

A Distance between two Points and Mid-Points

- 10 (a) P is the point $(-5, 3)$ and Q is the point $(2, -1)$.

Find the coordinates of the mid-point of PQ .

$$\begin{aligned} & \left(\frac{-5+2}{2}, \frac{3+(-1)}{2} \right) \\ &= \left(\frac{-3}{2}, \frac{2}{2} \right) \\ &= \left(\frac{-3}{2}, 1 \right) \end{aligned}$$

mid-point: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$

$\left(\frac{-3}{2}, 1 \right)$ [2]

- 8 A is the point $(-3, 5)$ and B is the point $(5, 2)$.

Find the coordinates of the midpoint of the line AB .

$$\begin{aligned} & \left(\frac{-3+5}{2}, \frac{5+2}{2} \right) \\ &= \left(\frac{2}{2}, \frac{7}{2} \right) = \left(1, \frac{7}{2} \right) \end{aligned}$$

$\left(1, \frac{7}{2} \right)$ [2]

- 3 A is the point $(0, 7)$ and B is the point $(-2, 1)$.
 M is the mid-point of AB .

Find the coordinates of M .

$$\begin{aligned} & \left(\frac{0+(-2)}{2}, \frac{7+1}{2} \right) \\ &= \left(\frac{-2}{2}, \frac{8}{2} \right) = (-1, 4) \end{aligned}$$

$(-1, 4)$ [2]

- 5 Point A has coordinates $(-3, 2)$.
Point B has coordinates $(5, -4)$.

(a) Find the mid-point of AB.

$$\left(\frac{-3+5}{2}, \frac{2+(-4)}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{-2}{2} \right) = (1, -1)$$

(..... 1, -1) [2]

(b) Find the length of AB.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(5 - (-3))^2 + (-4 - 2)^2}$$

$$= \sqrt{8^2 + (-6)^2}$$

$$= \sqrt{64 + 36}$$

$$= \sqrt{100}$$

$$= 10$$

..... 10 [3]

- 9 A is the point $(5, -5)$ and B is the point $(9, 3)$.

(a) Find the coordinates of the midpoint of AB.

$$\left(\frac{5+9}{2}, \frac{-5+3}{2} \right)$$

$$= \left(\frac{14}{2}, \frac{-2}{2} \right)$$

$$= (7, -1)$$

(..... 7, -1) [2]

(b) Find the length of AB.

$$d = \sqrt{(9-5)^2 + (3-(-5))^2}$$

$$= \sqrt{4^2 + 8^2}$$

$$= \sqrt{16 + 64}$$

$$= \sqrt{80}$$

$$= 4\sqrt{5}$$

..... $4\sqrt{5}$ [3]
(8.94 to 3sf)

16 A is the point $(5, 7)$ and B is the point $(9, -1)$.

(a) Find the length AB .

$$\begin{aligned}d &= \sqrt{(9-5)^2 + (-1-7)^2} \\&= \sqrt{4^2 + (-8)^2} \\&= \sqrt{16 + 64} \\&= \sqrt{80}\end{aligned}$$

$$\begin{aligned}&= \sqrt{16} \times \sqrt{5} \\&= 4\sqrt{5}\end{aligned}$$

$4\sqrt{5}$ [3]
(8.94 to 3sf)

10 The point A has coordinates $(2, 9)$ and the point B has coordinates $(5, 3)$.

Find the length of AB .

Give your answer in surd form.

$$\begin{aligned}d &= \sqrt{(5-2)^2 + (3-9)^2} \\&= \sqrt{3^2 + (-6)^2} \\&= \sqrt{9 + 36} \\&= \sqrt{45}\end{aligned}$$

$$\begin{aligned}&= \sqrt{9} \times \sqrt{5} \\&= 3\sqrt{5}\end{aligned}$$

$3\sqrt{5}$ [3]

8 A is the point $(-2, 4)$ and B is the point $(7, 1)$.

Find the length of AB giving your answer in its simplest surd form.

$$\begin{aligned}d &= \sqrt{(7-(-2))^2 + (1-4)^2} \\&= \sqrt{9^2 + (-3)^2} \\&= \sqrt{81 + 9} \\&= \sqrt{90}\end{aligned}$$

$$\begin{aligned}&= \sqrt{9} \times \sqrt{10} \\&= 3\sqrt{10}\end{aligned}$$

$3\sqrt{10}$ [4]

3 A line, l , joins point $F(3, 2)$ and point $G(-5, 4)$.

(a) Calculate the length of line l .

$$\begin{aligned} d &= \sqrt{(-5-3)^2 + (4-2)^2} = \sqrt{68} \\ &= \sqrt{(-8)^2 + 2^2} = 2\sqrt{17} \\ &= \sqrt{64 + 4} \end{aligned}$$

$2\sqrt{17}$ [3]

(c) A point H lies on the y -axis such that the distance $GH = 13$ units.

(8.25 to 3sf)

Find the coordinates of the two possible positions of H .

On the y -axis, $x=0$: $H: (0, y)$ $G: (-5, 4)$

$$d = \sqrt{(-5-0)^2 + (4-y)^2}$$

$$d = \sqrt{(-5)^2 + (4-y)^2}$$

$d=13$:

$$13 = \sqrt{25 + (4-y)(4-y)}$$

square both sides:

$$169 = 25 + 16 - 8y + y^2$$

$$169 = 41 - 8y + y^2$$

-169 -169

$$y^2 - 8y - 128 = 0$$

$$(y+8)(y-16) = 0$$

$$y = -8 \text{ or } y = 16$$

$(0, -8)$ and $(0, 16)$ [4]

9 A is the point $(0, 2)$, B is the point $(3, 3)$ and C is the point $(4, 0)$.

(a) Determine if triangle ABC is scalene, isosceles or equilateral.
You must show all your working.

AB: $d = \sqrt{(3-0)^2 + (3-2)^2}$

$$= \sqrt{3^2 + 1^2}$$

$$= \sqrt{9 + 1}$$

$$= \sqrt{10}$$

BC: $d = \sqrt{(4-3)^2 + (0-3)^2}$

$$= \sqrt{1^2 + (-3)^2}$$

$$= \sqrt{1 + 9}$$

$$= \sqrt{10}$$

AC: $d = \sqrt{(4-0)^2 + (0-2)^2}$

$$= \sqrt{4^2 + (-2)^2}$$

$$= \sqrt{16 + 4}$$

$$= \sqrt{20}$$

→ Triangle is isosceles because AB and BC are equal, but not AC .

[4]