

A Completing the Square

(c) $(x+a)^2 = x^2 + 22x + b$

Find the value of a and the value of b .

$$x^2 + 22x = (x+11)^2 - 121$$

+121 +121

$$x^2 + 22x + 121 = (x+11)^2$$

$a = 11$
 $b = 121$ [2]

24 (a) Write $x^2 - 4x + 7$ in the form $(x-a)^2 + b$.

$$(x-2)^2 - 4 + 7$$

$$= (x-2)^2 + 3$$

$(x-2)^2 + 3$ [2]

(b) Write down the coordinates of the turning point of the graph of $y = x^2 - 4x + 7$.

$$y = (x-2)^2 + 3$$

$(2, 3)$ [1]

→ smallest this can be is 0, when $x-2=0$, so $x=2$

when $x=2$
 $y = 0^2 + 3$
 $= 3$ $(2, 3)$

24 $x^2 - 16x + a$ can be written in the form $(x + b)^2$.

Find the value of a and the value of b .

$$x^2 - 16x = (x - 8)^2 - 64$$

$+64$ $+64$

$$x^2 - 16x + 64 = (x - 8)^2$$

$a = 64$
 $b = -8$ [2]

20 $x^2 - 12x + a = (x + b)^2$

Find the value of a and the value of b .

$$x^2 - 12x = (x - 6)^2 - 36$$

$+36$ $+36$

$$x^2 - 12x + 36 = (x - 6)^2$$

$a = 36$
 $b = -6$ [2]

12 Solve $x^2 - 2x - 6 = 0$.

Give your answer in the form $a \pm \sqrt{b}$ where a and b are integers.

$$x^2 - 2x - 6 = 0$$

$$(x - 1)^2 - 1 - 6 = 0$$

$$(x - 1)^2 - 7 = 0$$

$+7$ $+7$

$$(x - 1)^2 = 7$$

$\sqrt{}$ $\sqrt{}$

$$x - 1 = \pm \sqrt{7}$$

$$x = 1 \pm \sqrt{7}$$

$1 \pm \sqrt{7}$ [3]

$$x^2 - 14x + c = (x + d)^2$$

Find the value of c and the value of d .

$$x^2 - 14x = (x - 7)^2 - 49$$

+49 +49

$$x^2 - 14x + 49 = (x - 7)^2$$

$$c = \underline{49}$$

$$d = \underline{-7} \quad [3]$$

- 18 (a) Write $x^2 - 18x - 27$ in the form $(x + k)^2 + h$.

$$(x - 9)^2 - 81 - 27$$

$$= (x - 9)^2 - 108$$

$$\underline{(x - 9)^2 - 108} \quad [2]$$

- (b) Use your answer to part (a) to solve the equation $x^2 - 18x - 27 = 0$.

$$(x - 9)^2 - 108 = 0$$

+108 +108

$$(x - 9)^2 = 108$$

√ √

$$x - 9 = \pm \sqrt{108}$$

+9 +9

$$x = 9 \pm \sqrt{108}$$

$$= 9 \pm 6\sqrt{3}$$

$$x = \underline{9 + 6\sqrt{3}} \text{ or } x = \underline{9 - 6\sqrt{3}} \quad [2]$$

$$= \underline{19.4} \quad = \underline{-1.39}$$

- 9 (a) (i) Write $x^2 + 8x - 9$ in the form $(x+k)^2 + h$.

$$(x+4)^2 - 16 - 9$$

$$(x+4)^2 - 25$$

$$(x+4)^2 - 25 \quad [2]$$

- (ii) Use your answer to part (a)(i) to solve the equation $x^2 + 8x - 9 = 0$.

$$(x+4)^2 - 25 = 0$$

$$(x+4)^2 = 25$$

$$x+4 = \pm\sqrt{25}$$

$$x+4 = \pm 5$$

$$x = -4 \pm 5$$

$$x = 1 \text{ or } x = -9 \quad [2]$$

23 $x^2 + 8x + 10 = (x+p)^2 + q$

- (a) Find the value of p and the value of q .

$$(x+4)^2 - 16 + 10$$

$$(x+4)^2 - 6$$

$$p = 4$$

$$q = -6 \quad [2]$$

- (b) Solve.

$$x^2 + 8x + 10 = 30$$

$$(x+4)^2 - 6 = 30$$

$$(x+4)^2 = 36$$

$$x+4 = \pm\sqrt{36}$$

$$x+4 = \pm 6$$

$$x = -4 \pm 6$$

$$x = 2 \text{ or } x = -10 \quad [2]$$

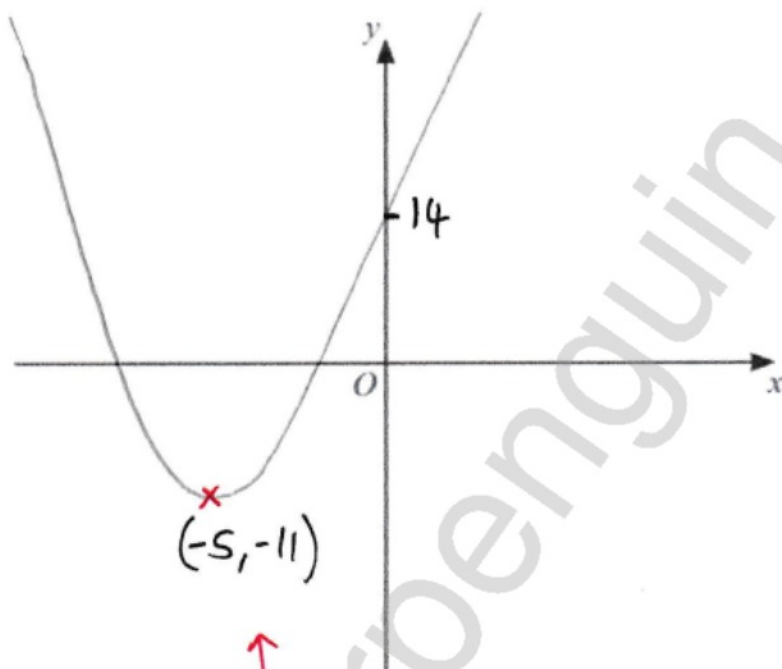
- (c) (i) Write $x^2 + 10x + 14$ in the form $(x+a)^2 + b$.

$$(x+5)^2 - 25 + 14$$

$$(x+5)^2 - 11$$

$$(x+5)^2 - 11 \quad [2]$$

- (ii) On the axes, sketch the graph of $y = x^2 + 10x + 14$, indicating the coordinates of the turning point.



$$y = (x+5)^2 - 11$$

$x+5=0$
 $x=-5$

$y=-11$

[3]

(b) (i) Write $x^2 + 4x + 1$ in the form $(x+p)^2 + q$.

$$(x+2)^2 - 4 + 1$$

$$(x+2)^2 - 3$$

$$\dots\dots\dots (x+2)^2 - 3 \dots\dots\dots [2]$$

(ii) Use your answer to part (b)(i) to solve the equation $x^2 + 4x + 1 = 0$.

$$(x+2)^2 - 3 = 0$$

$$\begin{array}{ccc} & +3 & +3 \\ (x+2)^2 & = & 3 \\ \downarrow & & \downarrow \end{array}$$

$$\begin{array}{ccc} x+2 & = & \pm\sqrt{3} \\ -2 & & -2 \end{array}$$

$$x = -2 \pm \sqrt{3}$$

$$\begin{array}{ccc} x = \frac{-2 + \sqrt{3}}{-0.268} & \text{or } x = & \frac{-2 - \sqrt{3}}{-3.73} \\ & & \end{array} [2]$$

(iii) Use your answer to part (b)(i) to write down the coordinates of the minimum point on the graph of $y = x^2 + 4x + 1$.

$$\begin{array}{ccc} y = (x+2)^2 - 3 & & \\ \underbrace{\hspace{1cm}}_{=0} & \uparrow & y = -3 \\ \text{so } x = -2 & & \end{array}$$

$$(\dots\dots\dots -2 \dots\dots\dots, \dots\dots\dots -3 \dots\dots\dots) [2]$$

18 $2x^2 + 12x - 2$ can be written in the form $a(x+b)^2 - c$.

Find the values of a , b and c .

$$2[x^2 + 6x] - 2$$

$$2[(x+3)^2 - 9] - 2$$

$$2(x+3)^2 - 18 - 2$$

$$2(x+3)^2 - 20$$

$$a = \dots 2 \dots, b = \dots 3 \dots, c = \dots 20 \dots [3]$$

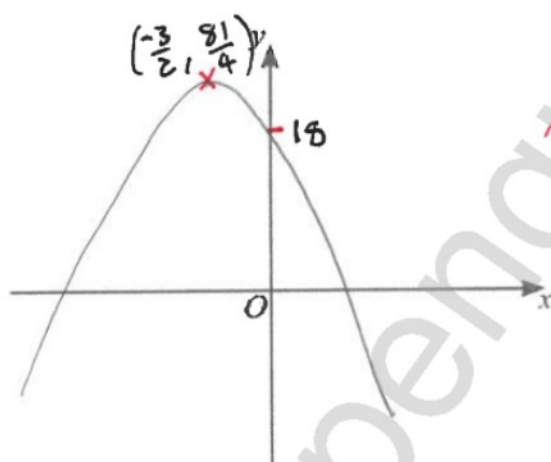
(b) $g(x) = 18 - 3x - x^2$

(i) Write $g(x)$ in the form $b - (a+x)^2$.

$$\begin{aligned}
 & -x^2 - 3x + 18 \\
 & -[x^2 + 3x] + 18 \\
 & -\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 18 \\
 & -\left(x + \frac{3}{2}\right)^2 + \frac{9}{4} + 18 \\
 & \qquad \qquad \qquad -\left(x + \frac{3}{2}\right)^2 + \frac{81}{4} \\
 & \qquad \qquad \qquad \frac{81}{4} - \left(x + \frac{3}{2}\right)^2 \\
 & \qquad \qquad \qquad \frac{81}{4} - \left(x + \frac{3}{2}\right)^2 \quad [3]
 \end{aligned}$$

(ii) Sketch the graph of $y = g(x)$.

On your sketch, show the coordinates of the turning point.



$$g(x) = 18 - 3x - x^2$$

negative quadratic:

max pt:

$$g(x) = -\left(x + \frac{3}{2}\right)^2 + \frac{81}{4}$$

= 0
when $x = -\frac{3}{2}$

$y = \frac{81}{4}$
(= 20.25)

(d) The expression $5 + 12x - 2x^2$ can be written in the form $q - 2(x+p)^2$.

(i) Find the value of p and the value of q .

$$-2x^2 + 12x + 5$$

$$-2[x^2 - 6x] + 5$$

$$-2[(x-3)^2 - 9] + 5$$

$$-2(x-3)^2 + 18 + 5$$

$$-2(x-3)^2 + 23$$

$$23 - 2(x-3)^2$$

$$p = -3, q = 23 \quad [3]$$

(ii) Write down the coordinates of the maximum point of the curve $y = 5 + 12x - 2x^2$.

$$y = -2(x-3)^2 + 23$$

= 0 when
 $x = 3$

$y = 23$

$$(3, 23) \quad [1]$$

- (b) The solutions of the equation $x^2 + bx + c = 0$ are $\frac{-7 + \sqrt{61}}{2}$ and $\frac{-7 - \sqrt{61}}{2}$.

Find the value of b and the value of c .

$$x = \frac{-7 \pm \sqrt{61}}{2}$$

$\times 2$ 2×2

$$2x = -7 \pm \sqrt{61}$$

$+7$ $+7$

$$2x + 7 = \pm \sqrt{61}$$

Square Square

$$(2x + 7)^2 = 61$$

$$(2x + 7)(2x + 7) = 61$$

$$4x^2 + 14x + 14x + 49 = 61$$

$$4x^2 + 28x + 49 = 61$$

-61 -61

$$4x^2 + 28x - 12 = 0 \quad \div 4$$

$$x^2 + 7x - 3 = 0$$

$$b = 7$$

$$c = -3 \quad [3]$$

- 16 The solutions to the equation $x^2 + gx + h = 0$ are $\frac{1 - \sqrt{17}}{2}$ and $\frac{1 + \sqrt{17}}{2}$.

Find the value of g and the value of h .

$$x = \frac{1 \pm \sqrt{17}}{2}$$

$\times 2$ 2×2

$$2x = 1 \pm \sqrt{17}$$

-1 -1

$$2x - 1 = \pm \sqrt{17}$$

Square Square

$$(2x - 1)^2 = 17$$

$$(2x - 1)(2x - 1) = 17$$

$$4x^2 - 2x - 2x + 1 = 17$$

$$4x^2 - 4x + 1 = 17$$

-17 -17

$$4x^2 - 4x - 16 = 0 \quad \div 4$$

$$x^2 - x - 4 = 0$$

$$g = -1$$

$$h = -4 \quad [3]$$