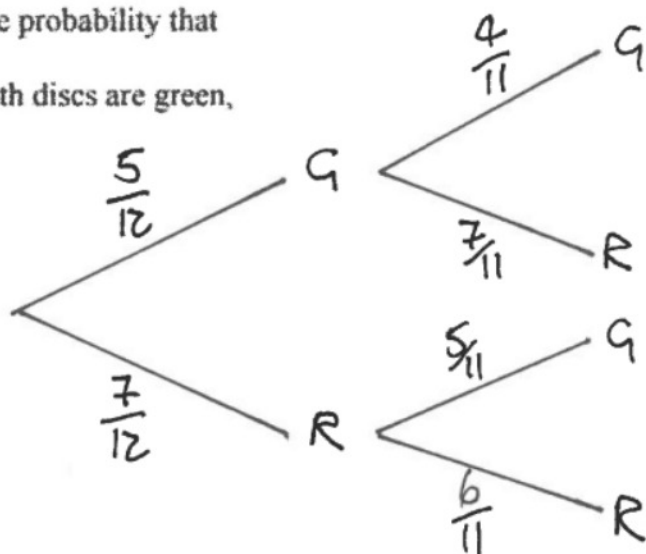
$$= \frac{8}{15}$$

$$\frac{8}{15} \dots \dots \dots [3]$$

- 12 A bag contains 12 discs.
 7 discs are red and 5 discs are green.
 A disc is picked at random and not replaced.
 A second disc is then picked at random.

Find the probability that

- (a) both discs are green,



$$\frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33}$$

$$\frac{5}{22}$$

[2]

- (b) at least one disc is green.

Easier to do $1 - P(RR)$

$$P(RR) = \frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$$

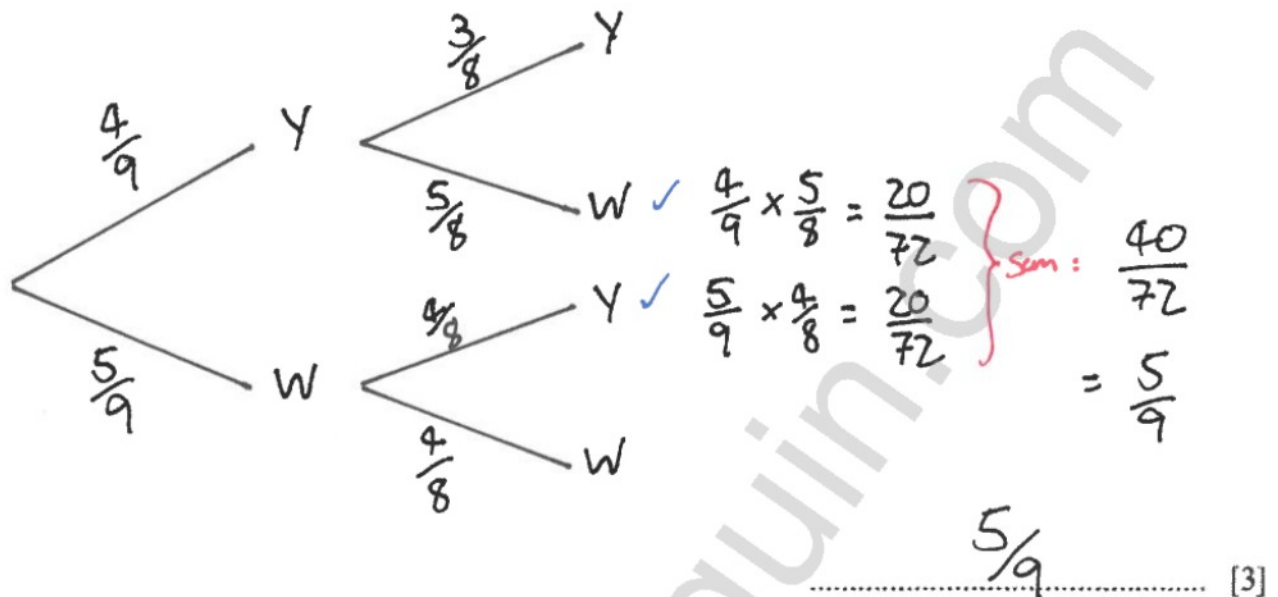
$$1 - \frac{7}{22} = \frac{15}{22}$$

$$\frac{15}{22}$$

[3]

- 13 Sara has a bag containing 4 yellow balls and 5 white balls.
Sara takes a ball from the bag at random without replacement.
She then takes a second ball from the bag at random.

Find the probability that the two balls are different colours.

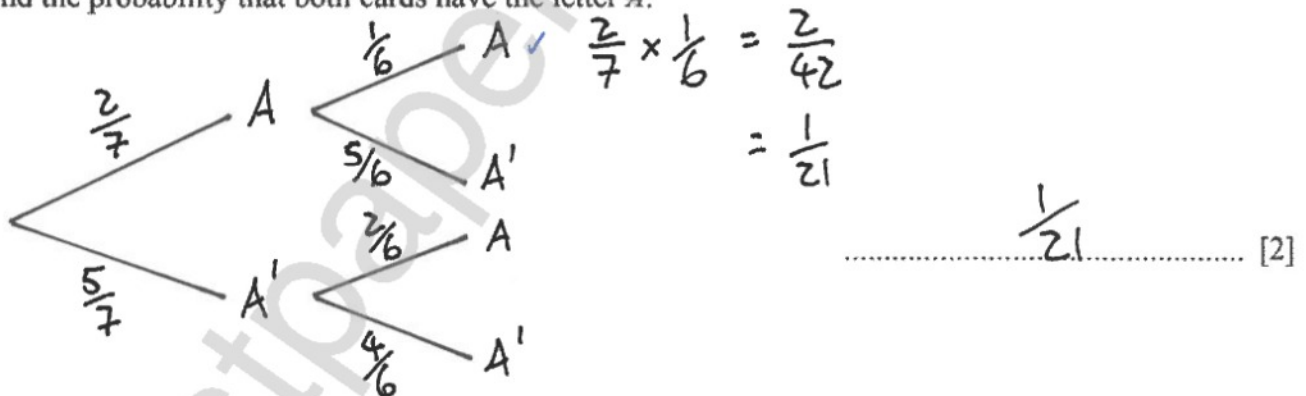


9



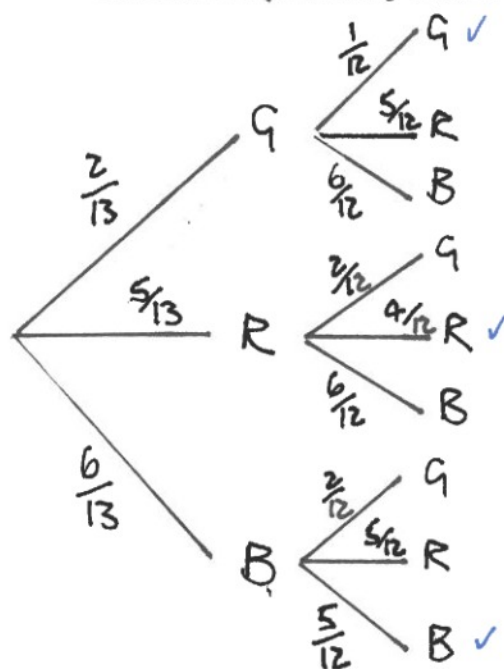
Ulrich has these cards.
He picks 2 cards at random without replacement.

Find the probability that both cards have the letter A.



- 25 A bag contains 2 green buttons, 5 red buttons and 6 blue buttons.
Two buttons are taken at random from the bag without replacement.

Calculate the probability that the two buttons are different colours.



Easier to find probability that they're the same colour and subtract from 1.

$$P(GG) = \frac{2}{13} \times \frac{1}{12} = \frac{2}{156}$$

$$P(RR) = \frac{5}{13} \times \frac{4}{12} = \frac{20}{156}$$

$$P(BB) = \frac{6}{13} \times \frac{5}{12} = \frac{30}{156}$$

$$\frac{52}{156} = \frac{1}{3}$$

Probability different colours:

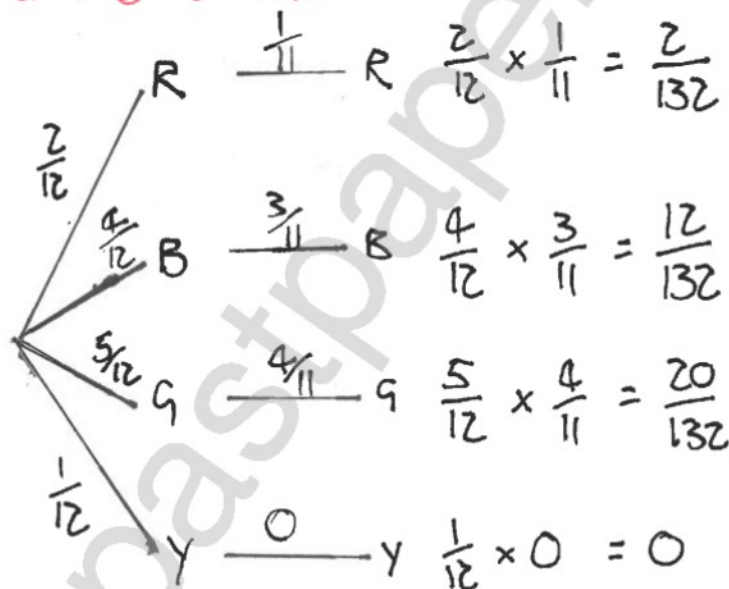
$$1 - \frac{1}{3} = \frac{2}{3}$$

[4]

- 13 A bag contains 12 discs.
There are 2 red discs, 4 blue discs, 5 green discs and 1 yellow disc.
A disc is chosen at random and not replaced.
A second disc is then chosen at random.

Find the probability that both discs are the same colour.

Whole tree diagram is too big to draw, so we'll just show the branches we need:



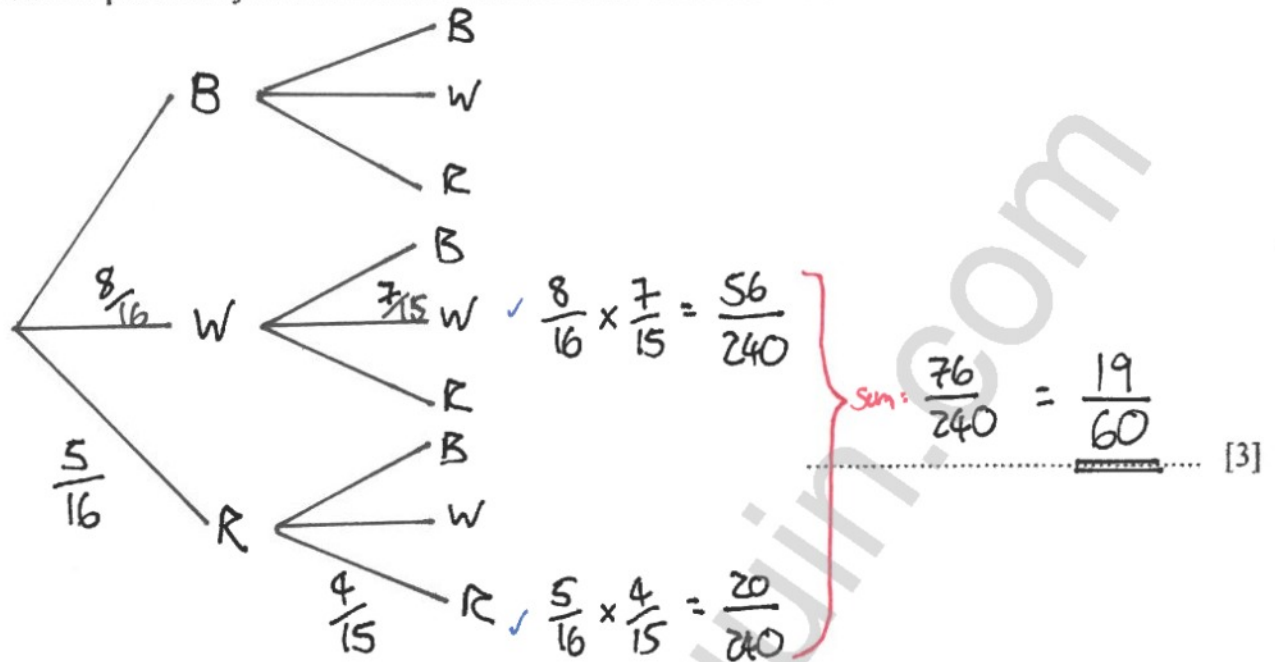
$$\text{sum: } \frac{34}{132} = \frac{17}{66}$$

$$\frac{17}{66}$$

[3]

- 17 A bag contains 3 blue buttons, 8 white buttons and 5 red buttons.
Two buttons are picked at random from the bag, without replacement.

Work out the probability that the two buttons are either both red or both white.



- 15 The height of each of 140 basketball players is recorded.
The table shows the results.

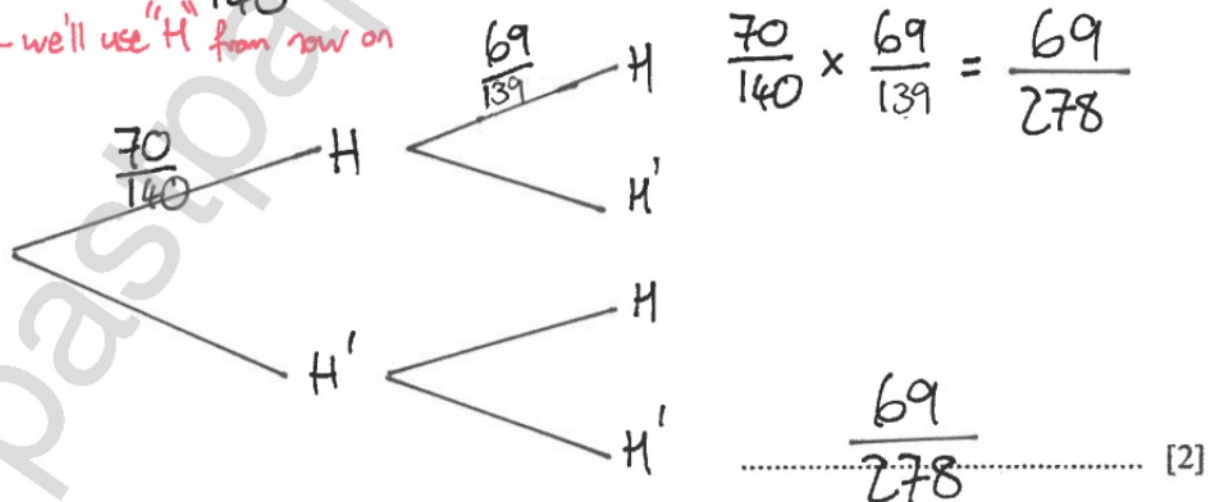
Height (h cm)	$160 < h \leq 180$	$180 < h \leq 185$	$185 < h \leq 190$	$190 < h \leq 200$	$200 < h \leq 210$
Frequency	7	12	31	70	20

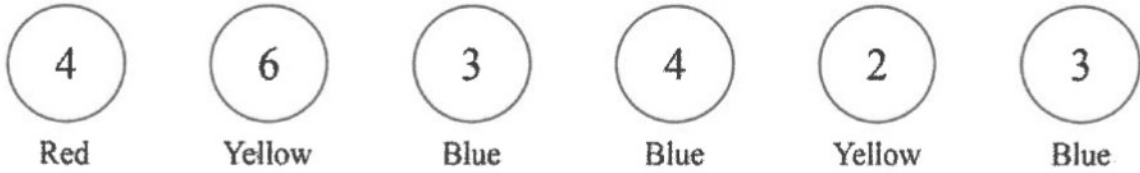
- (b) Two of the players are chosen at random.

Find the probability that both players have a height greater than 190 cm and no more than 200 cm.

$$P(190 < h \leq 200) = \frac{70}{140}$$

↑ we'll use "H" from now on



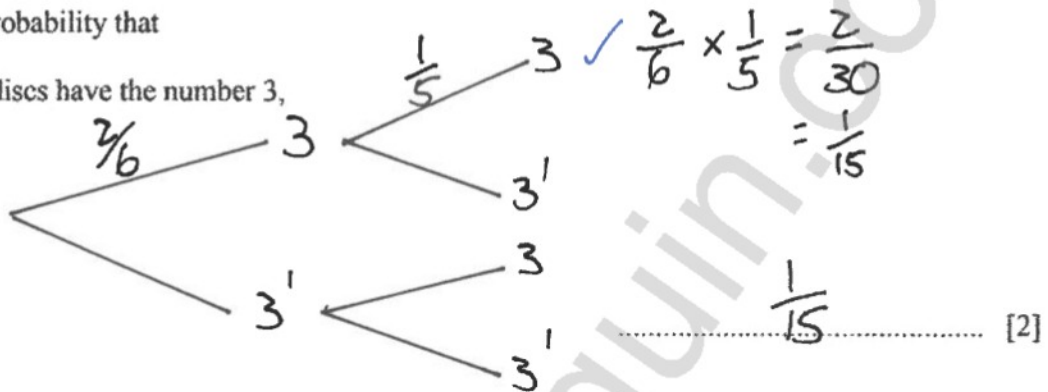


The diagram shows six discs.
Each disc has a colour and a number.

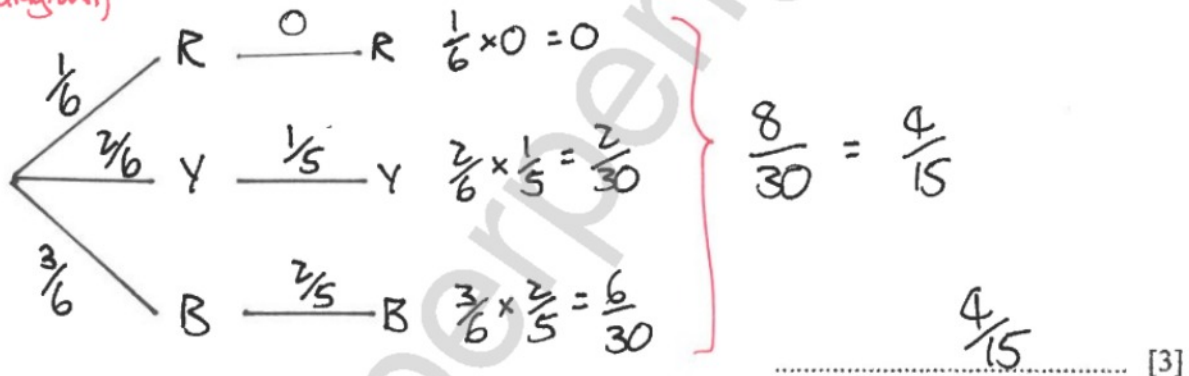
- (b) Two of the six discs are picked at random **without** replacement.

Find the probability that

- (i) both discs have the number 3,

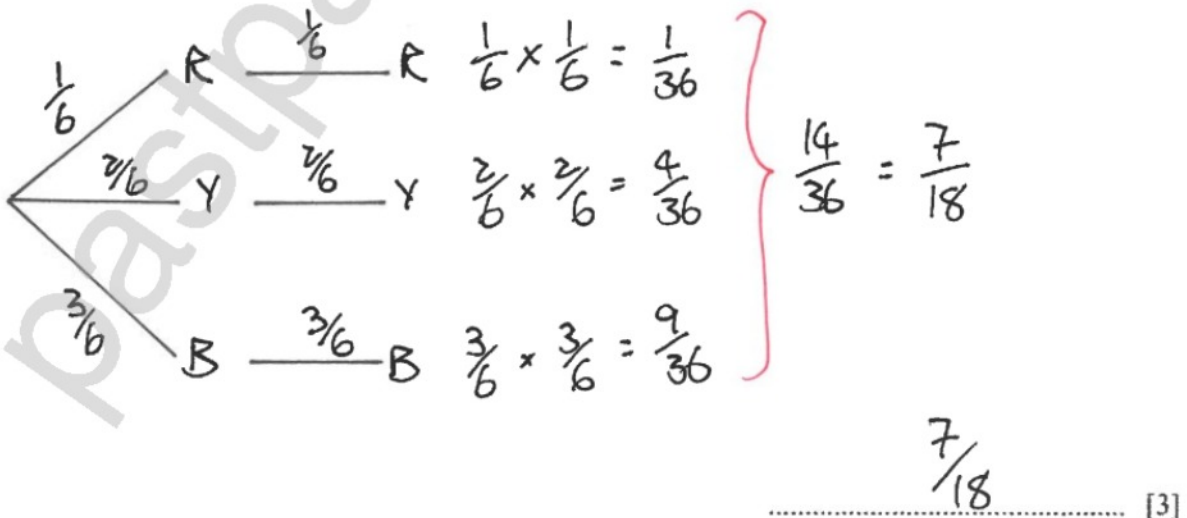


- (ii) both discs have the same colour.



- (c) Two of the six discs are picked at random **with** replacement.

Find the probability that both discs have the same colour.



- 24 The probability of Jamie hitting a target is $\frac{1}{3}$.

The probability that he hits the target for the first time on his n th attempt is $\frac{64}{2187}$.

Find the value of n .

e.g. hits it for the first time on fifth turn:

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2^4}{3^5}$$

On n th term:

$$\frac{2^{n-1}}{3^n} \rightarrow 3^n = 2187$$

trial and error:

$$3^7 = 2187, \text{ so he takes } \underline{7} \text{ turns.}$$

$$n = \underline{7} \quad [2]$$

- 23 Bag A and bag B each contain red sweets and yellow sweets.

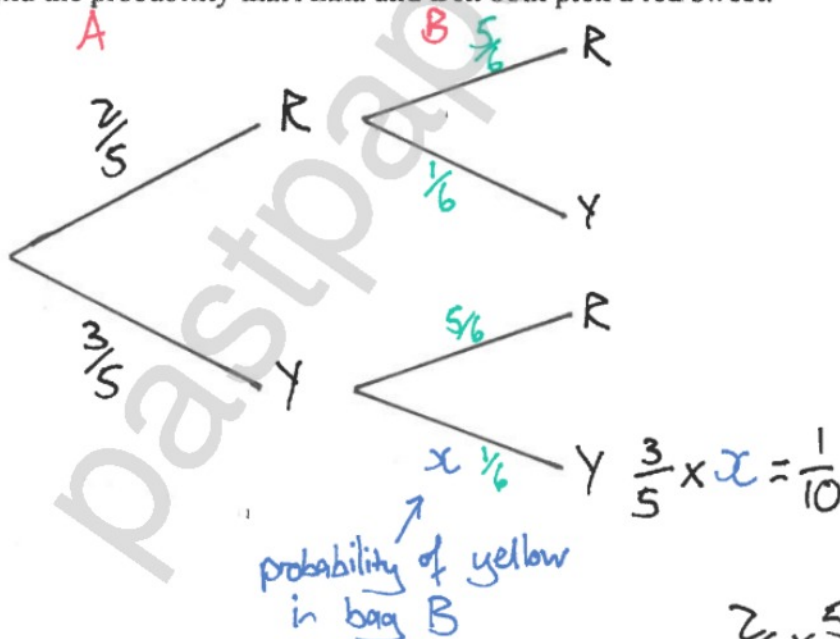
Anna picks a sweet at random from bag A.

Ben picks a sweet at random from bag B.

The probability that Anna picks a red sweet is $\frac{2}{5}$.

The probability Anna and Ben both pick a yellow sweet is $\frac{1}{10}$.

Find the probability that Anna and Ben both pick a red sweet.



$$\begin{aligned}
 x &= \frac{1}{10} \div \frac{3}{5} \\
 &= \frac{1}{10} \times \frac{5}{3} \\
 &= \frac{1}{6}
 \end{aligned}$$

so probability of Red in bag B = $\frac{5}{6}$

$$\frac{2}{5} \times \frac{5}{6} = \frac{10}{30} = \underline{\underline{\frac{1}{3}}} \quad [3]$$

25 A bag contains 5 red balls, 4 blue balls and 3 green balls.

- (a) (i) Megan picks a ball at random.

Write down the probability that the ball is red or blue.

$$\frac{9}{12} = \frac{3}{4} \quad [1]$$

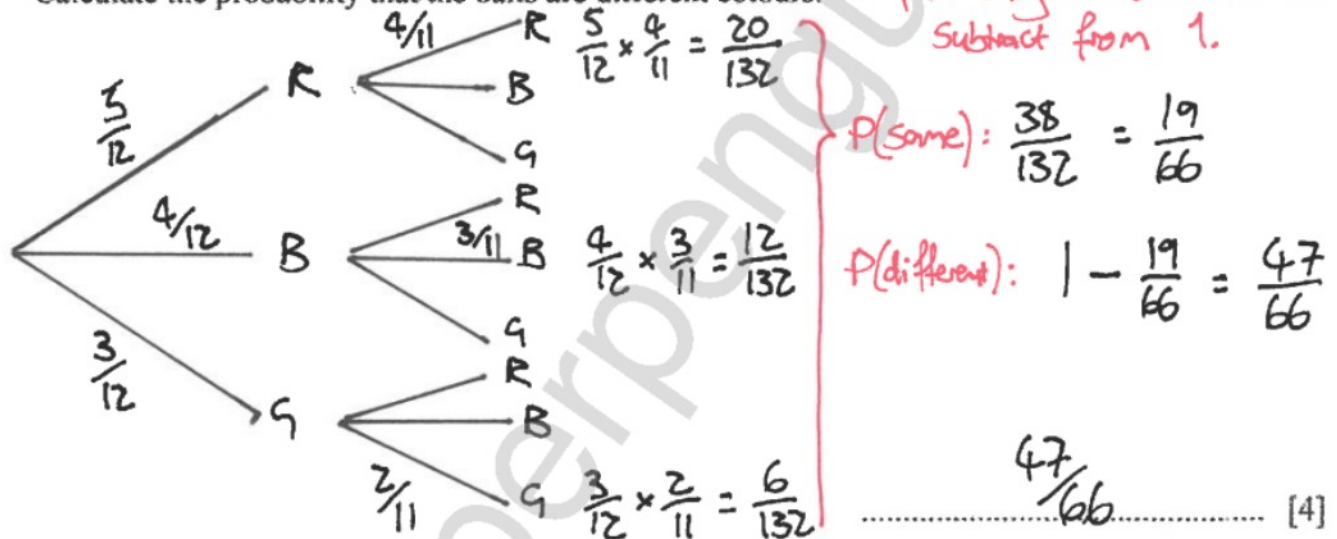
- (ii) Megan replaces the ball.
She picks a ball at random, notes the colour and replaces the ball.
She repeats this 60 times.

Calculate the number of times the ball is expected to be red or blue.

$$\frac{3}{4} \times 60 = 45 \quad [1]$$

- (b) Mick picks 2 of the 12 balls at random, without replacement.

Calculate the probability that the balls are different colours.



- (c) Marie picks balls at random, without replacement, from the 12 balls. When she picks a green ball she stops.

The probability that she picks a green ball on pick n is $\frac{21}{220}$.

Trial 1

Improvement: Find the value of n .

e.g. first green on first pick:

$$\frac{3}{12}$$

Second pick:

$$\frac{9}{12} \times \frac{3}{11} = \frac{9}{44}$$

not green

green

third:

$$\frac{9}{12} \times \frac{8}{11} \times \frac{3}{10} = \frac{9}{55}$$

fourth:

$$\frac{9}{12} \times \frac{8}{11} \times \frac{7}{10} \times \frac{3}{9} = \frac{7}{55}$$

fifth:

$$\frac{9}{12} \times \frac{8}{11} \times \frac{7}{10} \times \frac{6}{9} \times \frac{3}{8} = \frac{21}{220}$$

$$n = 5 \quad [2]$$

10 (a) Sarah spins a fair four-sided spinner numbered 0, 1, 1 and 3.

(i) What number is the spinner most likely to land on?

1 [1]

(ii) Sarah spins the spinner twice.

Find the probability that it lands on the number 1 both times.

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$\frac{1}{4}$ [2]

(iii) Sarah spins the spinner until it lands on the number 3.

The probability that this happens on the n th spin is $\frac{729}{16384}$.

Find the value of n .

$$P(3) = \frac{1}{4}$$

e.g. first 3 is on fifth spin:

$$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{3^4}{4^5}$$

$3' \quad 3' \quad 3' \quad 3' \quad 3$

n th spin:

$$\frac{3^{n-1}}{4^n}$$

$$4^n = 16384$$

trial and error:

$$4^7 = 16384$$

$n = 7$ [2]

- 7 Tanya plants some seeds.
The probability that a seed will produce flowers is 0.8 .
When a seed produces flowers, the probability that the flowers are red is 0.6 and the probability that the flowers are yellow is 0.3 .

(a) Tanya has a seed that produces flowers.

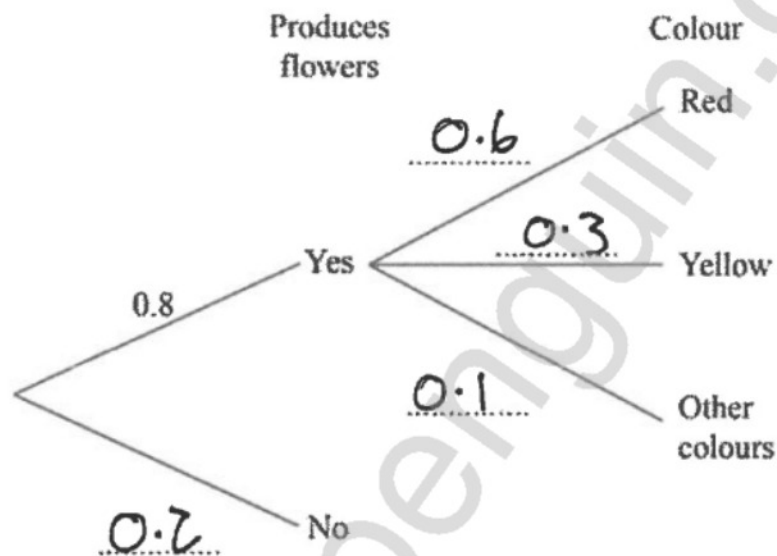
Find the probability that the flowers are not red and not yellow.

$$0.6 + 0.3 = 0.9$$

$$1 - 0.9 = 0.1$$

..... 0.1 [1]

(b) (i) Complete the tree diagram.



[2]

- (ii) Find the probability that a seed chosen at random produces red flowers.

$$0.8 \times 0.6 = 0.48$$

0.48 [2]

- (iii) Tanya chooses a seed at random.

Find the probability that this seed does not produce red flowers and does not produce yellow flowers.

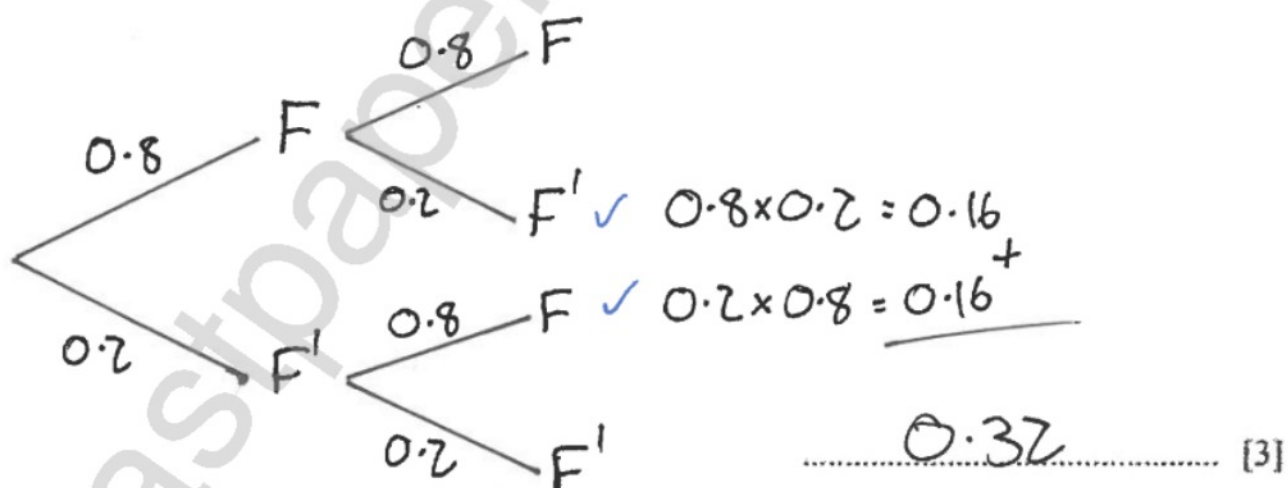
Yes, but other colours: $0.8 \times 0.1 = 0.08$

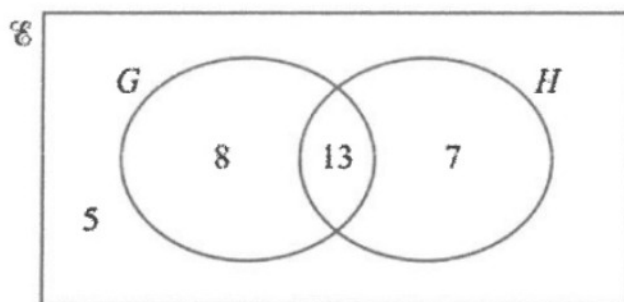
No flowers: 0.2

$$0.08 + 0.2 = 0.28 \quad [3]$$

- (c) Two of the seeds are chosen at random.

Find the probability that one produces flowers and one does not produce flowers.





$E = \{\text{number of students in a class}\}$

$G = \{\text{number of students who study geography}\}$

$H = \{\text{number of students who study history}\}$

The Venn diagram shows information about the 33 students in a class.

- (a) One of the students in the class is picked at random.

Find the probability that this student

- (i) does not study geography and does not study history

$$\frac{5}{33} \dots \dots \dots [1]$$

- (ii) studies geography and studies history.

$$\frac{13}{33} \dots \dots \dots [1]$$

- (b) Two of the students who study history are picked at random.

Find the probability that one student also studies geography and one student does not study geography.

$$\begin{array}{l}
 \begin{array}{l}
 \frac{13}{20} \rightarrow G \rightarrow \begin{array}{l} \frac{12}{19} \rightarrow G \\ \frac{7}{19} \rightarrow G' \end{array} \\
 \frac{7}{20} \rightarrow G' \rightarrow \begin{array}{l} \frac{13}{19} \rightarrow G \\ \frac{6}{19} \rightarrow G' \end{array}
 \end{array}
 \end{array}$$

$\frac{13}{20} \times \frac{7}{19} = \frac{91}{380}$
 $\frac{7}{20} \times \frac{13}{19} = \frac{91}{380}$

$\frac{182}{380} = \frac{91}{190}$
 $\frac{91}{190} \dots \dots \dots [3]$