

A Factorising Quadratics (and similar)

(d) Factorise the following completely.

(i) $x^2 - 3x - 28$

$$\begin{aligned} x^2 - 7x + 4x - 28 \\ x(x-7) + 4(x-7) \\ (x+4)(x-7) \end{aligned}$$

$$\underline{(x+4)(x-7)} \quad [2]$$

10 Factorise

(a) $x^2 - x - 6$

$$\begin{aligned} x^2 - 3x + 2x - 6 \\ x(x-3) + 2(x-3) \\ (x+2)(x-3) \end{aligned}$$

$$\underline{(x+2)(x-3)} \quad [2]$$

(b) $h^2 - 144$

↑ difference of two squares

$$\underline{(h-12)(h+12)} \quad [1]$$

(b) $9x^2 - 1$

$$\underline{(3x+1)(3x-1)} \quad [1]$$

12 Factorise fully.

(a) $(3y)^2 - 16$

$$9y^2 - 16$$

$$\underline{(3y+4)(3y-4)} \quad [1]$$

7 (a) Factorise fully.

(i) $27y^2 - 3$

Single bracket first:

$$3(9y^2 - 1)$$

$$3(3y+1)(3y-1)$$

$$\underline{3(3y+1)(3y-1)} \quad [3]$$

13 Factorise.

(a) $49 - 16u^2$

$$(7 + 4u)(7 - 4u) \quad [1]$$

18 (a) Factorise.

$$18a^2 - 98$$

single bracket first:

$$2(9a^2 - 49)$$

$$2(3a + 7)(3a - 7)$$

$$2(3a + 7)(3a - 7) \quad [2]$$

9 Factorise completely.

(b) $20x^2 - 45y^2$

$$5(4x^2 - 9y^2)$$

$$5(2x + 3y)(2x - 3y)$$

$$5(2x + 3y)(2x - 3y) \quad [3]$$

11 Factorise completely.

$$48x^2 - 75y^2$$

$$3(16x^2 - 25y^2)$$

$$3(4x + 5y)(4x - 5y)$$

$$3(4x + 5y)(4x - 5y) \quad [3]$$

(b) Factorise completely.

(i) $2x^2 - 288y^2$

$$2(x^2 - 144y^2)$$

$$2(x + 12y)(x - 12y)$$

$$2(x + 12y)(x - 12y) \quad [3]$$

(b) $2x^3 - 18xy^2$

$$2x(x^2 - 9y^2)$$

$$2x(x + 3y)(x - 3y)$$

$$\underline{2x(x + 3y)(x - 3y)} \quad [3]$$

(b) $5x^2 - 20y^2$

$$5(x^2 - 4y^2)$$

$$5(x + 2y)(x - 2y)$$

$$\underline{5(x + 2y)(x - 2y)} \quad [3]$$

(c) Factorise completely.

$$x^3 - 16xy^2$$

$$x(x^2 - 16y^2)$$

$$x(x + 4y)(x - 4y)$$

$$\underline{x(x + 4y)(x - 4y)} \quad [3]$$

6 (a) Factorise $a^2 - b^2$.

$$(a+b)(a-b) \quad [1]$$

(b) Work out $5.37^2 - 4.63^2$.

Factorise like part (a):

$$\begin{aligned} & (5.37 + 4.63)(5.37 - 4.63) \\ = & (10)(0.74) \\ = & 10 \times 0.74 \\ = & 7.4 \end{aligned}$$

$$7.4 \quad [2]$$

10 Factorise.

$$\begin{aligned} & \underline{2x+6} - \underline{3xy-9y} \\ & 2(x+3) - 3y(x+3) \\ & (2-3y)(x+3) \end{aligned}$$

$$(2-3y)(x+3) \quad [2]$$

9 Factorise completely.

$$\begin{aligned} & 2x-6y-ax+3ay \\ & 2(x-3y) - a(x-3y) \\ & (2-a)(x-3y) \end{aligned}$$

$$(2-a)(x-3y) \quad [2]$$

8 Factorise fully.

no obvious common factors, so use 1 or -1

$$\begin{aligned} & 2cx^2 - 2dx - cx + d \\ & 2xc(cx-d) - 1(cx-d) \\ & (2x-1)(cx-d) \end{aligned}$$

$$(2x-1)(cx-d) \quad [2]$$

(b) Factorise.

$$3x(a+4y) - ay - 4y^2$$

$$3x(a+4y) - y(a+4y)$$

$$(3x-y)(a+4y)$$

$$\dots\dots\dots (3x-y)(a+4y) \quad [1]$$

16 Factorise.

$$3x+6-2xy-4y$$

$$3(x+2) - 2y(x+2)$$

$$(3-2y)(x+2)$$

$$\dots\dots\dots (3-2y)(x+2) \quad [2]$$

10 Factorise.

(c) $6ax+9ay-8bx-12by$

$$3a(2x+3y) - 4b(2x+3y)$$

$$(3a-4b)(2x+3y)$$

$$\dots\dots\dots (3a-4b)(2x+3y) \quad [2]$$

20 Factorise completely.

(a) $1+x-y-xy$

use 1 as the common factor:

$$1(1+x) - y(1+x)$$

$$(1-y)(1+x)$$

$$\dots\dots\dots (1-y)(1+x) \quad [2]$$

21 Factorise completely.

$$1 - q - a + aq$$

$$1(1 - q) - a(1 - q)$$

$$(1 - a)(1 - q)$$

$$\dots (1 - a)(1 - q) \dots [2]$$

12 Factorise.

$$1 + a - c - ac$$

$$1(1 + a) - c(1 + a)$$

$$(1 - c)(1 + a)$$

$$\dots (1 - c)(1 + a) \dots [2]$$

10 (a) Write down all the factors of 18.

$$\begin{array}{r|l} 18 & \\ \hline 1 & 18 \\ 2 & 9 \\ 3 & 6 \end{array}$$

1, 2, 3, 6, 9, 18 [2]

(b) Factorise.

$$3y - xy + 15 - 5x$$

$$y(3-x) + 5(3-x)$$

$$(y+5)(3-x)$$

$(y+5)(3-x)$ [2]

(c) $3y - xy + 15 - 5x = 18$

where x and y are positive integers.

Using your answers to part (a) and part (b), find one possible value of x and the corresponding value of y .

from part (b): $(y+5)(3-x) = 18$

↑ ↑
must be factors of 18

either: $(y+5)(3-x) = 18$
9 × 2

so: $y+5=9$ and $3-x=2$
 $y=4$ $-x=-1$
 $x=1$

→ or: $(y+5)(3-x) = 18$
18 × 1

so: $y+5=18$ and $3-x=1$
 $y=13$ $-x=-2$
 $x=2$

$x = 1, y = 4$ [2]

(or $x=2, y=13$)

20 Factorise completely.

(a) $2m + 3p - 8km - 12kp$

$$1(2m + 3p) - 4k(2m + 3p)$$

$$(1 - 4k)(2m + 3p)$$

$$(1 - 4k)(2m + 3p) \quad [2]$$

7 Factorise.

(a) $12ax - 2by + 3ay - 8bx$

Doesn't work if you try to factorise this one as it is:

$$2(6ax - by) + 1(3ay - 8bx)$$

Not the same!

Need to re-order expression first:

$$12ax - 8bx + 3ay - 2by$$

$$4x(3a - 2b) + y(3a - 2b)$$

$$(4x + y)(3a - 2b)$$

$$(4x + y)(3a - 2b) \quad [2]$$

(b) $3ax + 2bx - 4by - 6ay$

$$x(3a + 2b) - 2y(2b + 3a)$$

same as $3a + 2b$

$$x(3a + 2b) - 2y(3a + 2b)$$

$$(x - 2y)(3a + 2b)$$

$$(x - 2y)(3a + 2b) \quad [2]$$

(b) $15ab - 1 - 3a + 5b$

re-order:

$$15ab - 3a + 5b - 1$$

$$3a(5b - 1) + 1(5b - 1)$$

$$(3a + 1)(5b - 1)$$

$$(3a + 1)(5b - 1) \quad [2]$$

13 Factorise completely.

(b) $mt - n - m + nt$

re-order:

$$\begin{aligned} mt - m + nt - n \\ m(t-1) + n(t-1) \\ (m+n)(t-1) \end{aligned}$$

$$\underline{(m+n)(t-1)} \quad [2]$$

(b) $1 + 4xy - 2x - 2y$

re-order:

$$\begin{aligned} 4xy - 2x - 2y + 1 \\ 2x(2y-1) - 1(2y-1) \\ (2x-1)(2y-1) \end{aligned}$$

$$\underline{(2x-1)(2y-1)} \quad [2]$$

(ii) $2m - pk + 2k - pm$

re-order:

$$\begin{aligned} 2m - pm + 2k - pk \\ m(2-p) + k(2-p) \\ (m+k)(2-p) \end{aligned}$$

$$\underline{(m+k)(2-p)} \quad [2]$$

(ii) $5x^2 + 17x - 40$

$ac = -200$

Numbers which $\times$ to -200
and $+$ to 17

25 and -8

Split the middle:

$$\begin{aligned} 5x^2 + 25x - 8x - 40 \\ 5x(x+5) - 8(x+5) \end{aligned}$$

$$\underline{(5x-8)(x+5)} \quad [2]$$

20 Factorise.

$$3x + 8y - 6ax - 16ay$$

re-order:

$$3x - 6ax + 8y - 16ay$$

$$3x(1 - 2a) + 8y(1 - 2a)$$

$$(3x + 8y)(1 - 2a)$$

$$(3x + 8y)(1 - 2a) \dots [2]$$

12 Factorise.

$$\begin{matrix} a & b & c \\ 2x^2 & -3x & -5 \end{matrix}$$

$$ac = -10$$

Find two numbers which multiply to ac (-10)
and add to give b (-3)

Two numbers: -5 and $+2$

Split the middle using these two numbers:

$$2x^2 + 2x - 5x - 5$$

Factorise:

$$2x(x+1) - 5(x+1)$$

$$(2x - 5)(x+1)$$

$$\dots [2]$$

17 Solve.

$$(5x - 3)(2x + 7) = 0$$

$$5x - 3 = 0 \text{ or } 2x + 7 = 0$$

$$5x = 3$$

$$2x = -7$$

$$x = \frac{3}{5}$$

$$x = -\frac{7}{2}$$

$$x = \frac{3}{5} \text{ or } x = -\frac{7}{2} \dots [1]$$

16 Factorise $6x^2 + 7x - 20$.

$$ac = -120$$

two numbers: $15, -8$

$$6x^2 + 15x - 8x - 20$$

$$3x(2x + 5) - 4(2x + 5)$$

$$(3x - 4)(2x + 5) \dots [2]$$

15 (a) Factorise fully.

$$6x^2 - 7x - 3$$

$$ac = -18$$

two numbers: 2, -9

$$6x^2 + 2x - 9x - 3$$
$$2x(3x + 1) - 3(3x + 1)$$

$$(2x - 3)(3x + 1) \quad [2]$$

(b) $5x^2 - 6x - 8$

$$ac = -40$$

two numbers: -10, 4

Split the middle:

$$5x^2 - 10x + 4x - 8$$
$$5x(x - 2) + 4(x - 2)$$

$$(5x + 4)(x - 2) \quad [2]$$

(b) (i) Factorise.

$$15x^2 - 2x - 8$$

$$ac = -120$$

two numbers: 10, -12

$$15x^2 + 10x - 12x - 8$$
$$5x(3x + 2) - 4(3x + 2)$$

$$(5x - 4)(3x + 2) \quad [2]$$

(ii) Solve the equation.

$$15x^2 - 2x - 8 = 0$$

$$(5x - 4)(3x + 2) = 0$$

$$5x - 4 = 0 \text{ or } 3x + 2 = 0$$

$$5x = 4$$

$$3x = -2$$

$$x = \frac{4}{5}$$

$$x = -\frac{2}{3}$$

$$x = \frac{4}{5} \text{ or } x = -\frac{2}{3} \quad [1]$$

(e) Solve by factorisation.

$$4x^2 + 8x - 5 = 0$$

$$ac = -20$$

two numbers: 10, -2

$$4x^2 + 10x - 2x - 5 = 0$$

$$2x(2x + 5) - 1(2x + 5) = 0$$

$$(2x - 1)(2x + 5) = 0$$

$$2x - 1 = 0 \quad \text{or} \quad 2x + 5 = 0$$

$$2x = 1$$

$$2x = -5$$

$$x = \frac{1}{2} \quad \text{or} \quad x = -\frac{5}{2} \quad [3]$$

11 Solve.

$$6x^2 - 5x - 6 = 0$$

$$ac = -36$$

two numbers: -9, 4

$$6x^2 - 9x + 4x - 6 = 0$$

$$3x(2x - 3) + 2(2x - 3) = 0$$

$$(3x + 2)(2x - 3) = 0$$

$$3x + 2 = 0 \quad \text{or} \quad 2x - 3 = 0$$

$$3x = -2$$

$$2x = 3$$

$$x = -\frac{2}{3}$$

$$x = \frac{3}{2}$$

$$x = -\frac{2}{3} \quad \text{or} \quad x = \frac{3}{2} \quad [3]$$

11 Solve.

$$(a) \quad 4x^2 - 5x - 6 = 0$$

$$ac = -24$$

two numbers: -8, 3

$$4x^2 - 8x + 3x - 6 = 0$$

$$4x(x - 2) + 3(x - 2) = 0$$

$$(4x + 3)(x - 2) = 0$$

$$4x + 3 = 0 \quad x - 2 = 0$$

$$4x = -3$$

$$x = 2$$

$$x = -\frac{3}{4}$$

$$x = -\frac{3}{4} \quad \text{or} \quad x = 2 \quad [3]$$

23 Solve $\frac{4}{x+1} + \frac{2}{2x-5} = 3$.

You must show all your working.

$$\frac{4}{x+1} \times \frac{(2x-5)}{(2x-5)} + \frac{2}{2x-5} \times \frac{(x+1)}{(x+1)} = 3$$

$$\frac{4(2x-5) + 2(x+1)}{(x+1)(2x-5)} = 3$$

$$\frac{8x - 20 + 2x + 2}{(x+1)(2x-5)} = 3$$

$$\frac{10x - 18}{(x+1)(2x-5)} = 3$$

$$\times (x+1)(2x-5)$$

$$10x - 18 = 3(x+1)(2x-5)$$

$$\rightarrow 10x - 18 = 3[2x^2 - 5x + 2x - 5]$$

$$10x - 18 = 3(2x^2 - 3x - 5)$$

$$10x - 18 = 6x^2 - 9x - 15$$

Set = 0:

$$6x^2 - 19x + 3 = 0$$

ac = 18, two numbers: -18, -1

$$6x^2 - 18x - 1x + 3 = 0$$

$$6x(x-3) - 1(x-3) = 0$$

$$(6x-1)(x-3) = 0$$

$$6x-1=0 \text{ or } x-3=0$$

$$6x=1$$

$$x=3$$

$$x = \frac{1}{6} \text{ or } x = 3 \quad [7]$$

Factorise.

(ii) $7(a+2b)^2 + 4a(a+2b)$

Factorise by taking $(a+2b)$ out in front:

$$(a+2b)[7(a+2b) + 4a]$$

$$(a+2b)[7a + 14b + 4a]$$

$$(a+2b)(11a + 14b)$$

$$(a+2b)(11a + 14b) \quad [2]$$

9 (a) Solve.

$$5x^2 = 12 - 17x$$

Set = 0:

$$5x^2 + 17x - 12 = 0$$

$$ac = -60$$

two numbers: 20, -3

$$5x^2 + 20x - 3x - 12 = 0$$

$$5x(x+4) - 3(x+4) = 0$$

$$(5x-3)(x+4) = 0$$

$$5x-3=0 \quad \text{or} \quad x+4=0$$

$$5x=3$$

$$x = \frac{3}{5}$$

$$x = -4$$

$$x = \frac{3}{5} \quad \text{or} \quad x = -4 \quad [4]$$

(b) $ax^2 + a = b$ where a and b are integers.

One solution of this equation is $x = 6$.

Write down the other solution.

$$ax^2 + a = b$$

$$ax^2 = b - a$$

$$x^2 = \frac{b-a}{a}$$

$$\text{so } x^2 = 36$$

only other possible solution is $x = -6$

$$x = -6 \quad [1]$$

If $x=6$:

$$36 = \frac{b-a}{a}$$

15 Factorise.

$$5x^2 - xy - 4y^2$$

$$5x^2 - yx - 4y^2$$

$$ac = -20y^2$$

Two numbers that multiply to $-20y^2$

and add to $-y$:

$$-5y \quad \text{and} \quad +4y$$

split middle:

$$5x^2 - 5yx + 4yx - 4y^2$$

$$5x(x-y) + 4y(x-y)$$

$$(5x+4y)(x-y)$$

..... [2]