

A Cubics

21 Expand and simplify $(x+3)(x-5)(3x-1)$.

$$\begin{aligned} & (x+3)[3x^2 - x - 15x + 5] \\ &= (x+3)[3x^2 - 16x + 5] \\ &= 3x^3 - 16x^2 + 5x + 9x^2 - 48x + 15 \\ &= \underline{3x^3 - 7x^2 - 43x + 15} \end{aligned}$$

[3]

20 Expand and simplify.

$$\begin{aligned} & (x-2)(2x+5)(x+3) \\ & (x-2)[2x^2 + 6x + 5x + 15] \\ &= (x-2)[2x^2 + 11x + 15] \\ &= 2x^3 + 11x^2 + 15x - 4x^2 - 22x - 30 \\ &= \underline{2x^3 + 7x^2 - 7x - 30} \end{aligned}$$

[3]

22 (a) Expand and simplify.

$$\begin{aligned} & (2x-1)(x+4)(x-3) \\ & (2x-1)[x^2 - 3x + 4x - 12] \\ &= (2x-1)[x^2 + x - 12] \\ &= 2x^3 + 2x^2 - 24x - x^2 - x + 12 \\ &= \underline{2x^3 + x^2 - 25x + 12} \end{aligned}$$

[3]

(c) Expand and simplify.

$$(y+3)(y-4)(2y-1)$$

$$(y+3)[2y^2 - y - 8y + 4]$$

$$= (y+3)[2y^2 - 9y + 4]$$

$$= 2y^3 - 9y^2 + 4y + 6y^2 - 27y + 12$$

$$= \underline{2y^3 - 3y^2 - 23y + 12}$$

[3]

(e) Expand and simplify.

$$(2x-3)(x+6)(x-4)$$

$$(2x-3)[x^2 - 4x + 6x - 24]$$

$$= (2x-3)[x^2 + 2x - 24]$$

$$= 2x^3 + 4x^2 - 48x - 3x^2 - 6x + 72$$

$$= \underline{2x^3 + x^2 - 54x + 72}$$

[3]

19 Expand and simplify.

$$\begin{aligned} & (2x+3)(x-2)^2 \\ & (2x+3)(x-2)(x-2) \\ & = (2x+3)[x^2 - 2x - 2x + 4] \\ & = (2x+3)[x^2 - 4x + 4] \\ & = 2x^3 - 8x^2 + 8x + 3x^2 - 12x + 12 \\ & = \underline{2x^3 - 5x^2 - 4x + 12} \end{aligned} \quad \dots\dots\dots [3]$$

21 Expand and simplify.

$$\begin{aligned} & (x-3)^2(2x+5) \\ & (x-3)(x-3)(2x+5) \\ & = (x-3)[2x^2 + 5x - 6x - 15] \\ & = (x-3)[2x^2 - x - 15] \\ & = 2x^3 - x^2 - 15x - 6x^2 + 3x + 45 \\ & = \underline{2x^3 - 7x^2 - 12x + 45} \end{aligned} \quad \dots\dots\dots [3]$$

20 $(x+a)(x+2)(2x+3)$ is equivalent to $2x^3 + bx^2 + cx - 18$.

Find the value of each of a , b and c .

$$(x+a)(2x^2 + 3x + 4x + 6)$$

$$= (x+a)(2x^2 + 7x + 6)$$

$$= 2x^3 + 7x^2 + 6x + 2ax^2 + 7ax + 6a$$

$$= 2x^3 + 2ax^2 + 7x^2 + 7ax + 6x + 6a$$

$$= 2x^3 + (2a+7)x^2 + (7a+6)x + 6a$$

$$= 2x^3 + bx^2 + cx - 18 \quad (\text{from above})$$

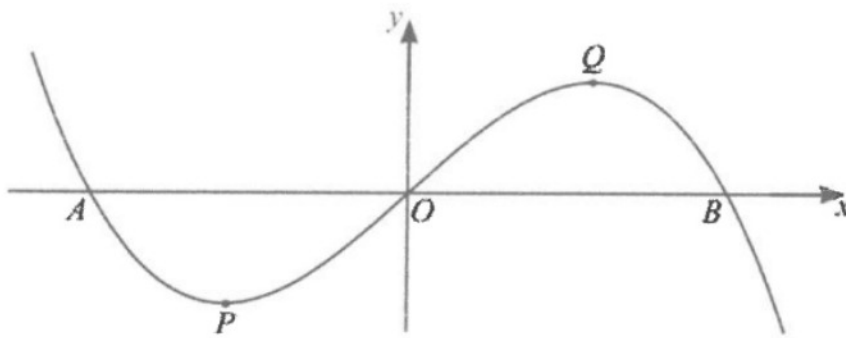
so: $6a = -18$
 $\div 6 \quad \div 6$
 $a = -3$

$\swarrow a = -3$
 $7a + 6 = c$
 $-21 + 6 = c$
 $-15 = c$

$\swarrow a = -3$
 $2a + 7 = b$
 $-6 + 7 = b$
 $1 = b$

$$\begin{aligned} a &= -3 \\ b &= 1 \\ c &= -15 \end{aligned}$$

[3]



The diagram shows the graph of $y = 3x - x^3$.
 The graph crosses the x -axis at A , at O and at B .
 The turning points of the graph are at P and at Q .

- (a) Find the x -coordinate of A and the x -coordinate of B .
 Give your answers as exact values.

Find roots by setting $y = 0$:

$$3x - x^3 = 0$$

$$x(3 - x^2) = 0$$

$$\underline{x = 0} \quad \text{or} \quad 3 - x^2 = 0$$

$$+x^2 \quad +x^2$$

$$3 = x^2$$

$$\underline{x = \pm\sqrt{3}}$$

$$x\text{-coordinate of } A \dots\dots\dots -\sqrt{3}$$

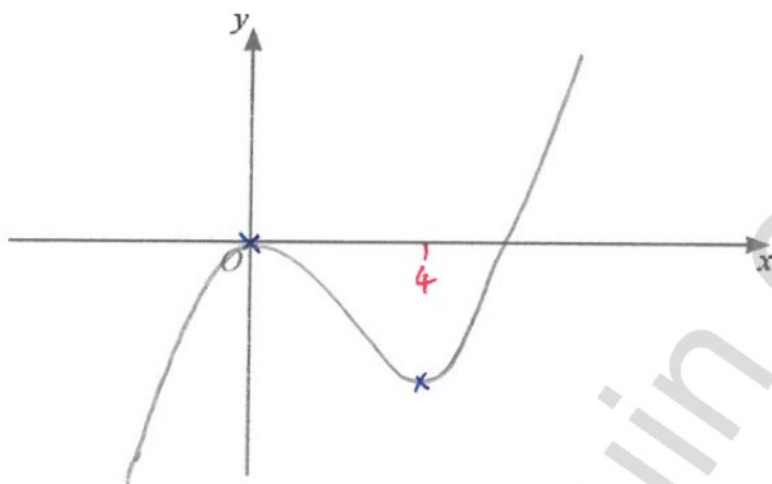
$$x\text{-coordinate of } B \dots\dots\dots \sqrt{3}$$

[3]

- 21 The graph of a cubic function has two turning points.
 When $x < 0$ and when $x > 4$ the gradient of the graph is positive.
 When $0 < x < 4$ the gradient of the graph is negative.
 The graph passes through the origin.

→ turning points at $x=0$
 and $x=4$

Sketch the graph.



[2]

9 $f(x) = x(x-1)(x-2)$

- (a) Find the coordinates of the points where the graph of $y = f(x)$ crosses the x -axis.

Set $y=0$:

$$x(x-1)(x-2) = 0$$

$$\underline{x=0} \quad \text{or} \quad \begin{array}{cc} +1 & +1 \\ x-1=0 & \\ \hline x=1 \end{array} \quad \text{or} \quad \begin{array}{cc} +2 & +2 \\ x-2=0 & \\ \hline x=2 \end{array}$$

$$(\dots\dots\dots 0 \dots\dots\dots, \dots\dots\dots 0 \dots\dots\dots)$$

$$(\dots\dots\dots 1 \dots\dots\dots, \dots\dots\dots 0 \dots\dots\dots)$$

$$(\dots\dots\dots 2 \dots\dots\dots, \dots\dots\dots 0 \dots\dots\dots) \quad [2]$$

- (b) Show that $f(x) = x^3 - 3x^2 + 2x$.

$$\begin{aligned} & x(x-1)(x-2) \\ &= x[x^2 - 2x - x + 2] \\ &= x[x^2 - 3x + 2] \\ &= \underline{\underline{x^3 - 3x^2 + 2x}} \end{aligned}$$

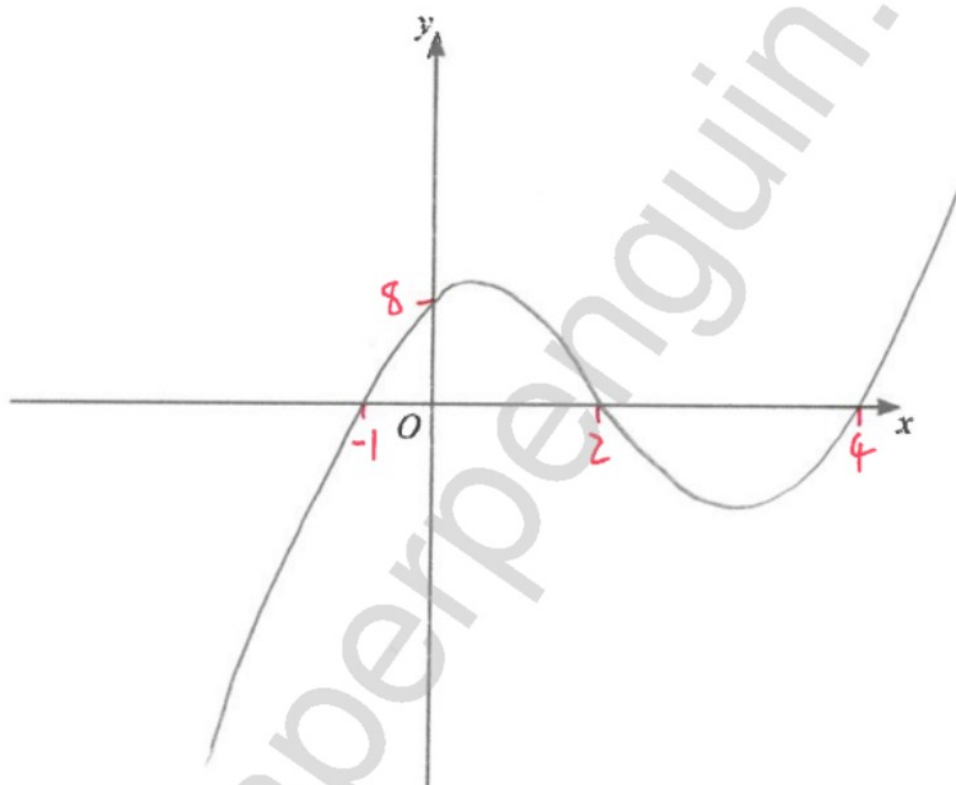
[2]

- 8 (a) (i) Show that the equation $y = (x-4)(x+1)(x-2)$ can be written as $y = x^3 - 5x^2 + 2x + 8$.

$$\begin{aligned}
 y &= (x-4)[x^2 - 2x + x - 2] \\
 &= (x-4)[x^2 - x - 2] \\
 &= x^3 - x^2 - 2x - 4x^2 + 4x + 8 \\
 &= \underline{x^3 - 5x^2 + 2x + 8}
 \end{aligned}$$

[2]

- (ii) On the diagram, sketch the graph of $y = x^3 - 5x^2 + 2x + 8$, indicating the values where the graph crosses the axes.



Roots: set $y=0$ in factorised form:

$$(x-4)(x+1)(x-2) = 0$$

$$\begin{array}{ccc}
 x-4=0 & \text{or } x+1=0 & \text{or } x-2=0 \\
 \begin{smallmatrix} +4 & +4 \end{smallmatrix} & \begin{smallmatrix} -1 & -1 \end{smallmatrix} & \begin{smallmatrix} +2 & +2 \end{smallmatrix} \\
 \underline{x=4} & \underline{x=-1} & \underline{x=2}
 \end{array}$$

y-intercept: set $x=0$:

$$\begin{aligned}
 y &= 0^3 - 5(0)^2 + 2(0) + 8 \\
 y &= 8
 \end{aligned}$$

[4]

Shape:

positive x^3 , so $y = x^3 - 5x^2 + 2x + 8$ as opposed to

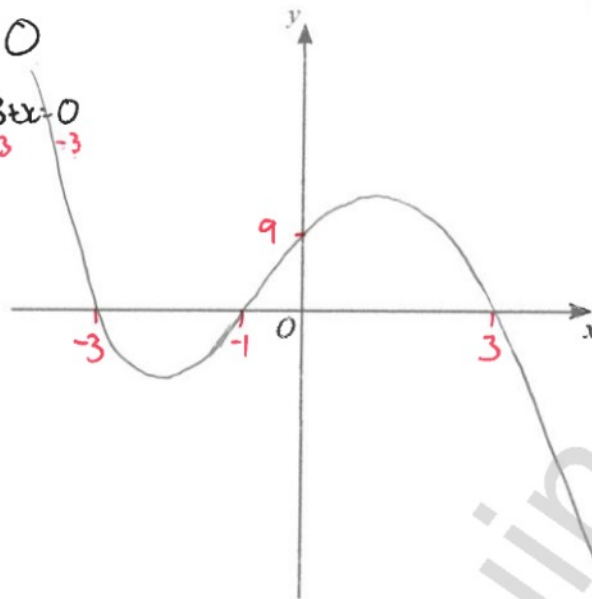
- 9 (a) Sketch the graph of $y = (x+1)(3-x)(3+x)$, indicating the coordinates of the points where the graph crosses the x-axis and the y-axis.

Roots: Set $y=0$:

$$(x+1)(3-x)(3+x) = 0$$

$$\underset{-1}{x+1}=0 \text{ or } \underset{+x}{3-x}=0 \text{ or } \underset{-3}{3+x}=0$$

$$\underline{x=-1} \quad \underline{x=3} \quad \underline{x=-3}$$



y-intercept: Set $x=0$ in
Expanded form from part (b):

$$y = 9 + 9(0) + 0^2 - 0^3$$

$$\underline{y = 9}$$

Shape:

$$y = 9 + 9x - x^2 - x^3$$

negative x^3 , so:



- (b) (i) Show that $y = (x+1)(3-x)(3+x)$ can be written as $y = 9 + 9x - x^2 - x^3$.

$$y = (x+1)[9 + 3x - 3x - x^2]$$

$$= (x+1)[9 - x^2]$$

$$= 9x - x^3 + 9 - x^2$$

$$= \underline{\underline{9 + 9x - x^2 - x^3}}$$

[2]

- 9 (a) (i) The equation $y = x^3 - 4x^2 + 4x$ can be written as $y = x(x-a)^2$.

Find the value of a .

$$\begin{aligned} y &= x[x^2 - 4x + 4] \\ &= x(x-2)(x-2) \\ &= x(x-2)^2 \end{aligned}$$

$$a = \dots\dots\dots 2 \dots\dots\dots [2]$$

- (ii) On the axes, sketch the graph of $y = x^3 - 4x^2 + 4x$, indicating the values where the graph meets the axes.

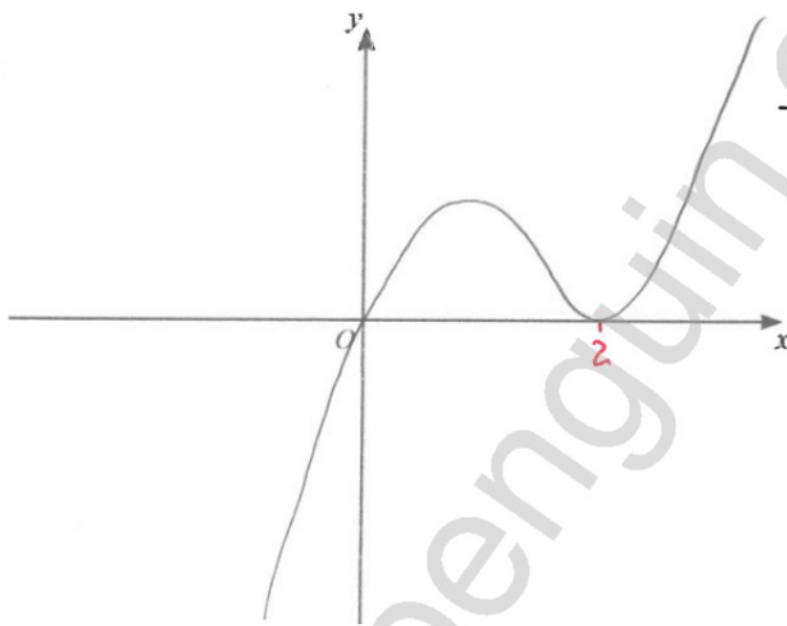
y-intercept:

$$\begin{aligned} y &= 0^3 - 4(0)^2 + 4(0) \\ y &= 0 \end{aligned}$$

Shape:

$$y = x^3 - 4x^2 + 4x$$

↑
positive cubic



roots:

$$\begin{aligned} x(x-2)^2 &= 0 \\ \underline{x=0} \quad \text{or} \quad \underline{x-2=0} \\ &\qquad\qquad\qquad \underline{x=2} \end{aligned}$$

↑
this is a repeated
root (because $(x-2)^2$)
so it touches axis
here, but doesn't cross

- 11 The graph of $y = (x-3)(x+b)(x+2)$ intersects the y-axis at -30 .

(a) Find the value of b .

$$y = (x-3)(x^2 + 2x + bx + 2b)$$

$$y = x^3 + 2x^2 + bx^2 + 2bx - 3x^2 - 6x - 3bx - 6b = \dots 5 \dots [2]$$

Set $x=0$ and the only term left is $-6b$:

$$y = -6b$$

$$-30 = -6b$$

$$\div -6 \quad \div -6$$

(b) When $x > 0$ the graph crosses the x-axis once.

Write down the coordinates of this point.

$$(x-3)(x+5)(x+2) = 0$$

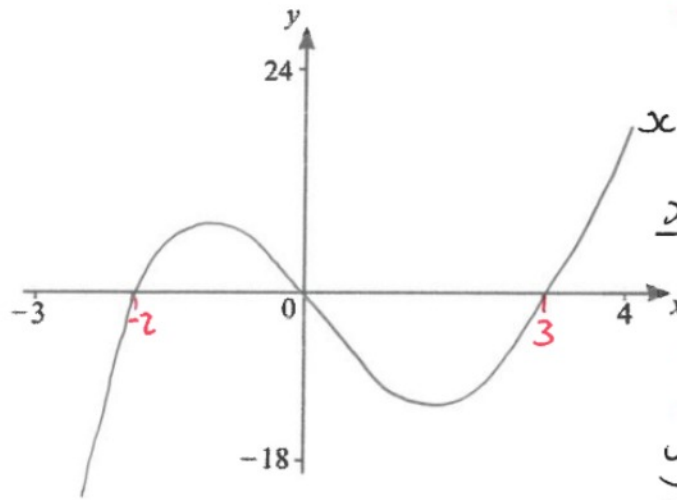
$$x-3=0 \quad x+5=0 \quad x+2=0$$

$$\begin{matrix} +3 & +3 \\ -5 & -5 \\ -2 & -2 \end{matrix}$$

$$\frac{x=3}{\checkmark} \quad \frac{x=-5}{\times} \quad \frac{x=-2}{\times}$$

(3, 0) [1]

Shape:
 $y = x^3 - x^2 - 6x$
 positive x^3 so



$$f(x) = x(x+2)(x-3)$$

roots:

$$x(x+2)(x-3) = 0$$

$$x=0 \text{ or } x+2=0 \text{ or } x-3=0$$

-2 -2 +3 +3

$$\underline{x=0} \quad \underline{x=-2} \quad \underline{x=3}$$

y-intercept: use expanded form in part (b):

$$y = 0^3 - 0^2 - 6(0)$$

$$\underline{y=0}$$

- (a) On the diagram, sketch the graph of $y = f(x)$ for $-3 \leq x \leq 4$. Show the values of the intersections with the axes.

[3]

- (b) Expand and simplify.

$$\begin{aligned} & x(x+2)(x-3) \\ &= x[x^2 - 3x + 2x - 6] \\ &= x[x^2 - x - 6] \end{aligned}$$

$$= x^3 - x^2 - 6x$$

$$\underline{x^3 - x^2 - 6x} \quad [3]$$

- (c) A is the point (1, -6).

The tangent to the graph of $y = f(x)$ at A meets the y-axis at B.

Find the coordinates of B.

(requires Differentiation)

$$\frac{dy}{dx} = 3x^2 - 2x - 6$$

sub. $x=1$ to find gradient at A:

$$\begin{aligned} m &= 3(1)^2 - 2(1) - 6 \\ &= 3 - 2 - 6 \\ &= \underline{-5} \end{aligned}$$

$$\rightarrow \underline{y = -5x + C}$$

sub. (1, -6):

$$-6 = -5(1) + C$$

$$-6 = -5 + C$$

+5 +5

$$\underline{-1 = C}$$

$$y = -5x - 1$$

← crosses y-axis at (0, -1)

$$(\underline{0}, \underline{-1}) \quad [5]$$