

A Functions: Composite and Inverse

11 $f(x) = \frac{1}{2x-5}, \quad x \neq 2.5$

(a) Find $f(2)$.

$$f(2) = \frac{1}{2(2)-5} = \frac{1}{4-5} = \frac{1}{-1} = -1 \quad [1]$$

(b) Solve $f(x) = 5$.

$$\begin{aligned} \frac{1}{2x-5} &= 5 \\ \times (2x-5) & \quad \times (2x-5) \\ 1 &= 5(2x-5) \\ 1 &= 10x - 25 \\ +25 & \quad +25 \\ 26 &= 10x \\ \div 10 & \quad \div 10 \\ 2.6 &= x \\ x &= 2.6 \quad [2] \end{aligned}$$

14 $f(x) = 5x+2$

Find $f^{-1}(x)$.

$$\begin{aligned} y &= 5x+2 \\ -2 & \quad -2 \\ y-2 &= 5x \\ \div 5 & \quad \div 5 \\ \frac{y-2}{5} &= x \end{aligned}$$

Swap x and y :

$$\begin{aligned} y &= \frac{x-2}{5} \\ f^{-1}(x) &= \frac{x-2}{5} \quad [2] \end{aligned}$$

14 $f(x) = x^{\frac{1}{7}}$

Find $f^{-1}(x)$.

$$\begin{aligned} y &= \sqrt[7]{x} \\ y^7 &= x \end{aligned}$$

Swap x and y :

$$\begin{aligned} y &= x^7 \\ f^{-1}(x) &= x^7 \end{aligned}$$

$$f^{-1}(x) = x^7 \quad [1]$$

$$f(x) = x^3 + 1$$

Find $f^{-1}(x)$.

$$y = x^3 + 1$$

$$y - 1 = x^3$$

$$\sqrt[3]{y-1} = x$$

Swap x and y :

$$y = \sqrt[3]{x-1}$$

$$f^{-1}(x) = \sqrt[3]{x-1}$$

$$f^{-1}(x) = \sqrt[3]{x-1} \quad [2]$$

8 (a) $f(x) = 3 - 5x$

(i) Find x when $f(x) = -5$.

$$3 - 5x = -5$$

$$-5x = -8$$

$$x = \frac{8}{5}$$

$$x = \frac{8}{5} \quad [2]$$

(ii) Find $f^{-1}(x)$.

$$y = 3 - 5x$$

$$5x + y = 3$$

$$5x = 3 - y$$

$$x = \frac{3-y}{5}$$

Swap x and y :

$$y = \frac{3-x}{5}$$

$$f^{-1}(x) = \frac{3-x}{5} \quad [2]$$

4

$$f(x) = 3x - 5$$

The domain of $f(x)$ is $\{-3, 0, 2\}$.

Find the range of $f(x)$.

$$f(-3) = 3(-3) - 5$$

$$= -14$$

$$f(0) = 3(0) - 5$$

$$= -5$$

$$f(2) = 3(2) - 5$$

$$= 1$$

$$\{-14, -5, 1\} \quad [2]$$

- 17 (a) $f(x) = 3x^2 + a$ where a is an integer.
 $f(-2) = 19$

Find the value of a .

$$\begin{aligned} f(-2) &= 3(-2)^2 + a \\ &= 3 \times 4 + a \\ &= 12 + a \end{aligned}$$

$$f(-2) = 19:$$

$$\begin{aligned} 12 + a &= 19 \\ -12 \quad -12 \\ \hline a &= 7 \end{aligned}$$

$$a = \underline{7} \quad [2]$$

- (b) $g(x) = 2x + 7$ $h(x) = 3x - 8$

(i) Find $gh(x)$ in its simplest form.

$$\begin{aligned} g(h(x)) &= 2(3x - 8) + 7 \\ &= 6x - 16 + 7 \\ &= 6x - 9 \end{aligned}$$

$$\underline{6x - 9} \quad [2]$$

(ii) Find $g^{-1}(x)$.

$$\begin{aligned} y &= 2x + 7 \\ -7 \quad -7 \end{aligned}$$

$$\begin{aligned} y - 7 &= 2x \\ \div 2 \quad \div 2 \end{aligned}$$

$$\frac{y - 7}{2} = x$$

Swap x & y :

$$y = \frac{x - 7}{2}$$

$$g^{-1}(x) = \underline{\frac{x - 7}{2}} \quad [2]$$

12

$f(x) = 11x + 2$

$g(x) = \sin x^\circ$

(a) Find $f^{-1}(x)$.

$$y = 11x + 2$$

-2 -2

$$y - 2 = 11x$$

$\div 11$ $\div 11$

$$\frac{y-2}{11} = x$$

Swap x/y :

$$y = \frac{x-2}{11}$$

$$f^{-1}(x) = \frac{x-2}{11} \quad [2]$$

(b) Find $g(f(8))$.

$$\begin{aligned} f(8) &= 11(8) + 2 \\ &= 88 + 2 \\ &= 90 \end{aligned}$$

$$\begin{aligned} g(90) &= \sin(90) \\ &= 1 \end{aligned}$$

$$1 \quad [2]$$

19

$f(x) = 7x - 8$

$g(x) = \frac{4}{x} + 5$

$h(x) = 2^x + 1$

(a) Find $f^{-1}(x)$.

$$y = 7x - 8$$

$+8$ $+8$

$$y + 8 = 7x$$

$\div 7$ $\div 7$

$$\frac{y+8}{7} = x$$

Swap x/y :

$$y = \frac{x+8}{7}$$

$$f^{-1}(x) = \frac{x+8}{7} \quad [2]$$

(b) Find the value of x when $h(x) = g\left(\frac{1}{3}\right)$.

$$g\left(\frac{1}{3}\right) = \frac{4 \times 3}{\frac{1}{3} \times 3} + 5$$

$$= 12 + 5$$

$$= 17$$

$$h(x) = 17:$$

$$2^x + 1 = 17$$

-1 -1

$$2^x = 16$$

$$2^4 = 16$$

$$\text{so } x = 4$$

$$x = 4 \quad [2]$$

14 (a) $f(x) = 4x + 3$ $g(x) = 5x - 4$

$fg(x) = 20x + p$

Find the value of p .

$$\begin{aligned} f(g(x)) &= 4(5x - 4) + 3 \\ &= 20x - 16 + 3 \\ &= 20x - 13 \end{aligned}$$

$$20x - 13 = 20x + p$$

$$\rightarrow p = -13$$

$p = \underline{-13}$ [2]

(b) $h(x) = \frac{5x-1}{3}$

Find $h^{-1}(x)$.

$$y = \frac{5x-1}{3} \quad \times 3$$

$$3y = 5x - 1 \quad +1$$

$$3y + 1 = 5x \quad \div 5$$

$$x = \frac{3y+1}{5}$$

$x/y:$

$$y = \frac{3x+1}{5}$$

$h^{-1}(x) = \underline{\frac{3x+1}{5}}$ [3]

18 $f(x) = 3 - 2x$ $g(x) = 2x + 3$ $h(x) = 2^x$

(a) (i) Find $f(-3)$. $f(-3) = 3 - 2(-3)$
 $= 3 + 6$
 $= 9$

..... 9 [1]

(ii) Find $gf(-3)$.

$g(9) = 2(9) + 3$
 $= 18 + 3$
 $= 21$

..... 21 [1]

(b) Find $f^{-1}(x)$.

$y = 3 - 2x$
 $+2x$ $+2x$
 $y + 2x = 3$
 $-y$ $-y$
 $2x = 3 - y$
 $\div 2$ $\div 2$

$x = \frac{3 - y}{2}$

$x/y:$
 $y = \frac{3 - x}{2}$

$f^{-1}(x) = \frac{3 - x}{2}$ [2]

(c) Find x when $gg(x) = 7$.

$g(g(x)) = 2(2x + 3) + 3$
 $= 4x + 6 + 3$
 $= 4x + 9$

$gg(x) = 7:$

$4x + 9 = 7$

$4x + 9 = 7$
 -9 -9

$4x = -2$
 $\div 4$ $\div 4$

$x = -\frac{1}{2}$

$x = -\frac{1}{2}$ [3]

(d) Find x when $h^{-1}(x) = 5$.

In questions like this, we can do $h(x)$ to both sides to cancel out the $h^{-1}(x)$:

$h^{-1}(x) = 5$

$h h^{-1}(x) = h(5)$
 $\underbrace{h h^{-1}}_{\text{cancel out}}(x) = h(5)$
 $x = h(5)$

$x = 2^5$
 $= 32$

$x = 32$ [2]

(a) Find $f(-2)$.

$$\begin{aligned} f(-2) &= 3(-2) - 4 \\ &= -6 - 4 \\ &= -10 \end{aligned}$$

..... -10 [1]

(b) Find $f^{-1}(x)$.

$$\begin{aligned} y &= 3x - 4 \\ y + 4 &= 3x \\ \frac{y+4}{3} &= x \end{aligned}$$

$x/y:$

$$y = \frac{x+4}{3} \quad f^{-1}(x) = \frac{x+4}{3}$$

..... $\frac{x+4}{3}$ [2]

(c) $fg(x) = ax + b$ Find the value of a , and the value of b .

$$\begin{aligned} f(g(x)) &= 3(4x + 1) - 4 \\ &= 12x + 3 - 4 \\ &= 12x - 1 \end{aligned}$$

$a = 12 \quad b = -1$ [2]

(d) Simplify.

$$\frac{2}{f(x)} - \frac{5}{g(x)}$$

Give your answer as a single fraction in terms of x .

$$\frac{2}{3x-4} - \frac{5}{4x+1}$$

$$= \frac{2}{3x-4} \times \frac{(4x+1)}{(4x+1)} - \frac{5}{4x+1} \times \frac{(3x-4)}{(3x-4)}$$

$$= \frac{2(4x+1) - 5(3x-4)}{(3x-4)(4x+1)}$$

$$= \frac{8x + 2 - 15x + 20}{(3x-4)(4x+1)} = \frac{-7x + 22}{(3x-4)(4x+1)}$$

..... [3]

$f(x) = x + 1$

$g(x) = 5 - 2x$

$h(x) = 2^x$

(a) Find $f(-3)$.

$$f(-3) = -3 + 1$$

$$= -2$$

$$\dots\dots\dots -2 \dots\dots\dots [1]$$

(b) The domain of $g(x)$ is $\{-3, 0, 2\}$.Find the range of $g(x)$.

$$g(-3) = 5 - 2(-3)$$

$$= 5 + 6$$

$$= 11$$

$$g(0) = 5 - 2(0)$$

$$= 5$$

$$g(2) = 5 - 2(2)$$

$$= 5 - 4$$

$$= 1$$

$$\{ \dots\dots\dots 11, 5, 1 \dots\dots\dots \} [2]$$

(c) Find x when $h(x) = \frac{1}{32}$.

$$2^x = \frac{1}{32}$$

$$2^5 = 32$$

$$2^{-5} = \frac{1}{32}$$

$$x = \dots\dots\dots -5 \dots\dots\dots [1]$$

(d) Find x when $h^{-1}(x) = 3$.apply h to both sides:

$$h^{-1}(x) = 3$$

$$hh^{-1}(x) = h(3)$$

cancel out

$$x = h(3)$$

$$x = 2^3$$

$$= 8$$

$$x = \dots\dots\dots 8 \dots\dots\dots [2]$$

$$f(x) = 5 - 4x$$

(a) Find $f(-3)$. $f(-3) = 5 - 4(-3)$
 $= 5 + 12$
 $= 17$

..... 17 [1]

(b) Find $f(3 - 2x)$.

Give your answer in its simplest form.

$$f(3 - 2x) = 5 - 4(3 - 2x)$$

$$= 5 - 12 + 8x$$

$$= -7 + 8x$$

..... $8x - 7$ [2]

(c) Find $f^{-1}(x)$.

$$y = 5 - 4x$$

$$+4x \quad +4x$$

$$4x + y = 5$$

$$-y \quad -y$$

$$4x = 5 - y$$

$$\div 4 \quad \div 4$$

$$x = \frac{5 - y}{4}$$

x/y :

$$y = \frac{5 - x}{4}$$

$f^{-1}(x) = \frac{5 - x}{4}$ [2]

$f(x) = 6x - 7$

$g(x) = x^{-3}$

(a) Find $f(x+2)$.

Give your answer in its simplest form.

$$\begin{aligned}
 f(x+2) &= 6(x+2) - 7 \\
 &= 6x + 12 - 7 \\
 &= 6x + 5
 \end{aligned}$$

$$\underline{\quad 6x + 5 \quad} [2]$$

(b) Find $f^{-1}(x)$.

$$y = 6x - 7$$

$$y + 7 = 6x$$

$$\frac{y+7}{6} = x$$

$$\rightarrow x/y: y = \frac{x+7}{6}$$

$$f^{-1}(x) = \underline{\quad \frac{x+7}{6} \quad} [2]$$

(c) Find x when $g(x) = f(22)$.

$$\begin{aligned}
 f(22) &= 6(22) - 7 \\
 &= 132 - 7 \\
 &= 125
 \end{aligned}$$

$$x^{-3} = 125$$

$$\frac{1}{x^3} = 125$$

$$1 = 125x^3$$

$$x^3 = \frac{1}{125}$$

$$x = \sqrt[3]{\frac{1}{125}}$$

$$x = \underline{\quad \frac{1}{5} \quad} [2]$$

$$f(x) = kx^2$$

$$g(x) = \frac{1}{x}$$

$$h(x) = \frac{7x-2}{5}$$

$$j(x) = \frac{3-10x}{14}$$

(a) $f(-5k) = 675$

Find the value of k .

$$\begin{aligned} f(-5k) &= k(-5k)^2 \\ &= k \times 25k^2 \\ &= 25k^3 \end{aligned}$$

$$f(-5k) = 675$$

$$25k^3 = 675$$

$$k^3 = 27$$

$$k = \sqrt[3]{27}$$

$$k = 3$$

$$k = \underline{3} \quad [2]$$

(b) Find $gh(x)$.

$$\begin{aligned} g(h(x)) &= \frac{1}{\frac{7x-2}{5}} \\ &= \frac{5}{7x-2} \end{aligned}$$

$$\underline{\frac{5}{7x-2}} \quad [1]$$

(c) Find $h^{-1}(x) + j(x)$.

Give your answer in its simplest form.

$$h^{-1}(x):$$

$$y = \frac{7x-2}{5}$$

$$5y = 7x - 2$$

$$5y + 2 = 7x$$

$$\frac{5y+2}{7} = x$$

$$x/y: y = \frac{5x+2}{7}$$

$$h^{-1}(x) = \frac{5x+2}{7}$$

$$h^{-1}(x) + j(x):$$

$$\frac{5x+2}{7} \times \frac{2}{2} + \frac{3-10x}{14}$$

$$= \frac{2(5x+2) + (3-10x)}{14}$$

$$= \frac{10x + 4 + 3 - 10x}{14}$$

$$= \frac{7}{14}$$

$$= \underline{\frac{1}{2}}$$

$$\underline{\frac{1}{2}} \quad [4]$$

$f(x) = 3x + 2$

$g(x) = x^2 + 1$

$h(x) = 4^x$

(a) Find $h(4)$.

$$h(4) = 4^4$$

$$= 256$$

$$\underline{\hspace{1cm} 256 \hspace{1cm}} \quad [1]$$

(b) Find $fg(1)$.

$$g(1) = 1^2 + 1$$

$$= 2$$

$$f(2) = 3(2) + 2$$

$$= 6 + 2$$

$$= 8$$

$$\underline{\hspace{1cm} 8 \hspace{1cm}} \quad [2]$$

(c) Find $gf(x)$ in the form $ax^2 + bx + c$.

$$g(f(x)) = (3x + 2)^2 + 1$$

$$= (3x + 2)(3x + 2) + 1$$

$$= 9x^2 + 6x + 6x + 4 + 1$$

$$= 9x^2 + 12x + 5$$

$$\underline{\hspace{1cm} 9x^2 + 12x + 5 \hspace{1cm}} \quad [3]$$

(d) Find x when $f(x) = g(7)$.

$$g(7) = 7^2 + 1$$

$$= 49 + 1$$

$$= 50$$

$$f(x) = 50$$

$$3x + 2 = 50$$

$$\quad \quad -2 \quad -2$$

$$3x = 48$$

$$\quad \quad \div 3 \quad \div 3$$

$$\underline{x = 16}$$

$$x = \underline{\hspace{1cm} 16 \hspace{1cm}} \quad [2]$$

(e) Find $f^{-1}(x)$.

$$y = 3x + 2$$

$$\quad \quad -2 \quad -2$$

$$y - 2 = 3x$$

$$\quad \quad \div 3 \quad \div 3$$

$$\frac{y - 2}{3} = x$$

$$\rightarrow x/y:$$

$$y = \frac{x - 2}{3}$$

$$f^{-1}(x) = \underline{\hspace{1cm} \frac{x - 2}{3} \hspace{1cm}} \quad [2]$$

(f) Find $\frac{g(x)}{f(x)} + x$.

Give your answer as a single fraction, in terms of x , in its simplest form.

$$\begin{aligned} & \frac{x^2 + 1}{3x + 2} + x \\ &= \frac{x^2 + 1}{3x + 2} + \frac{x}{1} \times \frac{(3x + 2)}{(3x + 2)} \\ &= \frac{x^2 + 1 + x(3x + 2)}{3x + 2} \\ &= \frac{x^2 + 1 + 3x^2 + 2x}{3x + 2} \end{aligned}$$

$$= \frac{4x^2 + 2x + 1}{3x + 2}$$

↖ doesn't factorise

$$\frac{4x^2 + 2x + 1}{3x + 2} \dots\dots\dots [3]$$

18 $f(x) = x^2 - 25$ $g(x) = x + 4$

Solve $fg(x+1) = gf(x)$.

$$\begin{aligned} g(x+1) &= (x+1) + 4 \\ &= x + 5 \end{aligned}$$

$$\begin{aligned} fg(x+1) &= f(x+5) \\ &= (x+5)^2 - 25 \\ &= (x+5)(x+5) - 25 \\ &= x^2 + 10x + 25 - 25 \\ &= x^2 + 10x \end{aligned}$$

$$\begin{aligned} gf(x) &= (x^2 - 25) + 4 \\ &= x^2 - 21 \end{aligned}$$

$fg(x+1) = gf(x)$:

$$\begin{array}{rcl} x^2 + 10x & = & x^2 - 21 \\ -x^2 & & -x^2 \end{array}$$

$$\begin{array}{rcl} 10x & = & -21 \\ \div 10 & & \div 10 \end{array}$$

$$\underline{x = -2.1}$$

$$x = \underline{-2.1} \dots\dots\dots [4]$$

$$f(x) = \frac{1}{x}, x \neq 0$$

$$g(x) = 3x - 5$$

$$h(x) = 2^x$$

(a) Find.

(i) $gf(2)$ $f(2) = \frac{1}{2}$ $g\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right) - 5$
 $= -\frac{7}{2}$

..... $-\frac{7}{2}$ [2]

(ii) $g^{-1}(x)$ $y = 3x - 5$
 $+5$ $+5$

$y + 5 = 3x$
 $\div 3$ $\div 3$

$\frac{y+5}{3} = x$

$y/x: y = \frac{x+5}{3}$

$g^{-1}(x) = \frac{x+5}{3}$ [2]

(b) Find in its simplest form $g(x-2)$.

$g(x-2) = 3(x-2) - 5$
 $= 3x - 6 - 5$
 $= 3x - 11$

..... $3x - 11$ [2]

(c) Find the value of x when

(i) $fg(x) = 0.1$

$f(g(x)) = \frac{1}{3x-5}$

$fg(x) = 0.1:$

$\frac{1}{3x-5} = \frac{1}{10}$

$10 = 3x - 5$
 $+5$ $+5$

$15 = 3x$
 $\div 3$ $\div 3$

$5 = x$

$x = 5$ [2]

(ii) $h(x) - g(7) = 0$.

$g(7) = 3(7) - 5$
 $= 21 - 5$
 $= 16$

$h(x) - g(7) = 0:$

$2^x - 16 = 0$
 $+16$ $+16$

$2^x = 16$

$2^4 = 16$

$x = 4$ [2]

$$f(x) = 7x - 4$$

$$g(x) = \frac{2x}{x-3}, x \neq 3$$

$$h(x) = x^2$$

(a) Find $g(6)$.

$$\begin{aligned} g(6) &= \frac{2(6)}{6-3} \\ &= \frac{12}{3} \end{aligned} \quad \text{red arrow} \quad = 4$$

$$\underline{4} \quad [1]$$

(b) Find $fg(4)$.

$$\begin{aligned} g(4) &= \frac{2(4)}{4-3} \\ &= \frac{8}{1} \\ &= 8 \end{aligned} \quad \left| \quad \begin{aligned} f(8) &= 7(8) - 4 \\ &= 56 - 4 \\ &= 52 \end{aligned} \right.$$

$$\underline{52} \quad [2]$$

(c) Find $fh(x)$.

$$\begin{aligned} f(h(x)) &= 7(x^2) - 4 \\ &= 7x^2 - 4 \end{aligned}$$

$$\underline{7x^2 - 4} \quad [1]$$

(d) Find $\frac{f(x)}{2} + g(x)$.

Give your answer as a single fraction, in terms of x , in its simplest form.

$$\begin{aligned} \frac{7x-4}{2} + \frac{2x}{x-3} &= \frac{7x^2 - 21x - 4x + 12 + 4x}{2(x-3)} \\ &= \frac{7x^2 - 21x + 12}{2(x-3)} \quad \leftarrow \text{doesn't factorise} \\ &= \frac{(7x-4)(x-3) + 4x}{2(x-3)} \end{aligned}$$

$$\underline{\hspace{10em}} \quad [3]$$

(e) Find the value of x when $f(x+2) = -11$.

$$\begin{aligned} f(x+2) &= 7(x+2) - 4 \\ &= 7x + 14 - 4 \\ &= 7x + 10 \end{aligned}$$

$$f(x+2) = -11$$

$$\begin{aligned} 7x + 10 &= -11 \\ -10 \quad -10 \\ 7x &= -21 \\ \div 7 \quad \div 7 \\ x &= -3 \end{aligned}$$

$$x = \underline{-3} \quad [2]$$

$f(x) = 2x - 1$

$g(x) = x^2 + 2x$

$h(x) = 4^x$

(a) Find the value of

$$(i) \quad h(3), \quad h(3) = 4^3 \\ = 64$$

$$\underline{\hspace{1cm} 64 \hspace{1cm}} \quad [1]$$

$$(ii) \quad fh(3), \quad f(h(3)) = f(64) = 128 - 1 \\ = 2(64) - 1 = 127$$

$$\underline{\hspace{1cm} 127 \hspace{1cm}} \quad [1]$$

(b) Solve the equation $gf(x) = 0$.

$$g(f(x)) = (2x - 1)^2 + 2(2x - 1) \\ = (2x - 1)(2x - 1) + 4x - 2 \\ = 4x^2 - 2x - 2x + 1 + 4x - 2 \\ = \underline{4x^2 - 1}$$

$$gf(x) = 0:$$

$$4x^2 - 1 = 0$$

$$\begin{matrix} +1 & +1 \end{matrix}$$

$$4x^2 = 1$$

$$\begin{matrix} \div 4 & \div 4 \end{matrix}$$

$$x^2 = \frac{1}{4}$$

$$x = \pm \sqrt{\frac{1}{4}}$$

$$x = \underline{\hspace{1cm} \frac{1}{2} \hspace{1cm}} \text{ or } x = \underline{\hspace{1cm} -\frac{1}{2} \hspace{1cm}} \quad [4]$$

$$(c) \quad p^{-1}(x) = f(x)$$

Find $p(x)$.

$$p^{-1}(x) = 2x - 1$$

$$y = 2x - 1$$

$$\begin{matrix} +1 & +1 \end{matrix} \\ y + 1 = 2x$$

$$\begin{matrix} \div 2 & \div 2 \end{matrix} \\ \frac{y + 1}{2} = x$$

$$\rightarrow y/x: \quad y = \frac{x + 1}{2}$$

$$\underline{\hspace{1cm} p(x) = \frac{x + 1}{2} \hspace{1cm}} \quad [2]$$

$$6 \quad f(x) = 5x - 3$$

$$g(x) = 64^x$$

$$h(x) = \frac{2}{x+1}, \quad x \neq -1$$

(a) Find the value of

$$\begin{aligned} \text{(i)} \quad f(2) \quad f(2) &= 5(2) - 3 \\ &= 10 - 3 \\ &= 7 \end{aligned}$$

$$\dots\dots\dots 7 \dots\dots\dots [1]$$

$$\begin{aligned} \text{(ii)} \quad gf(0.5) \\ f(0.5) &= 5(0.5) - 3 \\ &= 2.5 - 3 \\ &= -0.5 \end{aligned}$$

$$\begin{aligned} g(-0.5) &= 64^{-\frac{1}{2}} \\ &= \frac{1}{64^{\frac{1}{2}}} = \frac{1}{\sqrt{64}} = \frac{1}{8} \end{aligned}$$

$$\dots\dots\dots \frac{1}{8} \dots\dots\dots [2]$$

(b) Find $h^{-1}(x)$.

$$\begin{aligned} y &= \frac{2}{x+1} \\ x(x+1) & \quad x(x+1) \\ y(x+1) &= 2 \\ xy + y &= 2 \\ -y & \quad -y \end{aligned}$$

$$\begin{aligned} xy &= 2 - y \\ \div y & \quad \div y \\ x &= \frac{2-y}{y} \\ y &= \frac{2-x}{x} \end{aligned}$$

$$h^{-1}(x) = \frac{2-x}{x} \dots\dots\dots [3]$$

(c) Find x when $g(x) = \frac{1}{2^5}$.

$$\begin{aligned} 64^x &= \frac{1}{2^5} \\ 64^x &= 2^{-5} \\ 64 &= 2^6 \end{aligned}$$

$$\begin{aligned} (2^6)^x &= 2^{-5} \\ 2^{6x} &= 2^{-5} \\ 6x &= -5 \\ \div 6 & \quad \div 6 \\ x &= -\frac{5}{6} \end{aligned}$$

$$x = -\frac{5}{6} \dots\dots\dots [2]$$

(d) Write as a single fraction in its simplest form $\frac{1}{f(x)} - h(x)$.

$$\begin{aligned} &\frac{1}{5x-3} - \frac{2}{x+1} \\ &= \frac{1}{5x-3} \times \frac{(x+1)}{(x+1)} - \frac{2}{x+1} \times \frac{(5x-3)}{(5x-3)} \\ &= \frac{x+1 - 2(5x-3)}{(5x-3)(x+1)} \end{aligned}$$

$$\begin{aligned} &= \frac{x+1 - 10x + 6}{(5x-3)(x+1)} \\ &= \frac{-9x + 7}{(5x-3)(x+1)} \end{aligned}$$

$$\dots\dots\dots \frac{7-9x}{(5x-3)(x+1)} \dots\dots\dots [4]$$

$f(x) = x^2$

$g(x) = \frac{x+5}{2}$

$h(x) = 7x - 3$

(a) Find $f(-3)$.

$$f(-3) = (-3)^2$$

$$= 9$$

9

[1]

(b) Find $g^{-1}(x)$.

$$y = \frac{x+5}{2}$$

$$2y = x + 5$$

$$2y - 5 = x$$

$$x/y: y = 2x - 5$$

$$g^{-1}(x) = 2x - 5$$
 [2]

(c) Solve $gf(x) = hh^{-1}(63)$ where $x > 0$.

$$hh^{-1}(63):$$

$$hh^{-1}(63) = 63$$

cancel each other out

$$gf(x) = 63:$$

$$\frac{x^2+5}{2} = 63$$

$$x^2 + 5 = 126$$

$$x^2 = 121$$

$$x = \pm 11$$

$$x = 11$$
 [3]

but $x > 0$

$$f(x) = 2^{x-3}$$

$$g(x) = 2x - 1$$

$$h(x) = \frac{5}{x-4}$$

(a) Find $ff(6)$.

$$\begin{aligned} f(6) &= 2^{6-3} \\ &= 2^3 \\ &= 8 \end{aligned}$$

$$\begin{aligned} f(f(6)) &= f(8) \\ &= 2^{8-3} \\ &= 2^5 \\ &= 32 \end{aligned}$$

$$\underline{\hspace{1cm} 32 \hspace{1cm}} \quad [2]$$

(b) Find $g^{-1}g(x+21)$.

$$\underbrace{g^{-1}g}_{\text{cancel out}}(x+21) = x+21$$

$$\underline{\hspace{1cm} x+21 \hspace{1cm}} \quad [1]$$

(c) Find x when $f(x) = h(84)$.

$$\begin{aligned} h(84) &= \frac{5}{84-4} \\ &= \frac{5}{80} \\ &= \frac{1}{16} \end{aligned}$$

$$f(x) = h(84):$$

$$2^{x-3} = \frac{1}{16}$$

$$2^{x-3} = \frac{1}{2^4}$$

$$2^{x-3} = 2^{-4}$$

$$\begin{aligned} x-3 &= -4 \\ +3 & \quad +3 \\ \underline{x} &= -1 \end{aligned}$$

$$x = \underline{\hspace{1cm} -1 \hspace{1cm}} \quad [2]$$

$$f(x) = 5x - 3, x > 1$$

$$g(x) = \frac{10}{x-2}, x \neq 2$$

(a) Find $gf(x)$.

Give your answer in its simplest form.

$$g(f(x)) = \frac{10}{(5x-3)-2}$$

$$= \frac{10}{5x-3-2}$$

$$= \frac{10}{5x-5} \quad \div 5$$

$$= \frac{2}{x-1}$$

$$\frac{2}{x-1} \quad [2]$$

(b) Find $g^{-1}(x)$.

$$y = \frac{10}{x-2} \quad \times(x-2)$$

$$y(x-2) = 10$$

$$xy - 2y = 10$$

$$+2y \quad +2y$$

$$xy = 10 + 2y \quad \div y$$

$$x = \frac{10 + 2y}{y}$$

$$y = \frac{10 + 2x}{x} \quad \times/y$$

$$g^{-1}(x) = \frac{10 + 2x}{x} \quad [3]$$

(c) Find $ff^{-1}(x-1)$.

$$ff^{-1}(x-1) = x-1$$

cancel out

$$x-1 \quad [1]$$

$f(x) = 2x + 5$

$g(x) = x - 4$

$h(x) = 5^x$

(a) Find $f(3)$.

$$\begin{aligned} f(3) &= 2(3) + 5 \\ &= 6 + 5 \\ &= 11 \end{aligned}$$

$$\frac{11}{\dots\dots\dots} [1]$$

(b) Find $f^{-1}(x)$.

$$y = 2x + 5$$

$$y - 5 = 2x$$

$$\frac{y - 5}{2} = x$$

$$y = \frac{x - 5}{2}$$

$$f^{-1}(x) = \frac{x - 5}{2} \dots\dots\dots [2]$$

(c) Solve $fg(x) = 25$.

$$\begin{aligned} f(g(x)) &= 2(x - 4) + 5 \\ &= 2x - 8 + 5 \\ &= 2x - 3 \end{aligned}$$

$$f(g(x)) = 25:$$

$$2x - 3 = 25$$

$$2x = 28$$

$$x = 14$$

$$x = 14 \dots\dots\dots [3]$$

(d) Find x when $h^{-1}(x) = 2$.

$$h^{-1}(x) = 2$$

apply h to both sides:

$$hh^{-1}(x) = h(2)$$

cancel out

$$x = h(2)$$

$$\begin{aligned} x &= 5^2 \\ &= 25 \end{aligned}$$

$$x = 25 \dots\dots\dots [2]$$

11 $f(x) = 1 - 3x$ $g(x) = (x-1)^2$ $h(x) = \frac{3}{x}, x \neq 0$

(a) Find $g(3)$. $g(3) = (3-1)^2$
 $= 2^2 = 4$ [1]

(b) Find $f(x-2)$, giving your answer in its simplest form.

$f(x-2) = 1 - 3(x-2)$
 $= 1 - 3x + 6$
 $= 7 - 3x$ [2]

(c) Find $f^{-1}(x)$.

$y = 1 - 3x$
 $+3x$ $+3x$
 $y + 3x = 1$
 $-y$ $-y$
 $3x = 1 - y$
 $\div 3$ $\div 3$

$x = \frac{1-y}{3}$
 $x/y:$
 $y = \frac{1-x}{3}$
 $f^{-1}(x) = \frac{1-x}{3}$ [2]

(d) $gf(x) - g(x)f(x) = 3x^3 + ax^2 + bx + c$

Find the value of each of a , b and c .

$gf(x):$
 $g(f(x)) = [(1-3x)-1]^2$
 $= [-3x]^2$
 $= 9x^2$

$g(x)f(x):$
 $g(x)f(x) = (x-1)^2(1-3x)$
 $= (x-1)(x-1)(1-3x)$
 $= (x-1)[x-3x^2-1+3x]$
 $= (x-1)[-3x^2+4x-1]$
 $= -3x^3+4x^2-x+3x^2-4x+1$
 $= -3x^3+7x^2-5x+1$

$gf(x) - g(x)f(x):$
 $9x^2 - [-3x^3+7x^2-5x+1]$
 $= 9x^2 + 3x^3 - 7x^2 + 5x - 1$
 $= 3x^3 + 2x^2 + 5x - 1$

$a = 2$
 $b = 5$
 $c = -1$ [5]

(f) $h(x^n) = 3x^7$

Find the value of n .

$h(x^n) = \frac{3}{x^n}$
 $= 3x^{-n}$

$h(x^n) = 3x^7:$
 $3x^{-n} = 3x^7$
 $-n = 7$
 $n = -7$

$n = -7$ [1]

10 $f(x) = x^2 + 1$ $g(x) = 1 - 2x$ $h(x) = \frac{1}{x}, x \neq 0$ $j(x) = 5^x$

(a) Find the value of

(i) $f(3), f(3) = 3^2 + 1$
 $= 9 + 1$
 $= 10$

10 [1]

(ii) $gf(3), g(f(3)) = g(10)$
 $= 1 - 2(10) = -19$

-19 [1]

(b) Find $g^{-1}(x)$.

$y = 1 - 2x$
 $+2x \quad +2x$
 $y + 2x = 1$
 $-y \quad -y$
 $2x = 1 - y$
 $\div 2 \quad \div 2$
 $x = \frac{1 - y}{2}$
 $x(y): y = \frac{1 - x}{2}$

$g^{-1}(x) = \frac{1 - x}{2}$ [2]

(c) Find x when $h(x) = 2$.

$\frac{1}{x} = 2$

$1 = 2x$

$\frac{1}{2} = x$

$x = \frac{1}{2}$ [1]

(d) Find $g(x)g(x) - gg(x)$, giving your answer in the form $ax^2 + bx + c$.

$g(x)g(x):$

$g(x)g(x) = (1 - 2x)(1 - 2x)$
 $= 1 - 2x - 2x + 4x^2$
 $= 4x^2 - 4x + 1$

$g(x)g(x) - gg(x):$

$4x^2 - 4x + 1 - (4x - 1)$
 $= 4x^2 - 4x + 1 - 4x + 1$
 $= 4x^2 - 8x + 2$

$gg(x):$

$g(g(x)) = 1 - 2(1 - 2x)$
 $= 1 - 2 + 4x$
 $= -1 + 4x$
 $= 4x - 1$

$4x^2 - 8x + 2$ [4]

(e) Find $hh(x)$, giving your answer in its simplest form.

$h(h(x)) = \frac{1}{\frac{1}{x}} = \frac{x}{1} = x$

x [1]

(f) Find $j(5)$.

$j(5) = 5^5$
 $= 3125$

3125 [1]

9 $f(x) = (3x+1)(x+5)(x-4)$ $g(x) = 2x-3$ $h(x) = 4^{2x-1}$

(a) Find

(i) $f(0)$ $f(0) = (0+1)(0+5)(0-4)$
 $= 1 \times 5 \times -4$
 $= -20$

..... -20 [1]

(ii) $g^{-1}(x)$
 $y = 2x - 3$
 $+3 \quad +3$
 $y+3 = 2x$
 $\div 2 \quad \div 2$

$x = \frac{y+3}{2}$
 $x/y:$
 $y = \frac{x+3}{2}$

$g^{-1}(x) = \frac{x+3}{2}$ [2]

(iii) $gh(2)$.
 $h(2) = 4^{2(2)-1}$
 $= 4^{4-1}$
 $= 4^3 = 64$

$gh(2):$
 $g(h(2)) = g(64)$
 $= 2(64) - 3$
 $= 128 - 3$
 $= 125$

..... 125 [2]

(b) $g(2x) = 7$

Find the value of x .

$g(2x) = 2(2x) - 3$
 $= 4x - 3$

$g(2x) = 7:$
 $4x - 3 = 7$
 $+3 \quad +3$
 $4x = 10$
 $\div 4 \quad \div 4$

$x = \frac{10}{4}$
 $= \frac{5}{2}$
 $x = \frac{5}{2}$ [2]

(c) Simplify $g(x^2) + gg(x) + 1$.

$g(x^2):$
 $g(x^2) = 2(x^2) - 3$
 $= 2x^2 - 3$

$gg(x):$
 $g(g(x)) = 2(2x-3) - 3$
 $= 4x - 6 - 3$
 $= 4x - 9$

$g(x^2) + gg(x) + 1:$
 $2x^2 - 3 + 4x - 9 + 1$
 $= 2x^2 + 4x - 11$

..... $2x^2 + 4x - 11$ [3]

7

$$f(x) = 10 - x$$

$$g(x) = \frac{2}{x}, x \neq 0$$

$$h(x) = 2^x$$

$$j(x) = 5 - 2x$$

(a) (i) Find $g\left(\frac{1}{2}\right) \cdot g\left(\frac{1}{2}\right) = \frac{2}{\frac{1}{2} \times 2} = \frac{4}{1} = 4$

4

[1]

(ii) Find $hg\left(\frac{1}{2}\right) \cdot hg\left(\frac{1}{2}\right) = h(4)$
 $= 2^4 = 16$

16

[1]

(b) Find x when $f(x) = 7$.

$$\begin{array}{r} 10 - x = 7 \\ -10 \quad -10 \\ \hline -x = -3 \end{array} \quad \rightarrow \quad x = 3$$

x = 3

[1]

(c) Find x when $g(x) = h(3)$.

$$\begin{array}{l} h(3) = 2^3 \\ = 8 \end{array} \quad \left| \quad \begin{array}{l} g(x) = h(3) \\ \frac{2}{x} = 8 \\ 2 = 8x \\ \div 8 \quad \div 8 \end{array} \right. \quad \rightarrow \quad x = \frac{1}{4}$$

x = 1/4

[2]

(d) Find $j^{-1}(x)$.

$$\begin{array}{l} y = 5 - 2x \\ +2x \quad +2x \\ \hline 2x + y = 5 \\ -y \quad -y \\ \hline 2x = 5 - y \\ \div 2 \quad \div 2 \\ x = \frac{5 - y}{2} \end{array} \quad \xrightarrow{x/y} \quad \begin{array}{l} 2x = 5 - y \\ \div 2 \quad \div 2 \\ x = \frac{5 - y}{2} \\ y = \frac{5 - x}{2} \end{array}$$

$$j^{-1}(x) = \frac{5 - x}{2}$$

[2]

(e) Write $f(x) + g(x) + 1$ as a single fraction in its simplest form.

$$\begin{aligned} 10 - x + \frac{2}{x} + 1 &= \frac{10x - x^2 + 2 + x}{x} \\ = \frac{10 - x}{1} \times \frac{x}{x} + \frac{2}{x} + \frac{1}{1} \times \frac{x}{x} &= \frac{-x^2 + 11x + 2}{x} \quad \leftarrow \text{doesn't factorise} \\ = \frac{x(10 - x) + 2 + x}{x} & \end{aligned}$$

[3]

(f) $(f(x))^2 - ff(x) = ax^2 + bx + c$

Find the values of a , b and c .

$(f(x))^2$:

$$\begin{aligned}(f(x))^2 &= (10-x)^2 \\ &= (10-x)(10-x) \\ &= 100 - 10x - 10x + x^2 \\ &= \underline{x^2 - 20x + 100}\end{aligned}$$

$ff(x)$:

$$\begin{aligned}f(f(x)) &= 10 - (10-x) \\ &= 10 - 10 + x \\ &= \underline{x}\end{aligned}$$

$(f(x))^2 - ff(x)$:

$$\begin{aligned}x^2 - 20x + 100 - x \\ = \underline{x^2 - 21x + 100}\end{aligned}$$

$a = \underline{1}$

$b = \underline{-21}$

$c = \underline{100}$ [4]