

A Forming and Solving Quadratics using Formula or by Factorising

- 19 Solve the equation $x^2 + 5x - 7 = 0$.

You must show all your working and give your answers correct to 2 decimal places.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)}$$
$$= \frac{-5 \pm \sqrt{53}}{2}$$

$$x = 1.14 \text{ or } x = -6.14$$

$$x = \dots 1.14 \dots \text{ or } x = \dots -6.14 \dots [4]$$

- (c) Solve $x^3 + 4x^2 - 17x = x^3 - 9$.

You must show all your working and give your answers correct to 2 decimal places.

$$x^3 + 4x^2 - 17x = x^3 - 9$$

$$4x^2 - 17x = -9$$

$$4x^2 - 17x + 9 = 0$$

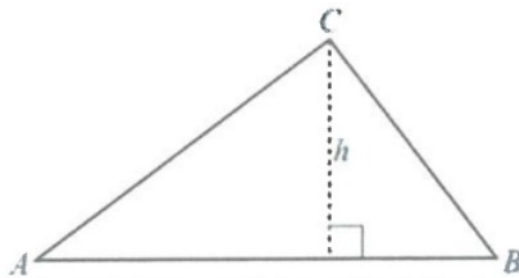
$$x = \frac{17 \pm \sqrt{(-17)^2 - 4(4)(9)}}{2(4)}$$

$$= \frac{17 \pm \sqrt{145}}{8}$$

$$x = 3.63 \text{ or } x = 0.62$$

$$x = \dots 3.63 \dots \text{ or } x = \dots 0.62 \dots [5]$$

(b)



NOT TO
SCALE

$AB = (2x + 3)$ cm and $h = (x + 5)$ cm.

The area of triangle $ABC = 50$ cm².

Find the value of x , giving your answer correct to 2 decimal places.
You must show all your working.

$$\text{Area} = \frac{1}{2} \times b \times h :$$

$$50 = \frac{1}{2} \times (2x + 3) \times (x + 5)$$

$\times 2 \qquad \times 2$

$$100 = (2x + 3)(x + 5)$$

$$100 = 2x^2 + 10x + 3x + 15$$

$$100 = 2x^2 + 13x + 15$$

$-100 \qquad -100$

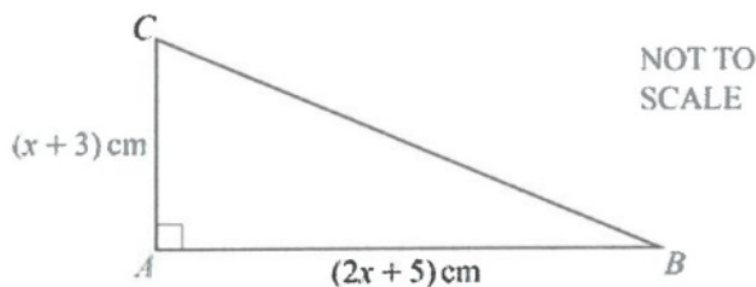
$$2x^2 + 13x - 85 = 0$$

$$x = \frac{-13 \pm \sqrt{13^2 - 4(2)(-85)}}{2(2)}$$
$$= \frac{-13 \pm \sqrt{849}}{4}$$

$$x = 4.03 \text{ or } x = -10.53$$

↑
can't be this
because lengths
can't be negative

$$x = 4.03 \dots \dots \dots [6]$$



The diagram shows a right-angled triangle ABC .

- (a) (i) The area of the triangle is 60 cm^2 .

Show that $2x^2 + 11x - 105 = 0$.

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$60 = \frac{1}{2} \times (2x+5) \times (x+3)$$

$\times 2 \quad \times 2$

$$120 = (2x+5)(x+3)$$

$$120 = 2x^2 + 6x + 5x + 15$$

$$120 = 2x^2 + 11x + 15$$

$-120 \quad -120$

$$2x^2 + 11x - 105 = 0$$

[3]

- (ii) Solve by factorisation.

$$2x^2 + 11x - 105 = 0$$

$$a \cdot c = -210$$

two numbers: 21, -10

$$\rightarrow 2x^2 - 10x + 21x - 105 = 0$$

$$2x(x-5) + 21(x-5) = 0$$

$$(2x+21)(x-5) = 0$$

$$2x+21=0 \quad x-5=0$$

$$2x = -21$$

$$x = 5$$

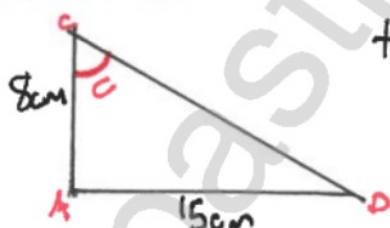
$$x = -10.5$$

$$x = -10.5 \text{ or } x = 5 \quad [3]$$

- (iii) Calculate angle ACB . (trigonometry required)

x can't be -10.5 because lengths can't be negative.

Sub. $x = 5$:

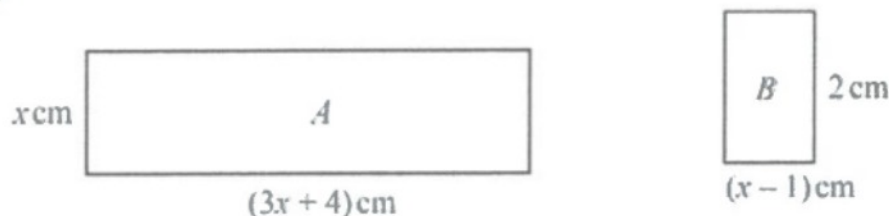


$$\tan C = \frac{15}{8}$$

$$C = \tan^{-1}\left(\frac{15}{8}\right)$$

$$C = 61.9^\circ \quad [3]$$

9 (a)



NOT TO
SCALE

The total of the areas of rectangles A and B is 20 cm^2 .

(i) Show that $3x^2 + 6x - 22 = 0$.

A: Area = $x(3x + 4)$
 $= 3x^2 + 4x$

B: Area = $2(x - 1)$
 $= 2x - 2$

$A + B = 20$

$3x^2 + 4x + 2x - 2 = 20$

$3x^2 + 6x - 2 = 20$

$3x^2 + 6x - 22 = 0$

[2]

(ii) Solve the equation $3x^2 + 6x - 22 = 0$, giving your answers correct to 4 significant figures. You must show all your working.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

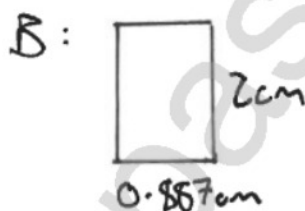
$$= \frac{-6 \pm \sqrt{300}}{6}$$

$x = 1.887$ or $x = -3.887$

$x = 1.887$ or $x = -3.887$ [4]

(iii) Find the perimeter of rectangle B.

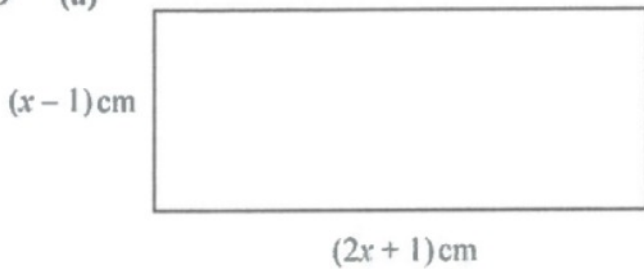
x can't be -3.887 , so sub. $x = 1.887$.



$P = 2 + 2 + 0.887 + 0.887$
 $= 5.774 \text{ cm}$

5.77 cm [1]
~~(34)~~

9 (a)



The area of the rectangle is 29 cm^2 greater than the area of the square.
The difference between the perimeters of the two shapes is $k\text{ cm}$.

Find the value of k .
You must show all your working.

Rectangle:

$$\begin{aligned}\text{Area} &= (x-1)(2x+1) \\ &= 2x^2 + x - 2x - 1 \\ &= 2x^2 - x - 1\end{aligned}$$

Square:

$$\text{Area} = x^2$$

Rectangle = Square + 29:

$$2x^2 - x - 1 = x^2 + 29$$

$-x^2$ $-x^2$

$$x^2 - x - 1 = 29$$

-29 -29

$$x^2 - x - 30 = 0$$

$$(x+5)(x-6) = 0$$

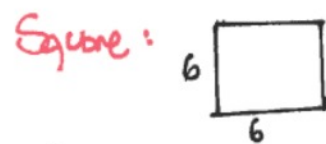
$$x+5=0 \text{ or } x-6=0$$

$$x=-5 \quad x=6$$

lengths can't
be negative



$$\begin{aligned}\text{Perimeter} &= 11+11+5+5 \\ &= 32\text{ cm}\end{aligned}$$

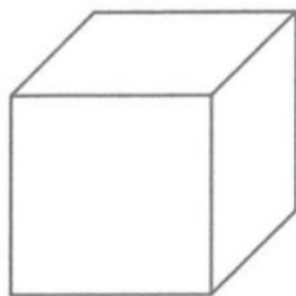


$$\text{Perimeter} = 4 \times 6 = 24\text{ cm}$$

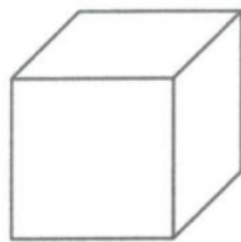
$$\text{Difference: } 32 - 24 = 8$$

$$k = 8 \quad [6]$$

(b)



$(y+1)$ cm



y cm

NOT TO
SCALE

The volume of the larger cube is 5 cm^3 greater than the volume of the smaller cube.

(i) Show that $3y^2 + 3y - 4 = 0$.

Large cube:

$$\begin{aligned}\text{Volume} &= (y+1)(y+1)(y+1) \\ &= (y+1)(y^2 + 2y + 1) \\ &= y^3 + 2y^2 + y + y^2 + 2y + 1 \\ &= y^3 + 3y^2 + 3y + 1\end{aligned}$$

Small cube:

$$\text{Volume} = y^3$$

→ large cube = small cube + 5

$$y^3 + 3y^2 + 3y + 1 = y^3 + 5$$

$$3y^2 + 3y + 1 = 5$$

$$3y^2 + 3y - 4 = 0$$

[4]

(ii) Find the volume of the smaller cube.

Show all your working and give your answer correct to 2 decimal places.

$$y = \frac{-3 \pm \sqrt{3^2 - 4(3)(-4)}}{2(3)}$$

$$= \frac{-3 \pm \sqrt{57}}{6}$$

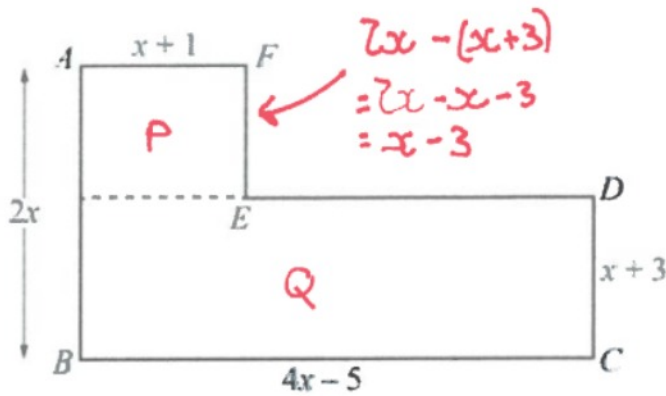
$$y = 0.758 \text{ STO} \quad \text{or} \quad y = -1.758$$

x length can't be negative.

$$\begin{aligned}V &= (0.758\dots)^3 \\ &= \underline{0.436 \text{ cm}^3}\end{aligned}$$

$$\underline{0.436} \text{ cm}^3 \quad [4]$$

5 All the lengths in this question are in centimetres.



NOT TO SCALE

The diagram shows a shape $ABCDEF$ made from two rectangles. The total area of the shape is 342 cm^2 .

(a) Show that $x^2 + x - 72 = 0$.

$$\begin{aligned} P: \text{Area} &= (x+1)(x-3) \\ &= x^2 - 3x + x - 3 \\ &= x^2 - 2x - 3 \end{aligned}$$

$$\begin{aligned} Q: \text{Area} &= (x+3)(4x-5) \\ &= 4x^2 - 5x + 12x - 15 \\ &= 4x^2 + 7x - 15 \end{aligned}$$

$$P + Q = 342:$$

$$x^2 - 2x - 3 + 4x^2 + 7x - 15 = 342$$

$$5x^2 + 5x - 18 = 342$$

$$5x^2 + 5x - 360 = 0 \quad \div 5$$

$$x^2 + x - 72 = 0$$

[5]

(b) Solve by factorisation.

$$x^2 + x - 72 = 0$$

$$(x+9)(x-8) = 0$$

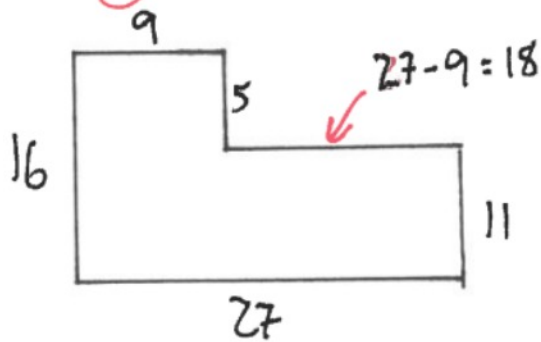
$$x+9=0 \quad \text{or} \quad x-8=0$$

$$x=-9 \quad x=8$$

$$x = -9 \quad \text{or} \quad x = 8 \quad [3]$$

(c) Work out the perimeter of the shape $ABCDEF$.

Lengths can't be negative, so use $x = 8$:



$$\begin{aligned}\text{Perimeter} &= 27 + 16 + 9 + 5 + 18 + 11 \\ &= 86 \text{ cm}\end{aligned}$$

86

cm [2]

- 18 One day, Anya runs 12 km at a speed of x km/h.
The next day she walks 10 km at a speed of $(x-4)$ km/h.

(a) Write down an expression, in terms of x , for the time she spends running.

$$\text{time} = \frac{\text{distance}}{\text{Speed}}$$

$$\text{time} = \frac{12}{x} \quad \dots\dots\dots \frac{12}{x} \dots\dots\dots \text{h [1]}$$

(b) Write down an expression, in terms of x , for the time she spends walking.

$$\text{time} = \frac{10}{x-4} \quad \dots\dots\dots \frac{10}{x-4} \dots\dots\dots \text{h [1]}$$

(c) The time Anya spends walking is 1 hour more than the time she spends running.

Write an equation in terms of x and show that it simplifies to $x^2 - 2x - 48 = 0$.

Walking = running + 1:

$$\frac{10}{x-4} = \frac{12}{x} + 1 \times \frac{x}{x}$$

$$\frac{10}{x-4} = \frac{12}{x} + \frac{x}{x}$$

$$\frac{10}{x-4} \times \frac{12+x}{x}$$

$$10x = (x-4)(12+x)$$

$$10x = 12x + x^2 - 48 - 4x$$

$$10x = x^2 + 8x - 48$$

$$x^2 - 2x - 48 = 0$$

[4]

(d) Use factorisation to solve the equation $x^2 - 2x - 48 = 0$.

$$(x+6)(x-8) = 0$$

$$x+6=0 \quad \text{or} \quad x-8=0$$

$$x=-6 \quad x=8$$

$$x = -6 \quad \text{or} \quad x = 8 \dots\dots\dots [3]$$

(e) Find the time Anya spends running.

Speed can't be negative, so $x=8$:

$$\text{time} = \frac{12}{x} = \frac{12}{8} \quad \dots\dots\dots 1.5 \dots\dots\dots \text{h [1]}$$

$$= 1.5 \text{ hours}$$

- 8 Darpan runs a distance of 12 km and then cycles a distance of 26 km. His running speed is x km/h and his cycling speed is 10 km/h faster than his running speed. He takes a total time of 2 hours 48 minutes.

(a) An expression for the time, in hours, Darpan takes to run the 12 km is $\frac{12}{x}$.

Write an equation, in terms of x , for the total time he takes in hours.

Cycling speed: $x + 10$

Cycling time: $\frac{26}{x+10}$

48 mins in hours: $48 \div 60 = 0.8$

So this 48m = 2.8 hours

Total time: $\frac{12}{x} + \frac{26}{x+10} = 2.8$

[3]

(b) Show that this equation simplifies to $7x^2 - 25x - 300 = 0$.

$\frac{12}{x} \times \frac{(x+10)}{(x+10)} + \frac{26}{x+10} \times \frac{x}{x} = 2.8$

$\frac{12(x+10) + 26x}{x(x+10)} = 2.8$

$\frac{12x + 120 + 26x}{x(x+10)} = 2.8$

$\frac{38x + 120}{x(x+10)} = 2.8$

$38x + 120 = 2.8x(x+10)$

$38x + 120 = 2.8x^2 + 28x$

$2.8x^2 - 10x - 120 = 0$

$7x^2 - 25x - 300 = 0$ [4]

(c) Use the quadratic formula to solve $7x^2 - 25x - 300 = 0$. You must show all your working.

$x = \frac{25 \pm \sqrt{(-25)^2 - 4(7)(-300)}}{2(7)}$

$x = \frac{25 \pm \sqrt{9025}}{14}$

$= \frac{25 \pm 95}{14}$

$x = \frac{60}{7}$ or $x = -5$

$x = \frac{60}{7}$ or $x = -5$ [4]

(d) Calculate the number of minutes Darpan takes to run the 12 km.

$x = \frac{60}{7}$ because speed can't be negative.

time = $\frac{12}{x} = \frac{84}{60}$

$= \frac{12 \times 7}{\frac{60}{7} \times 7} = 1.4$ hours

$1.4 \times 60 = 84$ mins

84 min [2]

- (a) Write down an expression, in terms of x , for the number of books Gaya buys.

$$\text{number} = \frac{\text{money}}{\text{cost}}$$

$$\text{number} = \frac{48}{x}$$

$$\frac{48}{x} \dots\dots\dots [1]$$

- (b) Myra spends \$60 to buy books that cost \$($x+2$) each.
Gaya buys 4 more books than Myra.

Show that $x^2 + 5x - 24 = 0$.

$$\text{Myra: } \frac{60}{x+2}$$

$$\text{Gaya} = \text{Myra} + 4:$$

$$\frac{48}{x} = \frac{60}{x+2} + \frac{4}{1} \times \frac{(x+2)}{(x+2)}$$

$$\frac{48}{x} = \frac{60 + 4(x+2)}{x+2}$$

$$\frac{48}{x} = \frac{60 + 4x + 8}{x+2}$$

$$\frac{48}{x} \rightarrow \frac{4x + 68}{x+2}$$

$$48(x+2) = x(4x + 68)$$

$$48x + 96 = 4x^2 + 68x$$

$$-48x \quad -96 \quad 96 = 4x^2 + 20x - 96$$

$$4x^2 + 20x - 96 = 0 \div 4$$

$$x^2 + 5x - 24 = 0 \quad [4]$$

- (c) Solve by factorisation.

$$x^2 + 5x - 24 = 0$$

$$(x+8)(x-3) = 0$$

$$x+8=0 \text{ or } x-3=0$$

$$x=-8 \quad x=3$$

$$x = -8 \text{ or } x = 3 \dots\dots\dots [3]$$

- (d) Find the number of books Myra buys.

x can't be negative, so $x=3$:

$$\text{Myra: } \frac{60}{3+2} = \frac{60}{5}$$

$$= 12$$

$$12 \dots\dots\dots [1]$$

- (c) Antonio travels 100 km at an average speed of x km/h.
He then travels a further 150 km at an average speed of $(x + 10)$ km/h.
The time taken for the whole journey is 4 hours 20 minutes.

(i) Show that $13x^2 - 620x - 3000 = 0$.

$$\rightarrow 20 \text{ mins} = \frac{20}{60} = \frac{1}{3} \text{ hour}$$

First 100 km: time = $\frac{100}{x}$

Next 150 km: time = $\frac{150}{x+10}$

Total Time: $\frac{100}{x} + \frac{150}{x+10} = 4\frac{1}{3}$

$$\frac{100}{x} \times \frac{(x+10)}{(x+10)} + \frac{150}{x+10} \times \frac{x}{x} = \frac{13}{3}$$

$$\frac{100(x+10) + 150x}{x(x+10)} = \frac{13}{3}$$

$$\frac{100x + 1000 + 150x}{x(x+10)} = \frac{13}{3}$$

$$\frac{250x + 1000}{x(x+10)} = \frac{13}{3}$$

$$3(250x + 1000) = 13x(x+10)$$

$$750x + 3000 = 13x^2 + 130x$$

$$-750x \quad -750x$$

$$3000 = 13x^2 - 620x$$

$$-3000 \quad -3000$$

$$13x^2 - 620x - 3000 = 0$$

[4]

- (ii) Solve $13x^2 - 620x - 3000 = 0$ to find the speed Antonio travels for the first 100 km of the journey.

You must show all your working and give your answer correct to 1 decimal place.

$$x = \frac{620 \pm \sqrt{(-620)^2 - 4(13)(-3000)}}{2(13)}$$

$$= \frac{620 \pm \sqrt{384400 + 156000}}{26}$$

$$= \frac{620 \pm \sqrt{540400}}{26}$$

$$x = 52.1 \quad \text{or} \quad x = -4.4$$

x can't be negative

..... 52.1 km/h [3]

- 5 (a) In a shop the cost of a fiction book is \$x and the cost of a reference book is \$(x+2).
The cost of 11 fiction books is the same as the cost of 10 reference books.

Find the value of x. $11x = 10(x+2)$

$$\begin{array}{r} 11x \\ -10x \\ \hline \end{array} = \begin{array}{r} 10x + 20 \\ -10x \\ \hline \end{array}$$

$$x = 20$$

$$x = \underline{20} \quad [2]$$

- (b) In another shop, the cost of a fiction book is \$y and the cost of a reference book is \$(y+2).
Maria spends \$95 on fiction books and \$147 on reference books.
She buys a total of 12 books.

(i) Show that $6y^2 - 109y - 95 = 0$.

Number of fiction books: $\frac{95}{y}$

Number of reference books: $\frac{147}{y+2}$

Total books = 12:

$$\frac{95}{y} + \frac{147}{y+2} = 12$$

$$\frac{95}{y} \times \frac{(y+2)}{(y+2)} + \frac{147}{y+2} \times \frac{y}{y} = 12$$

$$\frac{95(y+2) + 147y}{y(y+2)} = 12$$

$$\frac{95y + 190 + 147y}{y(y+2)} = 12$$

$$\frac{242y + 190}{y(y+2)} = 12$$

$$242y + 190 = 12y(y+2)$$

$$242y + 190 = 12y^2 + 24y$$

$$12y^2 - 218y - 190 = 0 \div 2$$

$$6y^2 - 109y - 95 = 0_{[4]}$$

(ii) Factorise $6y^2 - 109y - 95$.

$$ac = -570$$

two numbers: -114, 5

$$\rightarrow 6y^2 - 114y + 5y - 95$$

$$6y(y-19) + 5(y-19)$$

$$(6y+5)(y-19)$$

$$\underline{(6y+5)(y-19)} \quad [2]$$

(iii) Find the value of y.

$$6y+5=0 \text{ or } y-19=0$$

$$6y = -5$$

$$y = 19$$

$$y = \underline{19} \quad [1]$$

$$y = \frac{-5}{6}$$

x can't be negative

- 8 (a) Kaito runs along a 12 km path at an average speed of x km/h.

$$\text{time} = \frac{\text{distance}}{\text{Speed}}$$

- (i) Write down an expression, in terms of x , for the number of hours he takes.

$$\text{time} = \frac{12}{x}$$

$$\frac{12}{x} \text{ hours [1]}$$

- (ii) Yuki takes 1.5 hours longer to walk along the same path as Kaito. She walks at an average speed of $(x-4)$ km/h.

Write down an equation, in terms of x , and show that it simplifies to $x^2 - 4x - 32 = 0$.

$$\text{Yuki time} = \frac{12}{x-4}$$

$$\text{Yuki} = \text{Kaito} + 1.5:$$

$$\frac{12}{x-4} = \frac{12}{x} + \frac{3}{2}$$

$$\frac{12}{x-4} = \frac{12}{x} \times \frac{2}{2} + \frac{3}{2} \times \frac{x}{x}$$

$$\frac{12}{x-4} = \frac{24}{2x} + \frac{3x}{2x}$$

$$\frac{12}{x-4} = \frac{24 + 3x}{2x}$$

$$24x = (x-4)(24+3x)$$

$$24x = 24x + 3x^2 - 96 - 12x$$

$$3x^2 - 12x - 96 = 0 \div 3$$

$$x^2 - 4x - 32 = 0 \quad [4]$$

- (iii) Solve by factorisation.

$$x^2 - 4x - 32 = 0$$

$$(x-8)(x+4) = 0$$

$$x-8=0 \text{ or } x+4=0$$

$$x=8$$

$$x=-4$$

$$x = 8 \text{ or } x = -4 \quad [3]$$

- (iv) Find the number of hours it takes Yuki to walk along the 12 km path.

x can't be negative, so $x=8$:

$$\text{Yuki: } \frac{12}{8-4} = \frac{12}{4}$$

$$= 3 \text{ hrs}$$

$$3 \text{ hours [2]}$$

- (f) Alan invests \$200 at a rate of $r\%$ per year compound interest. After 2 years the value of his investment is \$206.46.

(i) Show that $r^2 + 200r - 323 = 0$.

$$200 \times \left(1 + \frac{r}{100}\right)^2 = 206.46$$

$$200 \times \left(1 + \frac{r}{100}\right)\left(1 + \frac{r}{100}\right) = 206.46$$

$$200 \times \left(1 + \frac{r}{100} + \frac{r}{100} + \frac{r^2}{10000}\right) = 206.46$$

$$200 \times \left(1 + \frac{r}{50} + \frac{r^2}{10000}\right) = 206.46$$

$$200 + 4r + \frac{r^2}{50} = 206.46$$

-206.46 -206.46

$$\frac{r^2}{50} + 4r - 6.46 = 0$$

$\times 50$ $\times 50$ $\times 50$ $\times 50$

$$r^2 + 200r - 323 = 0$$

[3]

- (ii) Solve the equation $r^2 + 200r - 323 = 0$ to find the rate of interest. Show all your working and give your answer correct to 2 decimal places.

$$r = \frac{-200 \pm \sqrt{200^2 - 4(1)(-323)}}{2(1)}$$

$$= \frac{-200 \pm \sqrt{40000 + 1292}}{2}$$

$$= \frac{-200 \pm \sqrt{41292}}{2}$$

$$r = 1.60 \text{ or } r = -201.60$$

$\times$
Can't be negative

$$r = 1.60\% \quad [3]$$