

A Direct and Inverse Proportion

- 14 y is directly proportional to the square of $(x+3)$.
When $x = 2, y = 5$.

Find y when $x = 1$.

$$\begin{aligned}y &\propto (x+3)^2 \\y &= k(x+3)^2 \\x=2, y=5: \\5 &= k(5)^2 \\5 &= 25k \\ \div 25 \quad \div 25 \quad k &= \frac{1}{5}\end{aligned}$$

$$\begin{aligned}y &= \frac{1}{5}(x+3)^2 \\x=1: \\y &= \frac{1}{5}(4)^2 \\&= \frac{1}{5} \times 16 \\&= \underline{3.2} \\y &= \underline{3.2} \quad [3]\end{aligned}$$

- 14 y is directly proportional to the square root of $(x-3)$.
When $x = 28, y = 20$.

Find y when $x = 39$.

$$\begin{aligned}y &\propto \sqrt{x-3} \\y &= k\sqrt{x-3} \\x=28, y=20: \\20 &= k\sqrt{25} \\20 &= 5k \\ \div 5 \quad \div 5 \\ \underline{k &= 4}\end{aligned}$$

$$\begin{aligned}y &= 4\sqrt{x-3} \\x=39: \\y &= 4\sqrt{36} \\&= 4 \times 6 \\&= \underline{24}\end{aligned}$$

$$y = \underline{24} \quad [3]$$

- 22 p is directly proportional to $(q+2)^2$.
When $q = 1$, $p = 1$.

Find p when $q = 10$.

$$p \propto (q+2)^2$$

$$p = k(q+2)^2$$

$$q=1, p=1:$$

$$1 = k(3)^2$$

$$1 = 9k$$

$$\div 9 \quad \div 9$$

$$k = \frac{1}{9}$$

$$p = \frac{1}{9}(q+2)^2$$

$$q=10:$$

$$p = \frac{1}{9}(12)^2$$

$$= \frac{1}{9} \times 144$$

$$= 16$$

$$p = \underline{16} \quad [3]$$

- 17 y is proportional to the square of $(x-7)$.
When $x = 12$, $y = 2$.

Find y when $x = 17$.

$$y \propto (x-7)^2$$

$$y = k(x-7)^2$$

$$x=12, y=2:$$

$$2 = k(5)^2$$

$$2 = 25k$$

$$\div 25 \quad \div 25$$

$$k = \frac{2}{25}$$

$$y = \frac{2}{25}(x-7)^2$$

$$x=17:$$

$$y = \frac{2}{25}(10)^2$$

$$= \frac{2}{25} \times 100$$

$$= 8$$

$$y = \underline{8} \quad [3]$$

- 11 y is directly proportional to the cube root of $(x+3)$.

When $x = 5$, $y = \frac{2}{3}$.

Find y when $x = 24$.

$$y \propto \sqrt[3]{x+3}$$

$$y = k\sqrt[3]{x+3}$$

$x=5, y=\frac{2}{3}$:

$$\frac{2}{3} = k\sqrt[3]{8}$$

$$\frac{2}{3} = 2k$$

$$\begin{array}{l} \div 2 \quad \div 2 \\ k = \frac{1}{3} \end{array}$$

$$y = \frac{1}{3}\sqrt[3]{x+3}$$

$x=24$:

$$y = \frac{1}{3}\sqrt[3]{27}$$

$$y = \frac{1}{3} \times 3$$

$$= 1$$

$y = \dots\dots\dots 1$ [3]

- 16 y is inversely proportional to x^2 .

When $x = 3$, $y = 2$.

Find y when $x = 2$.

$$y \propto \frac{1}{x^2}$$

$$y = \frac{k}{x^2}$$

$x=3, y=2$:

$$2 = \frac{k}{3^2}$$

$$2 = \frac{k}{9}$$

$\times 9$

$\times 9$

$$k = 18$$

$$y = \frac{18}{x^2}$$

$x=2$:

$$y = \frac{18}{2^2}$$

$$= \frac{18}{4}$$

$$= 4.5$$

$y = \dots\dots\dots 4.5$ [3]

- 22 x is inversely proportional to the square root of w .
When $w = 16$, $x = 3$.

Find x in terms of w .

$$x \propto \frac{1}{\sqrt{w}}$$

$$x = \frac{k}{\sqrt{w}}$$

$w=16, x=3:$

$$3 = \frac{k}{\sqrt{16}}$$

$$3_{\times 4} = \frac{k}{4_{\times 4}} \quad \underline{k=12}$$

$$x = \frac{12}{\sqrt{w}}$$

$$x = \frac{12}{\sqrt{w}} \quad [2]$$

- 13 y varies inversely as $\sqrt{x}$.
When $x = 9$, $y = 2$.

Find y in terms of x .

$$y \propto \frac{1}{\sqrt{x}}$$

$$y = \frac{k}{\sqrt{x}}$$

$x=9, y=2:$

$$2 = \frac{k}{\sqrt{9}}$$

$$2_{\times 3} = \frac{k}{3_{\times 3}}$$

$$\underline{k=6}$$

$$y = \frac{6}{\sqrt{x}}$$

$$y = \frac{6}{\sqrt{x}} \quad [2]$$

20

$$y \propto \frac{1}{\sqrt{x}}$$

When $y = 8, x = 4$.Find y when $x = 49$.

$$y = \frac{k}{\sqrt{x}}$$

$$y = 8, x = 4:$$

$$8 = \frac{k}{\sqrt{4}}$$

$$8 = \frac{k}{2}$$

$$k = 16$$

$$x = 49:$$

$$y = \frac{16}{\sqrt{x}}$$

$$y = \frac{16}{\sqrt{49}}$$

$$= \frac{16}{7}$$

$$y = \frac{16}{7} \dots \dots \dots [3]$$

- 19 m is inversely proportional to the square of $(t+2)$.
 $m = 0.64$ when $t = 3$.

Find m when $t = 8$.

$$m \propto \frac{1}{(t+2)^2}$$

$$m = \frac{k}{(t+2)^2}$$

$$m = 0.64, t = 3:$$

$$0.64 = \frac{k}{5^2}$$

$$0.64 = \frac{k}{25}$$

$$k = 16$$

$$m = \frac{16}{(t+2)^2}$$

$$t = 8:$$

$$m = \frac{16}{10^2} = \frac{16}{100} = \underline{\underline{0.16}}$$

- 14 y varies inversely as $(x-3)^2$.
When $x = 1, y = 4$.

Find y in terms of x .

$$y \propto \frac{1}{(x-3)^2}$$

$$y = \frac{k}{(x-3)^2}$$

$x=1, y=4$:

$$4 = \frac{k}{(-2)^2}$$

$$4 \times 4 = \frac{k}{4 \times 4}$$

$$\underline{k = 16}$$

$$y = \frac{16}{(x-3)^2}$$

$$y = \frac{16}{(x-3)^2} \quad [2]$$

- 19 y is inversely proportional to the square root of $(x+4)$.
When $x = 5, y = 2$.

Find y when $x = 77$.

$$y \propto \frac{1}{\sqrt{x+4}}$$

$$y = \frac{k}{\sqrt{x+4}}$$

$x=5, y=2$:

$$2 = \frac{k}{\sqrt{9}}$$

$$2 = \frac{k}{3} \times 3$$

$$\underline{k = 6}$$

$$y = \frac{6}{\sqrt{x+4}}$$

$x=77$:

$$y = \frac{6}{\sqrt{81}}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

$$y = \frac{2}{3} \quad [3]$$

- 18 y is inversely proportional to the cube root of $(x + 5)$.
When $x = 3, y = 12$.

Find y when $x = 22$.

$$y \propto \frac{1}{\sqrt[3]{x+5}}$$

$$y = \frac{k}{\sqrt[3]{x+5}}$$

$x=3, y=12:$

$$12 = \frac{k}{\sqrt[3]{8}}$$

$$12 = \frac{k}{2} \quad k=24$$

$$y = \frac{24}{\sqrt[3]{x+5}}$$

$x=22:$

$$y = \frac{24}{\sqrt[3]{27}}$$

$$= \frac{24}{3}$$

$$= 8$$

$y = 8$ [3]

- 24 y is inversely proportional to the cube of $(x - 1)$.
 $y = 9.45$ when $x = 3$.

Find y when $x = 4$.

$$y \propto \frac{1}{(x-1)^3}$$

$$y = \frac{k}{(x-1)^3}$$

$y=9.45, x=3:$

$$9.45 = \frac{k}{2^3}$$

$$9.45 = \frac{k}{8} \quad k=75.6$$

$$k = 75.6$$

$$y = \frac{75.6}{(x-1)^3}$$

$x=4:$

$$y = \frac{75.6}{3^3}$$

$$= \frac{75.6}{27}$$

$$= 2.8$$

$y = 2.8$ [3]

- 22 y is inversely proportional to the square of $(x+3)$.
When $x = 5$, $y = 0.375$.

Find y in terms of x .

$$y \propto \frac{1}{(x+3)^2}$$

$$y = \frac{k}{(x+3)^2}$$

$x=5, y=0.375:$

$$0.375 = \frac{k}{8^2}$$

$$0.375 = \frac{k}{64 \times 64}$$

$$k = 24$$

$$y = \frac{24}{(x+3)^2}$$

$$y = \frac{24}{(x+3)^2} \quad [2]$$

20 $y \propto \frac{1}{\sqrt{x}}$

- (a) When $x = 9$, $y = 2$.

Find the value of y when $x = 36$.

$$y = \frac{k}{\sqrt{x}}$$

$x=9, y=2:$

$$2 = \frac{k}{\sqrt{9}}$$

$$2 = \frac{k}{3} \quad k=6$$

$$y = \frac{6}{\sqrt{x}}$$

$x=36:$

$$y = \frac{6}{\sqrt{36}}$$

$$= \frac{6}{6} = 1$$

$$y = 1 \quad [3]$$

- (b) When x is increased by a factor of 4, the value of y changes by a factor of p .

Find the value of p .

Increased by a factor of 4:

$$x \rightarrow 4x$$

Sub. $4x:$

$$y = \frac{k}{\sqrt{4x}}$$

$$y = \frac{k}{2\sqrt{x}}$$

$$y = \frac{1}{2} \times \frac{k}{\sqrt{x}}$$

so y has changed by a factor of $\frac{1}{2}$

$$p = \frac{1}{2} \quad [1]$$

- 7 y varies inversely as x .
When $x = 3$, $y = 16$.

Find x when $y = 6$.

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

$x=3, y=16$:

$$16 = \frac{k}{3} \times 3$$

$$k = 48$$

$$y = \frac{48}{x}$$

$y=6$:

$$6 = \frac{48}{x}$$

$$6x = 48$$

$$x = 8$$

$$x = \underline{8} \quad [3]$$

- 21 The force of attraction, F Newtons, between two magnets is inversely proportional to the square of the distance, d cm, between the magnets.

When $d = 1.5$, $F = 48$.

(a) Find an expression for F in terms of d .

$$F \propto \frac{1}{d^2}$$

$$F = \frac{k}{d^2}$$

$d=1.5, F=48$:

$$48 = \frac{k}{1.5^2}$$

$$48 = \frac{k}{2.25}$$

$$108 = k$$

$$F = \frac{108}{d^2}$$

$$F = \underline{\frac{108}{d^2}} \quad [2]$$

(b) When the distance between the two magnets is doubled the new force is n times the original force.

Work out the value of n .

distance is doubled:

$$d \rightarrow 2d$$

sub. $2d$:

$$F = \frac{108}{(2d)^2}$$

$$F = \frac{108}{4d^2}$$

$$F = \frac{1}{4} \times \frac{108}{d^2}$$

so F is now $\frac{1}{4}$ of the original force

$$n = \underline{\frac{1}{4}} \quad [1]$$

- 12 y is inversely proportional to the square root of x .
 v is directly proportional to y^2 .
 When $x = 9$, $y = 2$ and $v = 12$.

Find v in terms of x .

Give your answer in its simplest form.

$$\begin{aligned}
 y &\propto \frac{1}{\sqrt{x}} \\
 y &= \frac{k}{\sqrt{x}} \\
 x=9, y=2: \\
 2 &= \frac{k}{\sqrt{9}} \\
 2 &= \frac{k}{3} \quad \times 3 \\
 \underline{k = 6}
 \end{aligned}
 \quad \rightarrow \quad
 \begin{aligned}
 y &= \frac{6}{\sqrt{x}} \quad (1) \\
 v &\propto y^2 \\
 v &= Ky^2 \\
 y=2, v=12: \\
 12 &= K \times 2^2 \\
 12 &= K \times 4 \\
 \div 4 \quad \div 4 \\
 \underline{K = 3}
 \end{aligned}
 \quad \rightarrow \quad
 \begin{aligned}
 v &= 3y^2 \quad (2) \\
 \text{Sub. (1) into (2):} \\
 v &= 3 \left(\frac{6}{\sqrt{x}} \right)^2 \\
 &= 3 \times \frac{36}{x} \\
 v &= \frac{108}{x} \\
 v &= \frac{108}{x} \quad [4]
 \end{aligned}$$

- 18 (a) y is directly proportional to the cube root of $(x+1)$.
 When $x = 7$, $y = 1$.

Find the value of y when $x = 124$.

$$\begin{aligned}
 y &\propto \sqrt[3]{x+1} \\
 y &= k\sqrt[3]{x+1} \\
 x=7, y=1: \\
 1 &= k\sqrt[3]{8} \\
 1 &= 2k \\
 \div 2 \quad \div 2 \\
 \underline{k = \frac{1}{2}}
 \end{aligned}
 \quad \rightarrow \quad
 \begin{aligned}
 y &= \frac{1}{2} \sqrt[3]{x+1} \\
 x=124: \\
 y &= \frac{1}{2} \sqrt[3]{125} \\
 &= \frac{1}{2} \times 5 \\
 &= 2.5 \\
 y &= 2.5 \quad [3]
 \end{aligned}$$

- (b) F is inversely proportional to the square of d .

Explain what happens to F when d is halved.

$$\begin{aligned}
 F &\propto \frac{1}{d^2} \\
 F &= \frac{k}{d^2} \\
 d \rightarrow \frac{d}{2}: \\
 F &= \frac{k}{\left(\frac{d}{2}\right)^2} \\
 &= \frac{k \times 4}{\frac{d^2}{4} \times 4} \\
 F &= \frac{4k}{d^2} \\
 F &= 4 \times \frac{k}{d^2} \quad [1]
 \end{aligned}$$

$\rightarrow$ So when d is halved, F increases by a factor of 4.

- 25 w is proportional to the square root of y .
 y is inversely proportional to x .
 When $x = 4$, $y = 16$ and $w = 8$.

Find w in terms of x .

$$w \propto \sqrt{y}$$

$$w = k\sqrt{y}$$

$w = 8, y = 16:$

$$8 = k\sqrt{16}$$

$$8 = 4k$$

$$\div 4 \quad \div 4$$

$$k = 2$$

$$w = 2\sqrt{y} \quad (1)$$

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

$x = 4, y = 16:$

$$16 = \frac{k}{4}$$

$$\times 4 \quad \times 4$$

$$64 = k$$

$$y = \frac{64}{x} \quad (2)$$

Sub. (2) into (1):

$$w = 2\sqrt{\frac{64}{x}}$$

$$= 2 \times \frac{\sqrt{64}}{\sqrt{x}}$$

$$= 2 \times \frac{8}{\sqrt{x}}$$

$$w = \frac{16}{\sqrt{x}} \quad [3]$$

- 23 y is inversely proportional to $\sqrt{x}$ and x is directly proportional to w^2 .
 When $w = 12$, $y = 12$.

Find y in terms of w .

$$y \propto \frac{1}{\sqrt{x}}$$

$$x \propto w^2$$

$$x = kw^2 \quad (1)$$

$$y = \frac{k}{\sqrt{x}} \quad (2)$$

Sub. (1) into (2):

$$y = \frac{k}{\sqrt{kw^2}}$$

$$y = \frac{k}{\sqrt{k}} \times \frac{1}{\sqrt{w^2}}$$

can use a single letter to represent this constant: C

$$y = \frac{C}{w}$$

$w = 12, y = 12:$

$$12 = \frac{C}{12}$$

$$\times 12 \quad \times 12$$

$$C = 144$$

$$y = \frac{144}{w}$$

- (e) The energy of a moving object is directly proportional to the square of its speed.
 The speed of the object is increased by 30%. $\rightarrow 100\% + 30\% = 130\%$.

Calculate the percentage increase in the energy of the object.

$$E \propto v^2$$

$v \rightarrow 1.3v$

$$E = kv^2$$

$$E = k \times (1.3v)^2$$

$$= k \times 1.69v^2$$

$$= 1.69 \times kv^2$$

so E has increased by 69% $(100\% + 69\% = 169\%)$

$$\underline{69} \quad \% [2]$$