

A Index Laws (Non-Calc)

1 Work out.

$$\begin{aligned} 2^4 &= 2 \times 2 \times 2 \times 2 \\ &= 4 \times 4 \\ &= 16 \end{aligned}$$

.....16 [1]

5 Write down the value of 9^0 .

.....1 [1]

5 Simplify $d^8 \div d^2$.

..... d^6 [1]

6 Simplify.

(a) $p^2 \times p^4$

..... p^6 [1]

(b) $m^{15} \div m^5$

..... m^{10} [1]

(c) $(k^3)^5$

..... k^{15} [1]

11 Find the value of p when $6^p \times 6^4 = 6^{28}$.

$$6^{p+4} = 6^{28}$$

$$\begin{array}{r} p+4 = 28 \\ -4 \quad -4 \end{array}$$

$$p = 24$$

$p =$ 24 [1]

4 Simplify.

$$a^2 \div a^6$$

$$\frac{a^{-4}}{\dots\dots\dots} \quad [1]$$

4 Simplify $\frac{a^3 \times a^7}{a^4} = \frac{a^{10}}{a^4}$

$$\frac{a^6}{\dots\dots\dots} \quad [1]$$

10 $5^w \div 5^{13} = 25$ $\leftarrow 5^2$

Find the value of w .

$$5^{w-13} = 5^2$$

$$\begin{array}{ccc} w-13 & = & 2 \\ +13 & & +13 \end{array}$$

$$w = 15$$

$$w = \frac{15}{\dots\dots\dots} \quad [1]$$

3 Simplify.

$$(x^8 y^7) \div (x^{-1} y^3)$$

$$\frac{x^8 y^7}{x^{-1} y^3}$$

$$\frac{x^9 y^4}{\dots\dots\dots} \quad [2]$$

6 Simplify.

(a) $y^3 \div y^5$

$$\frac{y^{-2}}{\dots\dots\dots} \quad [1]$$

(b) $7x^0 = 7 \times x^0$
 $= 7 \times 1$
 $= 7$

$$\frac{7}{\dots\dots\dots} \quad [1]$$

10 Simplify.

(a) $n^5 \times n^1$

n^6 [1]

(b) $8x^6 \div 2x^2$

$4x^4$ [2]

11 Simplify $3x^3 \times 4x^4$.

$12x^7$ [2]

9 Simplify $4x^4 \times 5x^5$.

$20x^9$ [2]

13 Simplify $8t^8 \div 4t^4$.

$2t^4$ [2]

7 Simplify.

$$30r^{30} \div 5r^5$$

$$\underline{6t^{25}} \quad [2]$$

10 Simplify $18x^{18} \div 9x^9$.

$$\underline{2x^9} \quad [2]$$

7 Simplify.

$$32g^{32} \div 4g^4$$

$$\underline{8g^{28}} \quad [2]$$

3 (a) Simplify, giving your answer as a single power of 7.

(i) $7^5 \times 7^6$

7^{11}

[1]

(ii) $7^{15} \div 7^5$

7^{10}

[1]

(iii) $42 \div 7 = 49$
 $= 7^2$

7^2

[1]

(b) Simplify.

$(5x^2 \times 2xy^4)^3$

$(10x^3y^4)^3$

$= 10^3 \times x^9 \times y^{12}$

$= 1000x^9y^{12}$

$1000x^9y^{12}$

[3]

9 Simplify $(3w^3)^3 = 3^3 \times w^9$
 $= 27w^9$

$27w^9$

[2]

5 Find the value of

(a) 64^0 ,

1

[1]

(b) $64^{\frac{1}{3}}$.

$= \sqrt[3]{64}$
 $= 4$

4

[1]

8 Find the value of $49^{\frac{1}{2}}$.

$= \sqrt{49}$
 $= 7$

7

[1]

10 Work out $16^{\frac{1}{4}}$.

$$= \sqrt[4]{16}$$
$$= 2$$

2

[1]

12 $\sqrt{\frac{10^{400}}{10^{220} \times 10^{80}}} = 10^k$

Find the value of k .

$$\sqrt{\frac{10^{400}}{10^{300}}}$$

$$= \sqrt{10^{100}}$$

$$= (10^{100})^{\frac{1}{2}}$$

$$= 10^{50}$$

$$k = 50$$

$$k = 50 \quad [3]$$

12 Work out the value of $32^{\frac{2}{5}}$.

$$(32^{\frac{1}{5}})^2$$

$$= (\sqrt[5]{32})^2$$

$$= 2^2 = 4$$

4

[1]

11 Find the value of $64^{\frac{2}{3}}$.

$$(64^{\frac{1}{3}})^2$$

$$= (\sqrt[3]{64})^2$$

$$= 4^2 = 16$$

16

[2]

(b) $7^n = \sqrt[5]{7}$

Find the value of n .

$$\sqrt[5]{7} = 7^{\frac{1}{5}}$$

$$n = \frac{1}{5} \quad [1]$$

$$\text{so } n = \frac{1}{5}$$

5 Work out $4^{\frac{3}{2}}$.

$$\begin{aligned} & (4^{\frac{1}{2}})^3 \\ & = (\sqrt{4})^3 \\ & = 2^3 = 8 \end{aligned}$$

$$8 \quad [1]$$

15 Simplify $(3125w^{3125})^{\frac{1}{5}}$.

$$\begin{aligned} & \sqrt[5]{3125} \times w^{625} \\ & 5w^{625} \end{aligned}$$

$$5w^{625} \quad [2]$$

18 Simplify $(343x^9)^{\frac{2}{3}}$.

$$\begin{aligned} & (343^{\frac{1}{3}})^2 \times x^{\frac{2}{3} \times 9} \\ & 7^2 \times x^6 \\ & 49x^6 \end{aligned}$$

$$49x^6 \quad [2]$$

17 Simplify $(81x^{12})^{\frac{3}{4}}$.

$$(81^{\frac{1}{4}})^3 \times x^{\frac{3}{4} \times 12}$$

$$3^3 \times x^9$$

$$27x^9$$

$$\underline{27x^9} \quad [2]$$

22 Simplify fully $(216y^{216})^{\frac{2}{3}}$.

$$(216^{\frac{1}{3}})^2 \times y^{216 \times \frac{2}{3}}$$

$$6^2 \times y^{144}$$

$$36y^{144}$$

$$\underline{36y^{144}} \quad [2]$$

14 Simplify $(25x^{16})^{\frac{3}{2}}$.

$$(25^{\frac{1}{2}})^3 \times x^{16 \times \frac{3}{2}}$$

$$5^3 \times x^{24}$$

$$125x^{24}$$

$$\underline{125x^{24}} \quad [2]$$

17 (a) Simplify.

$$(64y^{27})^{\frac{2}{3}}$$

$$(64^{\frac{1}{3}})^2 \times y^{27 \times \frac{2}{3}}$$

$$4^2 \times y^{18}$$

$$16y^{18}$$

$$\underline{16y^{18}} \quad [2]$$

21 Simplify fully.

$$(243y^{10})^{\frac{3}{5}}$$

$$(243^{\frac{1}{5}})^3 \times y^{10 \times \frac{3}{5}}$$

$$3^3 \times y^6$$

$$27y^6$$

$$\underline{27y^6} \quad [2]$$

21 Simplify.

(a) $(5x^4)^3$

$$5^3 \times x^{12}$$

$$125x^{12}$$

$$\underline{125x^{12}} \quad [2]$$

(b) $(256x^{256})^{\frac{3}{8}}$

$$(256^{\frac{1}{8}})^3 \times x^{256 \times \frac{3}{8}}$$

$$2^3 \times x^{96}$$

$$8x^{96}$$

$$\underline{8x^{96}} \quad [2]$$

7 Simplify.

(a) $\frac{15w^{15}}{3w^3}$

$$\underline{5w^{12}} \quad [2]$$

(b) $(125y^6)^{\frac{2}{3}}$

$$(125^{\frac{1}{3}})^2 \times y^{6 \times \frac{2}{3}}$$

$$5^2 \times y^4$$

$$25y^4$$

$$\underline{25y^4} \quad [2]$$

26 $\sqrt[3]{y^2} = \sqrt[n]{x}$ and $y = \sqrt[n]{x}$.

Find the value of n .

$$(y^2)^{\frac{1}{3}} = x^{\frac{1}{6}}$$

$$y^{\frac{2}{3}} = x^{\frac{1}{6}}$$

$$(y^{\frac{2}{3}})^{\frac{3}{2}} = (x^{\frac{1}{6}})^{\frac{3}{2}}$$

$$y^{\frac{6}{6}} = x^{\frac{3}{12}}$$

$$y^1 = x^{\frac{1}{4}}$$

$$y = \sqrt[4]{x}$$

$$n = 4 \quad [2]$$

21 (a) $\sqrt[5]{3} = 3^h$

Write down the value of h .

$$\sqrt[5]{3} = 3^{\frac{1}{5}}$$

$$h = \frac{1}{5} \quad [1]$$

(b) Simplify $(4x^3)^3$.

$$4^3 \times x^9$$

$$64x^9$$

$$64x^9 \quad [2]$$

23 (a) Simplify $\left(\frac{x^2}{4}\right)^{\frac{3}{2}}$.

$$\frac{x^{2 \times \frac{3}{2}}}{(4^{\frac{1}{2}})^3} = \frac{x^3}{2^3} = \frac{x^3}{8}$$

$$\frac{x^3}{8} \quad [2]$$

8 Write 4^{-2} as a fraction.

$$4^{-2} = \frac{1}{4^2} \\ = \frac{1}{16}$$

$$\frac{1}{16} \quad [1]$$

15 $4^x = \frac{1}{64}$

Find the value of x .

$$\frac{1}{64} = \frac{1}{4^3} \\ = 4^{-3}$$

$$x = -3 \quad [1]$$

13 (a) Simplify $h^2 \times h^5$.

$$h^7 \quad [1]$$

(b) Simplify $\left(\frac{7}{x}\right)^{-3}$.

$$\left(\frac{7}{x}\right)^{-3} = \left(\frac{x}{7}\right)^3$$

$$= \frac{x^3}{7^3} = \frac{x^3}{343}$$

$$\frac{x^3}{343} \quad [1]$$

(c) $a^8 \div a^p = a^2$

Find the value of p .

$$a^{8-p} = a^2$$

$$8-p = 2$$

$$-p = -6$$

$$p = 6$$

$$p = 6 \quad [1]$$

8 Find the value of

(a) $\left(\frac{1}{2}\right)^{-3}$,

$$\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3$$

$$= \frac{2^3}{1^3} = 8$$

$$8 \quad [1]$$

8 Find the value of $125^{-\frac{1}{3}}$.

$$125^{-\frac{1}{3}} = \frac{1}{125^{\frac{1}{3}}}$$

$$= \frac{1}{\sqrt[3]{125}}$$

$$= \frac{1}{5}$$

$$\frac{1}{5} \quad [1]$$

11 Work out $4^{-\frac{3}{2}}$.

$$= \frac{1}{4^{\frac{3}{2}}} = \frac{1}{(4^{\frac{1}{2}})^3}$$

$$= \frac{1}{2^3}$$

$\frac{1}{8}$ [2]

19 Find the value of $27^{-\frac{2}{3}}$.

$$= \frac{1}{27^{\frac{2}{3}}} = \frac{1}{(27^{\frac{1}{3}})^2}$$

$$= \frac{1}{3^2}$$

$\frac{1}{9}$ [2]

4 Work out $(64)^{-\frac{2}{3}}$.

$$= \frac{1}{64^{\frac{2}{3}}} = \frac{1}{(64^{\frac{1}{3}})^2}$$

$$= \frac{1}{4^2}$$

$\frac{1}{16}$ [2]

17 Find the value of

(a) $125^{\frac{2}{3}}$

$$(125^{\frac{1}{3}})^2 = 5^2$$

$$= 25$$

25 [2]

(b) $4^{-\frac{5}{2}}$.

$$\frac{1}{4^{\frac{5}{2}}} = \frac{1}{(4^{\frac{1}{2}})^5}$$

$$= \frac{1}{2^5}$$

$$= \frac{1}{32}$$

$\frac{1}{32}$ [2]

20 Simplify fully.

(a) $(81x^{16})^{\frac{3}{4}}$

$$(81^{\frac{1}{4}})^3 \times x^{16 \times \frac{3}{4}}$$
$$3^3 \times x^{12} = 27x^{12}$$

$27x^{12}$ [2]

(b) $\left(\frac{1}{y^2}\right)^{-\frac{1}{2}} = \left(\frac{y^2}{1}\right)^{\frac{1}{2}}$

$$= (y^2)^{\frac{1}{2}} = y^{2 \times \frac{1}{2}} = y$$

y [1]

2 (a) Simplify fully.

(i) $p^3 \times p^{11}$

p^{14} [1]

(ii) $\frac{18m^6}{3m^2}$

$6m^4$ [2]

(iii) $\left(\frac{27x^9y^{27}}{64}\right)^{-\frac{1}{3}}$

$$= \left(\frac{64}{27x^9y^{27}}\right)^{\frac{1}{3}}$$

$$= \frac{64^{\frac{1}{3}}}{27^{\frac{1}{3}} \times x^3 \times y^9}$$

$$= \frac{4}{3x^3y^9}$$

$\frac{4}{3x^3y^9}$ [3]

9 (a) Simplify.

(i) $(3x^2y^4)^3$

$$3^3 \times x^6 \times y^{12}$$
$$27x^6y^{12}$$

$$\underline{27x^6y^{12}} \quad [2]$$

(ii) $\left(\frac{16}{x^{16}y^8}\right)^{-\frac{3}{2}}$

$$\left(\frac{x^{16}y^8}{16}\right)^{\frac{3}{2}} = \frac{x^{16 \times \frac{3}{2}} \times y^{8 \times \frac{3}{2}}}{(16^{\frac{1}{2}})^3}$$

$$= \frac{x^{24} \times y^{12}}{4^3}$$

$$= \frac{x^{24}y^{12}}{64}$$

$$\underline{\frac{x^{24}y^{12}}{64}} \quad [3]$$

(b) $2^{12} \div 2^{\frac{k}{2}} = 32$

Find the value of k .

$$2^{12 - \frac{k}{2}} = 2^5$$

$$\underset{-12}{12} - \frac{k}{2} = \underset{-12}{5}$$

$$\underset{\times 2}{-\frac{k}{2}} = \underset{\times 2}{-7}$$

$$\underset{\times -1}{-k} = \underset{\times -1}{-14}$$

$$\underline{k = 14}$$

$$k = \underline{14} \quad [2]$$

13 (a) $3^{3p} \times 3^{2p} = 729 \leftarrow 3^6$

Find the value of p .

$$3^{3p+2p} = 3^6$$

$$3^{5p} = 3^6$$

$$5p = 6$$

$$\begin{array}{r} 5p = 6 \\ \div 5 \quad \div 5 \\ \hline p = \frac{6}{5} \end{array}$$

$$p = \frac{6}{5} \quad [2]$$

(b) Simplify.

$$(32x^{10})^{\frac{1}{5}}$$

$$32^{\frac{1}{5}} \times x^{10 \times \frac{1}{5}}$$

$$2 \times x^2$$

$$2x^2$$

$$\dots\dots\dots [2]$$

(c) $(243y^{20})^{\frac{2}{5}}$

$$(243^{\frac{1}{5}})^2 \times y^{20 \times \frac{2}{5}}$$

$$3^2 \times y^8$$

$$9y^8$$

$$9y^8$$

$$\dots\dots\dots [2]$$

6 Find the value of p when $2^6 \div 4^p = 2^7$.

$$2^6 \div 4^p = 2^7$$

$$\leftarrow 4 = 2^2$$

$$2^6 \div (2^2)^p = 2^7$$

$$2^6 \div 2^{2p} = 2^7$$

$$2^{6-2p} = 2^7$$

$$6-2p = 7$$

$$-2p = 1$$

$$\div -2 \quad \div -2$$

$$p = -\frac{1}{2}$$

$$p = -\frac{1}{2} \quad [3]$$

11 (a) Simplify fully.

$$(4ab^5)^4$$

$$4^4 \times a^4 \times b^{20}$$

$$256a^4b^{20}$$

$$256a^4b^{20}$$

[2]

(b) $2p^{\frac{1}{3}} = 6$

Find the value of p .

$$2p^{\frac{1}{3}} = 6$$

$$\div 2$$

$$p^{\frac{1}{3}} = 3$$

$$\sqrt[3]{p} = 3$$

$$\text{cube}$$

$$p = 3^3$$

$$= 27$$

$$p = 27$$

[1]

(c) $81^2 \div 3^t = 9$

Find the value of t .

$$81 = 3^4 \text{ and } 9 = 3^2 \text{ so:}$$

$$81^2 \div 3^t = 9 \text{ becomes...}$$

$$(3^4)^2 \div 3^t = 3^2$$

$$3^8 \div 3^t = 3^2$$

$$3^{8-t} = 3^2$$

$$8 - t = 2$$

$$-8$$

$$-t = -6$$

$$t = 6$$

$$t = 6$$

[2]

17 $3^x = 27^{x+2}$

Find the value of x .

$$27 = 3^3$$

$$3^x = (3^3)^{x+2}$$

$$3^x = 3^{3(x+2)}$$

$$x = 3(x+2)$$

$$x = 3x + 6$$

$$-x$$

$$0 = 2x + 6$$

$$0 = 2x + 6$$

$$-6$$

$$-6 = 2x$$

$$\div 2$$

$$-3 = x$$

$$x = -3$$

[2]

16

$$8^{\frac{4}{3}} = 32^x$$

$$\swarrow 32 = 2^5$$

Find the value of x .

$$8^{\frac{4}{3}} = (8^{\frac{1}{3}})^4$$

$$= 2^4$$

$$2^4 = 32^x$$

$$2^4 = (2^5)^x$$

$$2^4 = 2^{5x}$$

$$4 = 5x \quad x = \frac{4}{5} \quad [2]$$

$$\div 5 \quad \div 5$$

$$\frac{4}{5} = x$$

13 (a) Write 243×27^{2n} as a single power of 3 in terms of n .

$$= 3^5 \times (3^3)^{2n}$$

$$= 3^5 \times 3^{6n}$$

$$= \underline{3^{5+6n}}$$

$$\underline{3^{5+6n}} \quad [2]$$

18 (a) Simplify. $(4xy^2)^3$

$$4^3 \times x^3 \times y^6$$
$$64x^3y^6$$

$$\underline{64x^3y^6} \quad [2]$$

(b) $25 = 125^k$

Find the value of k .

$$25 = 5^2 \quad 125 = 5^3 :$$

$$5^2 = (5^3)^k$$

$$5^2 = 5^{3k}$$

$$2 = 3k$$

$$\div 3 \quad \div 3$$

$$\underline{\frac{2}{3} = k}$$

$$k = \underline{\frac{2}{3}} \quad [1]$$

$$m^{-\frac{1}{4}} = 27m^{-1}$$

Find the value of m .

$$m^{-\frac{1}{4}} = 27 \times \frac{1}{m}$$

$$m^{-\frac{1}{4}} = \frac{27}{m}$$

$\times m \quad \times m$

$$m^{-\frac{1}{4}} \times m^1 = 27$$

$$m^{\frac{3}{4}} = 27$$

$$(\sqrt[4]{m})^3 = 27$$

$\sqrt[4]{} \quad \sqrt[3]{}$

$$\sqrt[4]{m} = \sqrt[3]{27}$$

$$\sqrt[4]{m} = 3$$

$\times 4 \quad \times 4$

$$m = 3^4 = 81$$

$$m = 81 \quad [3]$$

(d) $5^n + 5^n + 5^n + 5^n + 5^n = 5^m$

$5 \text{ lots of } 5^n$

Find an expression for m in terms of n .

$$5 \times 5^n = 5^m$$

$$5^1 \times 5^n = 5^m$$

$$5^{1+n} = 5^m$$

$$\underline{1+n = m}$$

$$m = n + 1 \quad [2]$$

22 (a) Simplify.

$$\begin{aligned}\frac{x^{\frac{2}{3}}}{x^{\frac{8}{3}}} &= x^{\frac{2}{3}} \div x^{\frac{8}{3}} \\ &= x^{\frac{2}{3} - \frac{8}{3}} \\ &= x^{-\frac{6}{3}} \\ &= x^{-2} \quad \text{or } \frac{1}{x^2}\end{aligned}$$

x^{-2} [1]

(b) $16 = 64^k$

Find the value of k .

$$\begin{aligned}16 &= 4^2 & 64 &= 4^3 \\ 4^2 &= (4^3)^k \\ 4^2 &= 4^{3k} \\ 2 &= 3k \\ \div 3 & \quad \div 3 \\ \frac{2}{3} &= k\end{aligned}$$

$k = \frac{2}{3}$ [1]

(c) Solve.

$$3^{3x} \times \left(\frac{1}{9}\right)^{4-3x} = 3$$

$$9 = 3^2$$

$$3^{3x} \times \left(\frac{1}{3^2}\right)^{4-3x} = 3^1$$

$$3^{3x} \times (3^{-2})^{4-3x} = 3^1$$

$$3^{3x} \times 3^{-8+6x} = 3^1$$

$$3^{3x-8+6x} = 3^1$$

$$3^{9x-8} = 3^1$$

$$9x - 8 = 1$$

$$+8 \quad +8$$

$$9x = 9$$

$$\div 9 \quad \div 9$$

$$x = 1$$

$x = 1$ [3]

$$\frac{16^{5m}}{4} = 64^{2n}$$

Find m in terms of n .

$$16 = 4^2 \quad 64 = 4^3$$

$$\frac{(4^2)^{5m}}{4} = (4^3)^{2n}$$

$$\frac{4^{10m}}{4^1} = 4^{6n}$$

$$4^{10m-1} = 4^{6n}$$

$$10m - 1 = 6n$$

$$+1 \quad +1$$

$$10m = 6n + 1$$

$$\div 10 \quad \div 10$$

$$m = \frac{6n+1}{10}$$

$$m = \frac{6n+1}{10} \quad [3]$$

(b) $16^x \times \left(\frac{1}{2}\right)^x = 4^{x+3}$

Find the value of x .

$$16 = 2^4 \quad 4 = 2^2$$

$$(2^4)^x \times (2^{-1})^x = (2^2)^{x+3}$$

$$2^{4x} \times 2^{-x} = 2^{2(x+3)}$$

$$2^{3x} = 2^{2(x+3)}$$

$$3x = 2(x+3)$$

$$3x = 2x + 6$$

$$-2x \quad -2x$$

$$\underline{x = 6}$$

$$x = \underline{6} \quad [4]$$

(e) $3^{2x-1} = \frac{1}{9^x} \times 3^{2y-x}$

Find an expression for y in terms of x .

$9 = 3^2 :$

$$3^{2x-1} = \frac{1}{(3^2)^x} \times 3^{2y-x}$$

$$3^{2x-1} = \frac{1}{3^{2x}} \times 3^{2y-x}$$

$$3^{2x-1} = 3^{-2x} \times 3^{2y-x}$$

$$3^{2x-1} = 3^{-2x+2y-x}$$

$$3^{2x-1} = 3^{-3x+2y}$$

$$2x-1 = -3x+2y$$

$$5x-1 = 2y$$

$$\frac{5x-1}{2} = y$$

$$y = \frac{5x-1}{2} \quad [4]$$