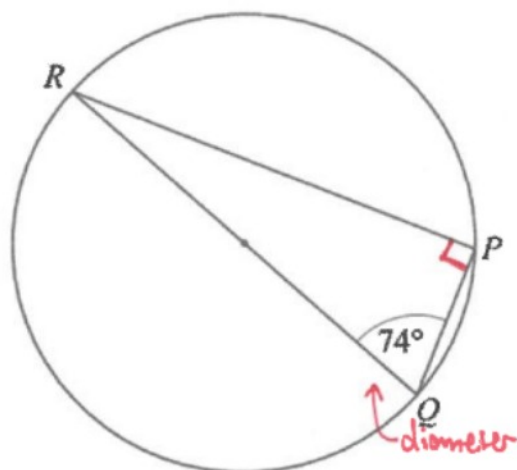


12

NOT TO
SCALE

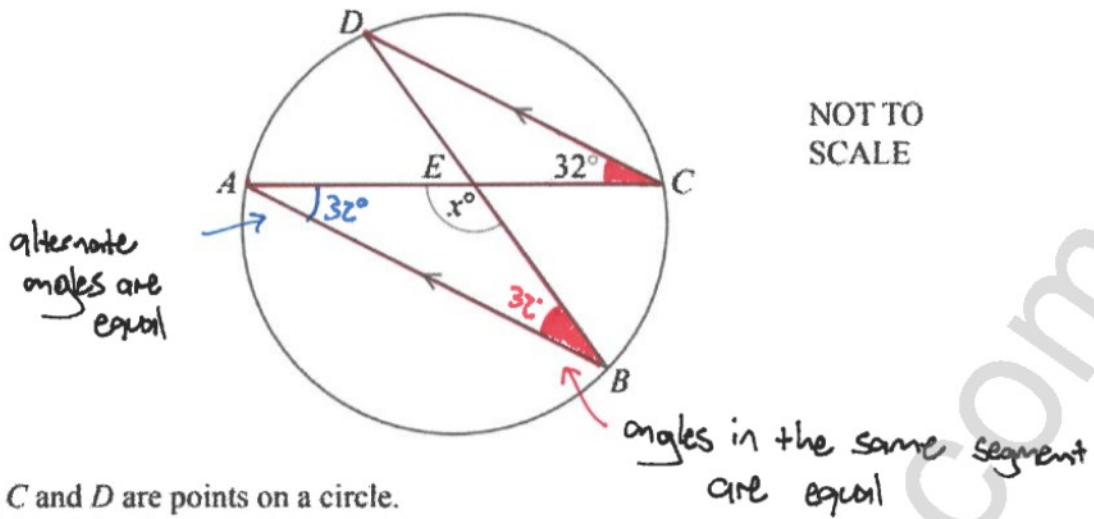
P , Q and R lie on a circle.
 QR is a diameter.

$$180 - (90 + 74) = 16^\circ$$

Find angle PRQ .
 Give geometrical reasons for your answer.

Angle $PRQ = 16^\circ$ because angle in a semi-circle = 90° and
 angles in a triangle sum to 180° . [2]

13



A, B, C and D are points on a circle.
 AB is parallel to DC and angle $ACD = 32^\circ$.
 Chords AC and DB intersect at E .

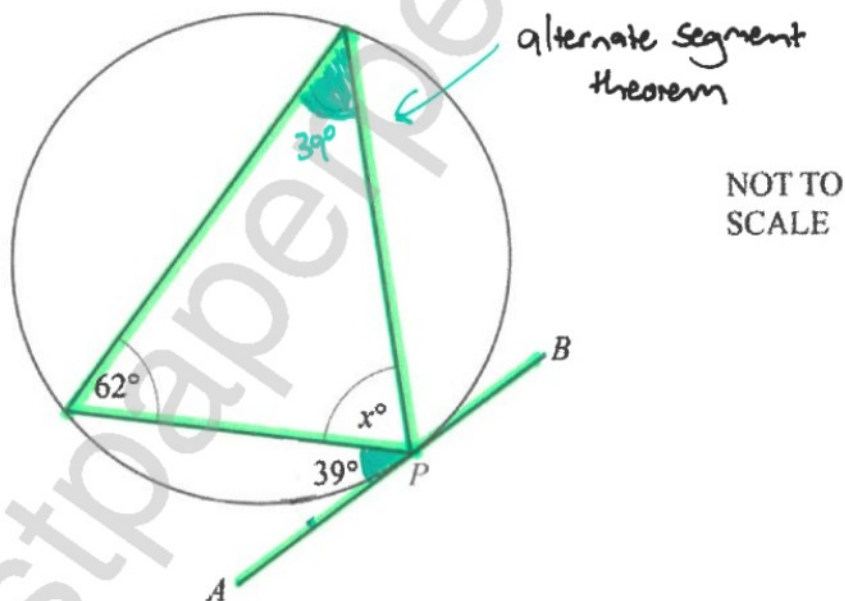
Find the value of x .

$$180 - (32 + 32) = 116^\circ$$

angles in a triangle

$$x = 116^\circ \quad [2]$$

10

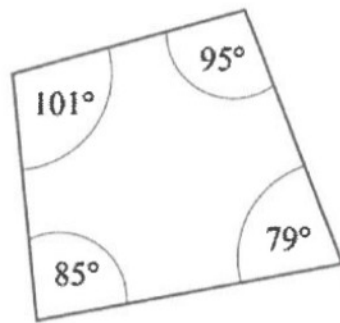


APB is a tangent to the circle at P .

Work out the value of x .

$$180 - (62 + 39) = 79^\circ$$

$$x = 79^\circ \quad [2]$$



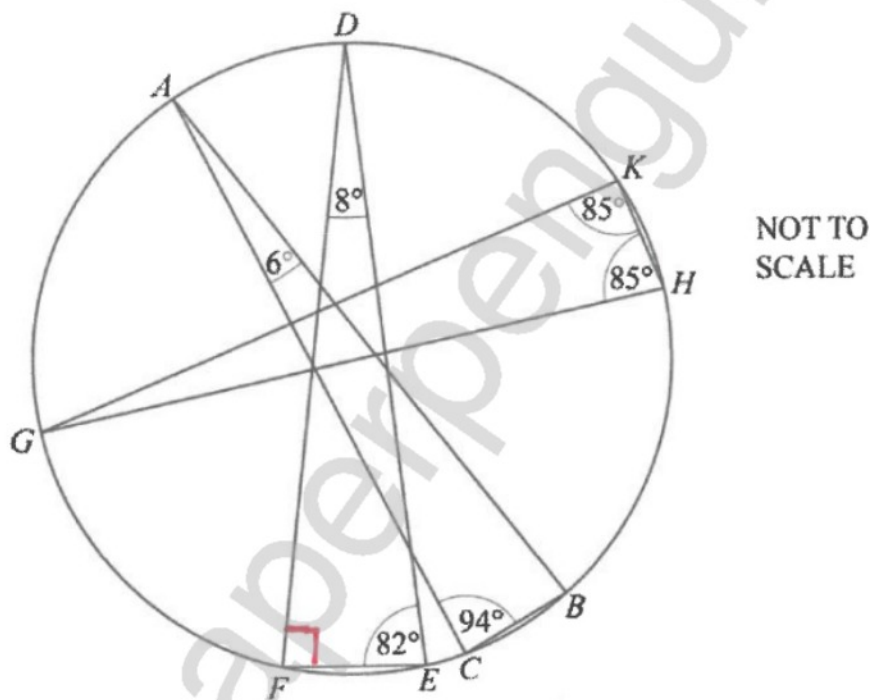
NOT TO
SCALE

The diagram shows a quadrilateral.

Give a geometrical reason why this is a cyclic quadrilateral.

$85 + 95 = 180$ and $101 + 79 = 180$: opposite angles sum to 180° [1]

- 11 ABC , DEF and GKH are triangles with all vertices on the circumference of a circle.



NOT TO
SCALE

From the list, draw a ring around the line that is a diameter of the circle.

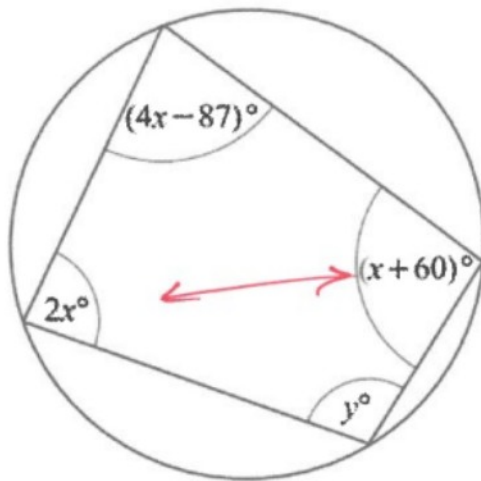
AB AC DE DF GH GK

[1]

DEF: $8 + 82 = 90$

$180 - 90 = 90$

(angle in a semi-circle is 90°)



NOT TO
SCALE

The diagram shows a cyclic quadrilateral.

Find the value of y .

$$2x + x + 60 = 180$$

$$3x + 60 = 180$$

$$\begin{array}{r} -60 \\ -60 \end{array}$$

$$3x = 120$$

$$\begin{array}{r} :3 \\ :3 \end{array}$$

$$x = 40$$

Find value of $4x - 87$ by sub. $x = 40$:

$$4(40) - 87$$

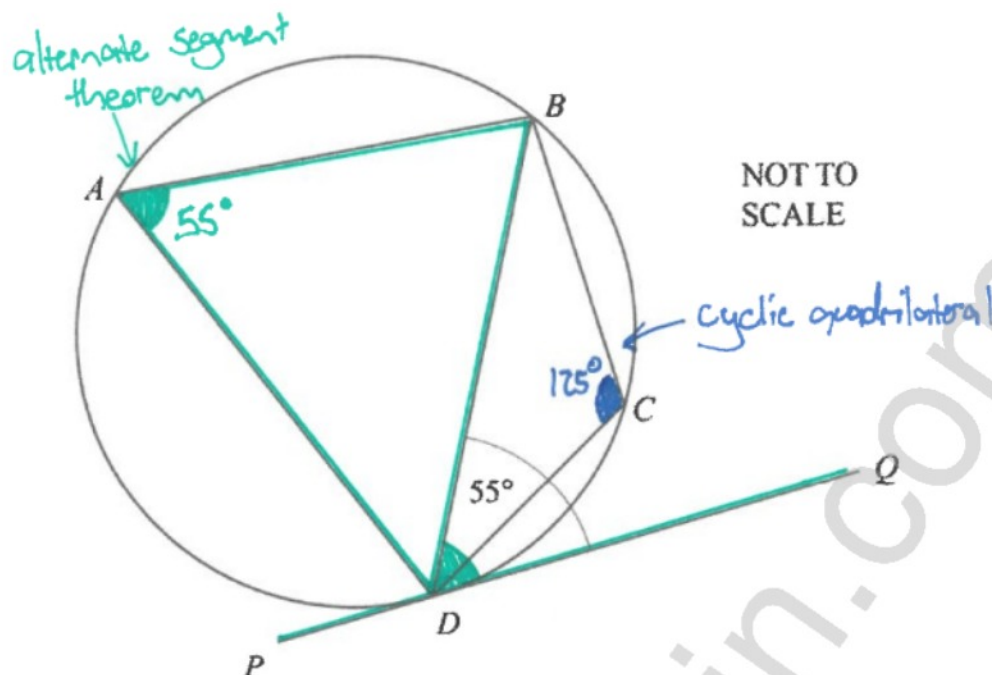
$$= 160 - 87$$

$$= 73^\circ$$

Angle y :

$$180 - 73 = 107^\circ$$

$$y = \underline{107^\circ} \quad [4]$$

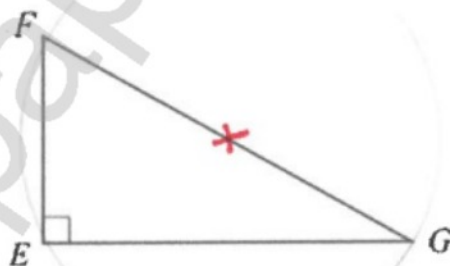


A, B, C and D are points on the circle.
 PQ is a tangent to the circle at D .
 Angle $BDQ = 55^\circ$.

Complete these statements giving a reason for each answer.

- (a) Angle $BAD = 55^\circ$ because angles in the alternate segment are equal. [2]
- (b) Angle $BCD = 125^\circ$ because opposite angles in a cyclic quadrilateral sum to 180° . $(180 - 55 = 125)$ [2]

(b)



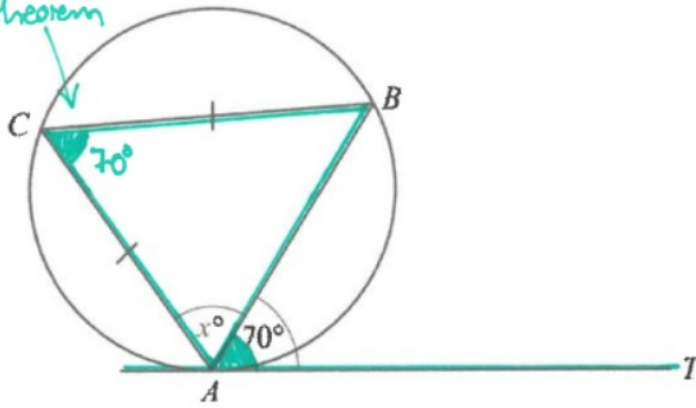
EFG is a right-angled triangle.
 A circle can be drawn that passes through the three vertices of the triangle.

On the diagram, mark the position of the centre of the circle with a cross.
 Explain how you decide.

Angle in a semi-circle = 90° , so FG is the diameter.

[2]

alternate segment
theorem



NOT TO
SCALE

A , B and C are points on a circle.
 TA is a tangent to the circle at A .
 $CA = CB$ and angle $BAT = 70^\circ$.

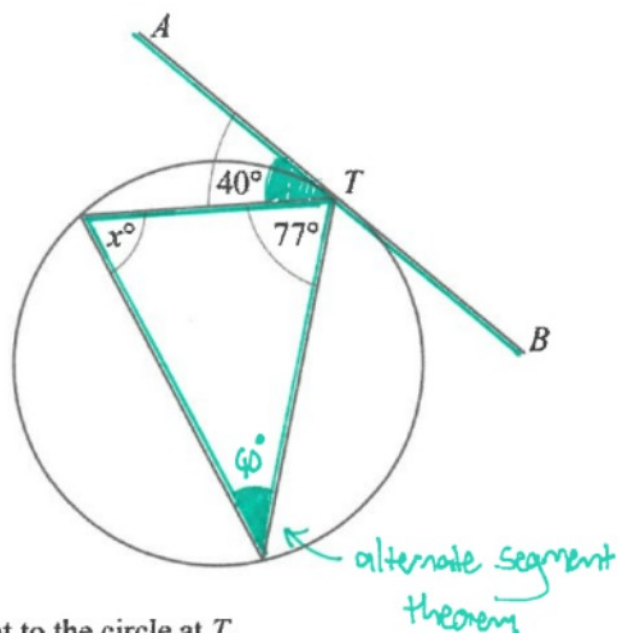
Work out the value of x .

isosceles triangle:

$$180 - 70 = 110$$

$$110 \div 2 = 55^\circ$$

$$x = \dots\dots\dots 55^\circ \dots\dots\dots [2]$$



NOT TO
SCALE

AB is a tangent to the circle at T .

Find the value of x .

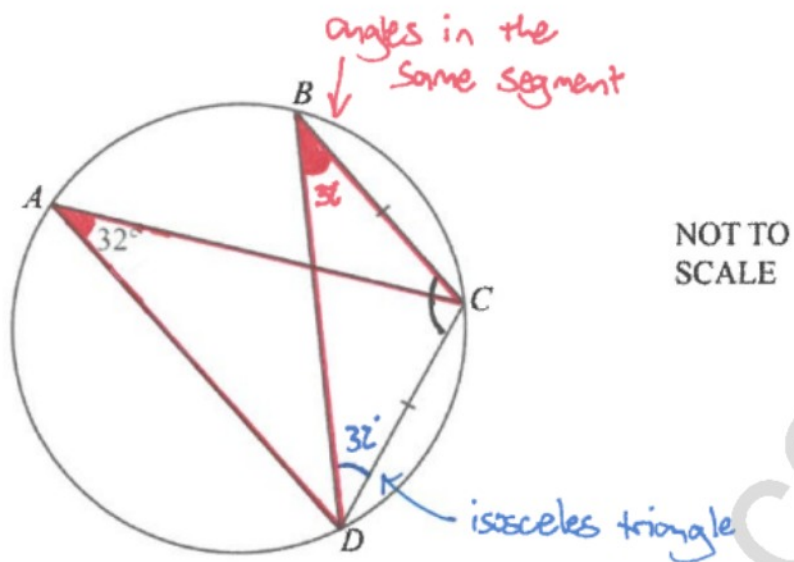
$$180 - (40 + 77)$$

$$= 63^\circ$$

(angles in a triangle)

$$x = 63^\circ \quad [2]$$

12 (a)



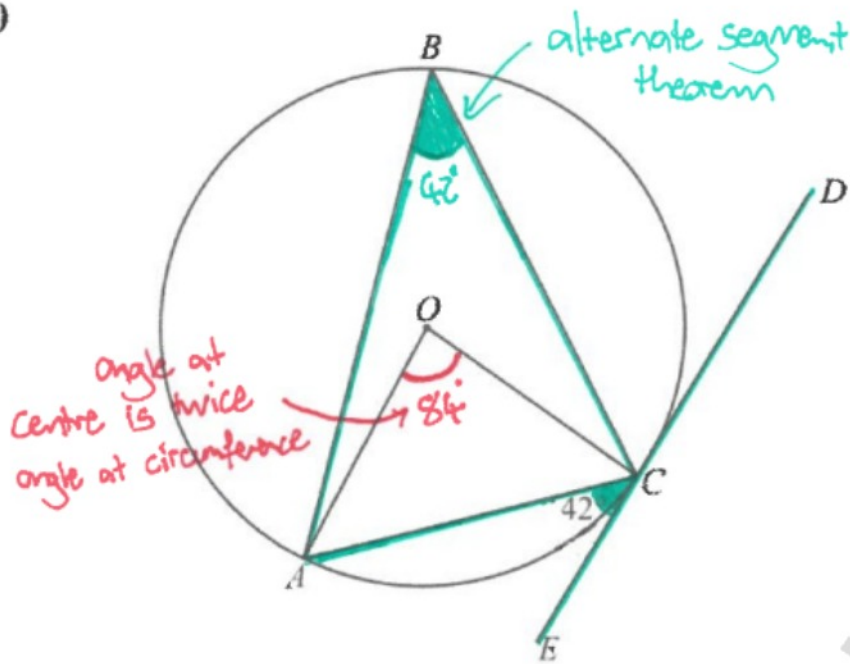
$A, B, C,$ and D are points on a circle.
Angle $DAC = 32^\circ$.
 $BC = DC$.

Find angle BCD .

$$\begin{aligned} BCD &= 180 - (32 + 32) \\ &= 116 \end{aligned}$$

Angle $BCD = 116^\circ$ [2]

(b)



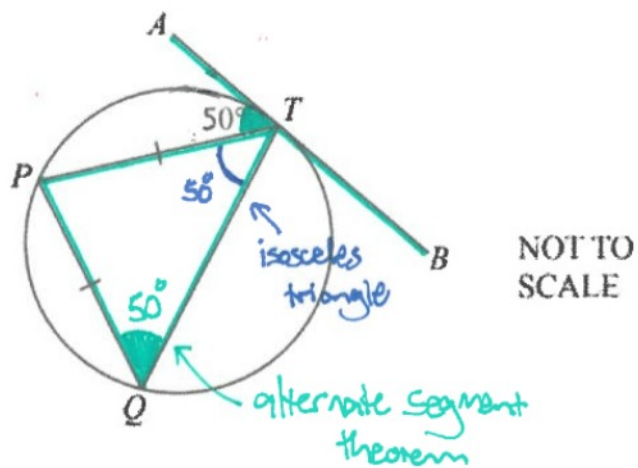
NOT TO
SCALE

A , B and C are points on the circle centre O .
 ECD is a tangent to the circle at C .
Angle $ACE = 42^\circ$.

Find angle AOC .

Angle $AOC = 84^\circ$ [2]

19 (a)



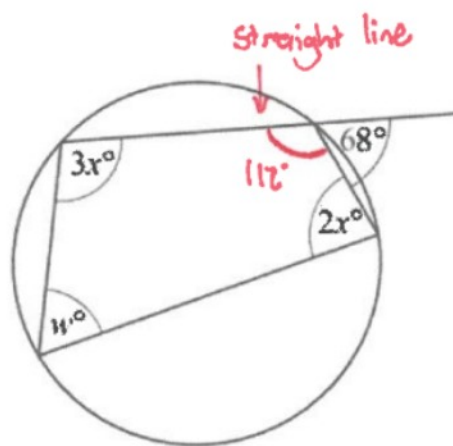
P , Q and T are points on a circle.
 ATB is a tangent to the circle at T and $PT = PQ$.

Find angle TPQ .

$$180 - (50 + 50) = 80^\circ$$

Angle $TPQ = 80^\circ$ [2]

(b)



NOT TO
SCALE

The diagram shows a cyclic quadrilateral with an exterior angle of 68° .

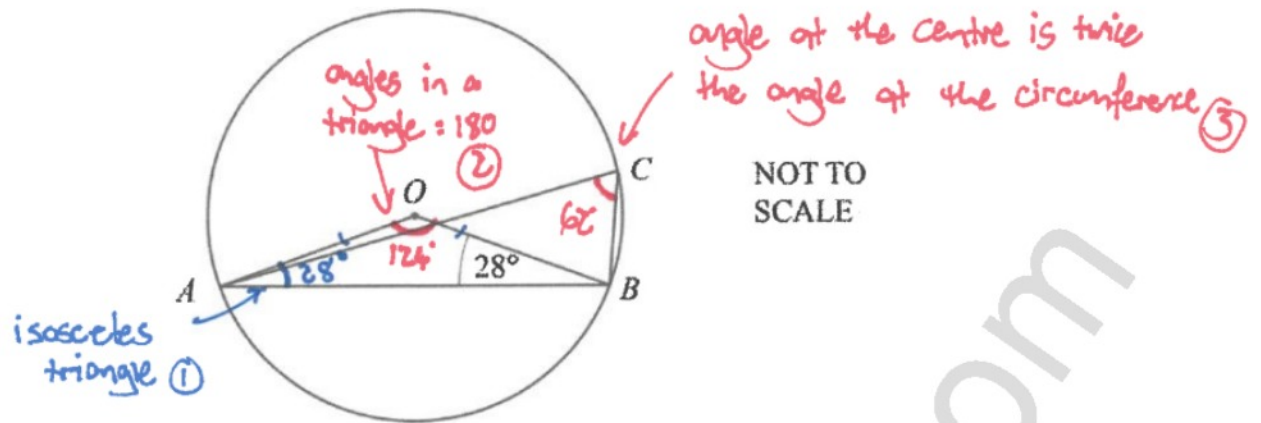
Find the value of w and the value of x .

$$\begin{aligned}w + 112 &= 180 \\- 112 & \quad - 112 \\ \hline w &= 68\end{aligned}$$

$$\begin{aligned}3x + 2x &= 180 \\5x &= 180 \\ \div 5 & \quad \div 5 \\ \hline x &= 36\end{aligned}$$

$$\begin{aligned}w &= 68 \\x &= 36\end{aligned} \quad [3]$$

17 (a)

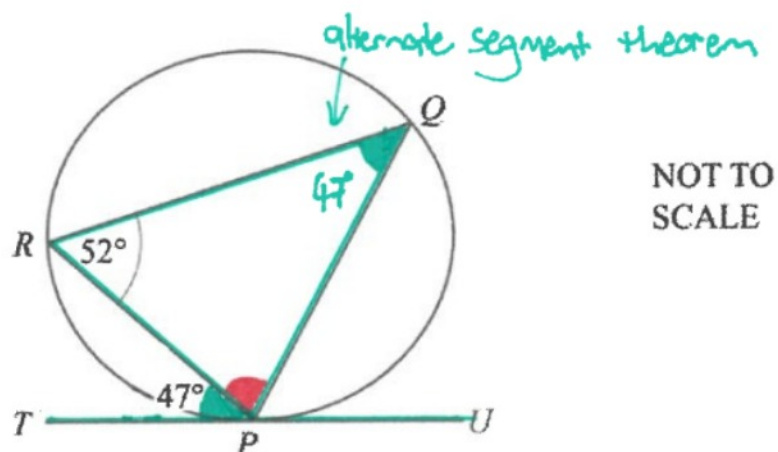


A, B and C are points on a circle, centre O .
Angle $OBA = 28^\circ$.

Find angle ACB .

Angle $ACB = 62^\circ$ [2]

(b)



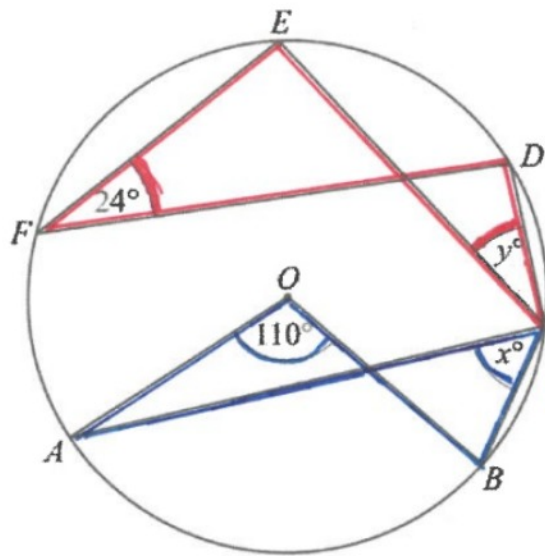
P , Q and R are points on a circle.
 TU is a tangent to the circle at P .
Angle $TPR = 47^\circ$ and angle $PRQ = 52^\circ$.

Find angle RPQ .

$$180 - (52 + 47) = 81^\circ$$

↑
angles in a triangle

Angle $RPQ = 81^\circ$ [2]



NOT TO
SCALE

$y = 24^\circ$
angles in the same segment

$x = 55^\circ$
angle at the centre is twice
the angle at the circumference

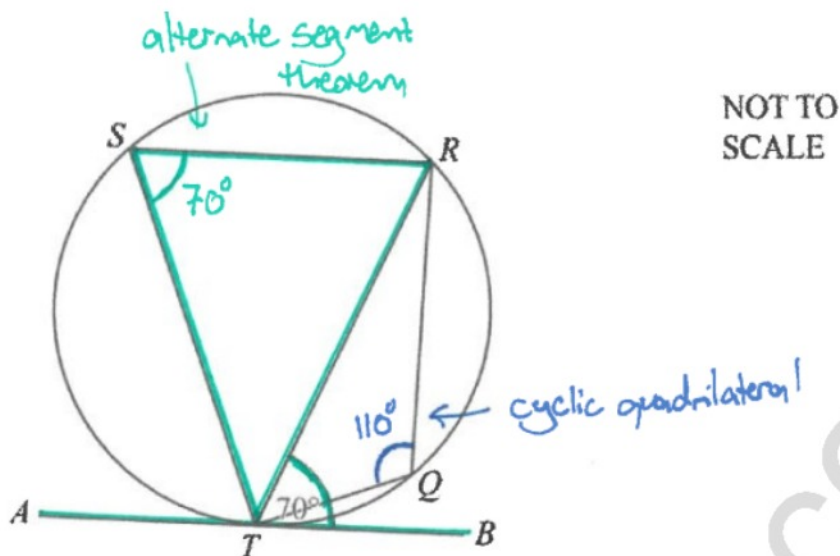
Points A, B, C, D, E and F lie on the circle, centre O .

Find the value of x and the value of y .

$$x = 55^\circ$$

$$y = 24^\circ \quad [2]$$

15

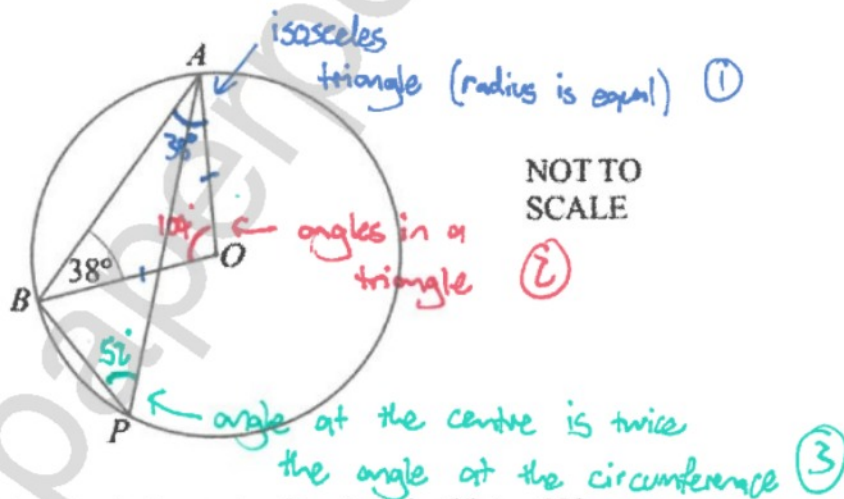


Points Q , R , S and T lie on the circle.
 AB is a tangent to the circle at T .
 Angle $RTB = 70^\circ$.

Find angle RQT .

Angle $RQT = 110^\circ$ [2]

2 (a)

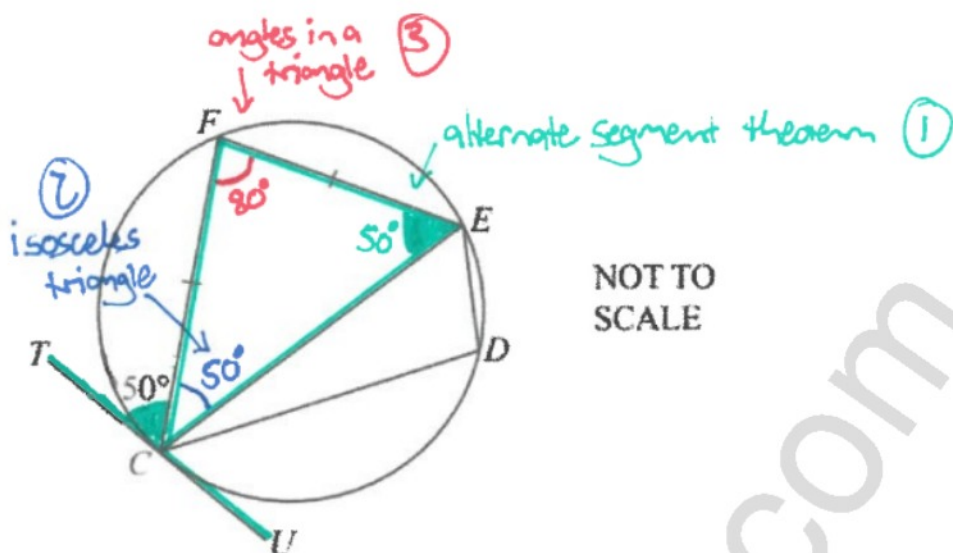


A , B and P are points on a circle, centre O and angle $OBA = 38^\circ$.

Find angle APB .

Angle $APB = 52^\circ$ [3]

(b)



$CDEF$ is a cyclic quadrilateral and $FC = FE$.
 TU is a tangent to the circle at C and angle $TCF = 50^\circ$.

Find

(i) angle EFC ,

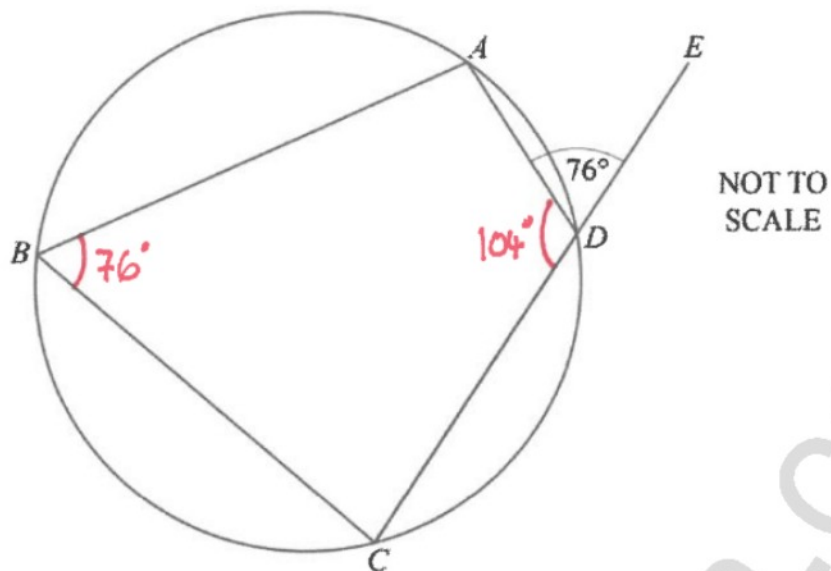
Angle $EFC = 80^\circ$ [2]

(ii) angle CDE .

$$100 - 80 = 100^\circ$$

(opposite angles in a cyclic quadrilateral) Angle $CDE = 100^\circ$ [1]

9 (a)



$A, B, C,$ and D are points on a circle.

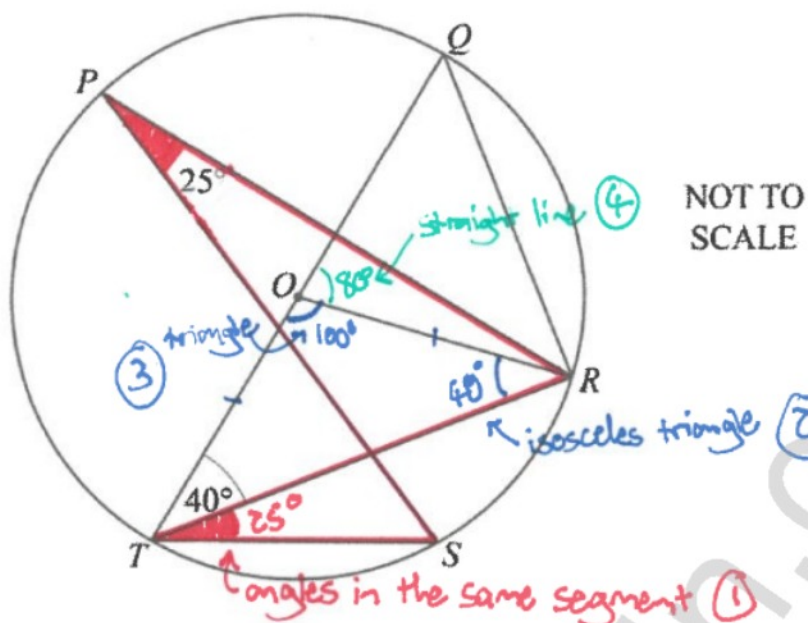
CDE is a straight line.

Find angle ABC .

$$180 - 104 = 76^\circ \quad (\text{opposite angles in a cyclic quadrilateral})$$

Angle $ABC = 76^\circ$ [1]

(b)



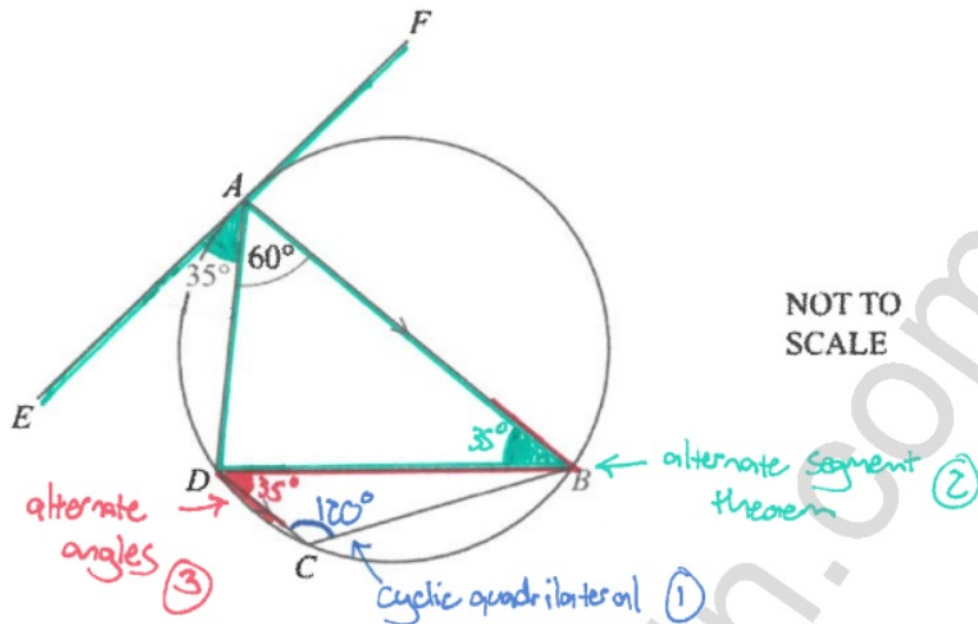
P, Q, R, S and T are points on the circle centre O .
 TOQ is a straight line.

(i) Find angle STR .

Angle $STR = 25^\circ$ [1]

(ii) Find angle QOR .

Angle $QOR = 80^\circ$ [1]



A, B, C and D are points on a circle.
 EF is a tangent to the circle at A .
 AB is parallel to DC .

- (a) Find angle DCB , giving a geometrical reason.

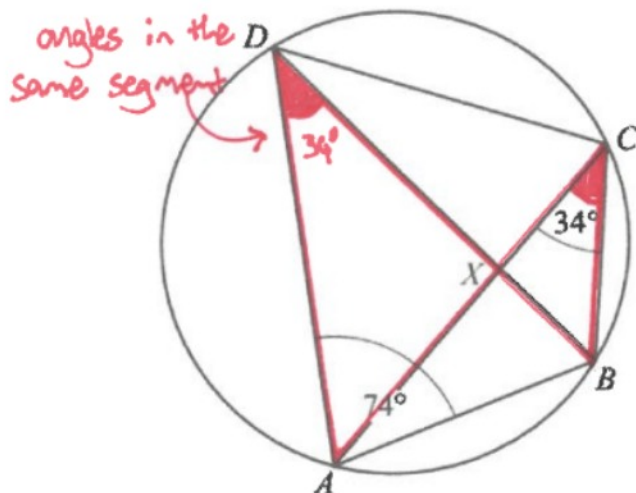
Angle $DCB = 120^\circ$ because opposite angles in a cyclic quadrilateral sum to 180° . [2]

- (b) Find angle DBC .

$$180 - (120 + 35) = 25^\circ$$

angles in a triangle
sum to 180°

Angle $DBC = 25^\circ$ [2]



NOT TO
SCALE

The diagram shows a cyclic quadrilateral $ABCD$.
 BD and AC intersect at X .

- (a) Angle $BAD = 74^\circ$ and angle $BCA = 34^\circ$.

Find

- (i) angle BDA

Angle $BDA = 34^\circ$ [1]

- (ii) angle BCD

$180 - 74 = 106^\circ$

(cyclic quadrilateral)

Angle $BCD = 106^\circ$ [1]

- (iii) angle ABD .

$180 - (34 + 74) = 72^\circ$

(angles in a triangle)

Angle $ABD = 72^\circ$ [1]

- (b) In the diagram, triangle ADX is similar to triangle BCX .
 $BC = 4.5$ cm, $AD = 9$ cm and $CX = 3.3$ cm.

Work out XD .

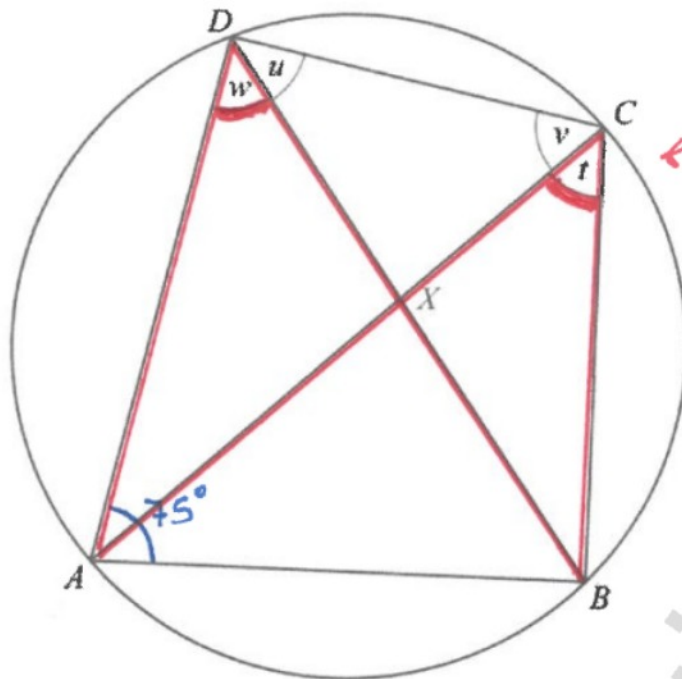
XD is similar to XC .

Use AD and BC to find scale factor.

S.F. = $\frac{9}{4.5} = 2$

$XD = 3.3 \times 2$
 $= 6.6$

$XD = 6.6$ cm [2]



← angles in the same segment

NOT TO SCALE

$ABCD$ is a cyclic quadrilateral and the diagonals AC and BD intersect at X .

(a) Complete the statement using two of t , u , v and w .

Angle t is equal to angle w

[1]

(b) Angle $DAB = 75^\circ$.

Find angle DCB .

$$180 - 75 = 105^\circ$$

(cyclic quadrilateral)

Angle $DCB = \dots\dots\dots 105^\circ \dots\dots\dots$ [1]

(c) $AB = 8$ cm, $AX = 6$ cm, $BX = 4$ cm and $DC = 5$ cm.

Work out CX .

CX is similar to BX .

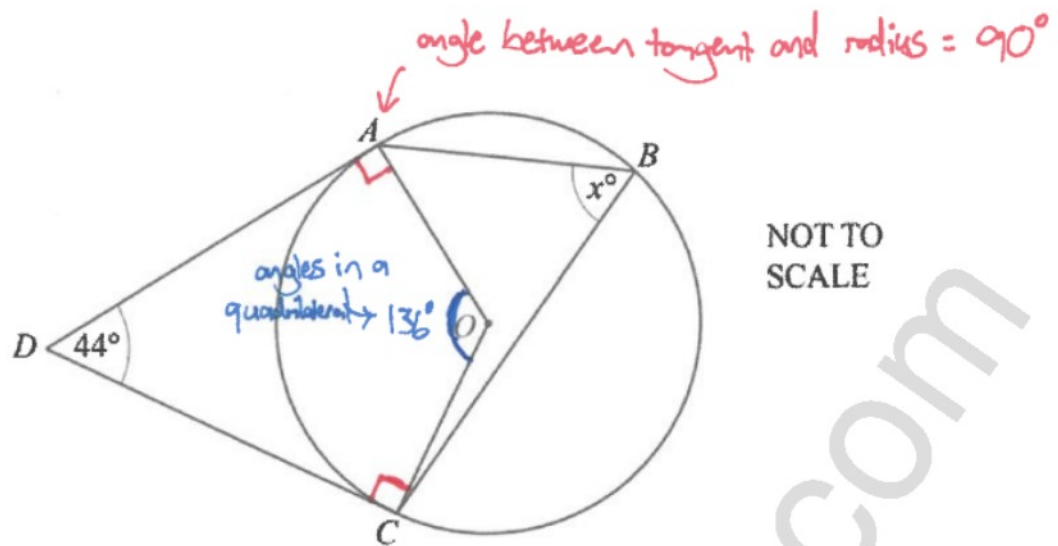
Use AB and DC to find scale factor:

$$S.F. = \frac{8}{5}$$

$$CX = 4 \div \frac{8}{5} = 2.5 \text{ cm}$$

↑
divide because going
from big triangle to small triangle.

$CX = \dots\dots\dots 2.5 \dots\dots\dots$ cm [2]



A , B and C are points on a circle, centre O .
 DA and DC are tangents.
 Angle $ADC = 44^\circ$.

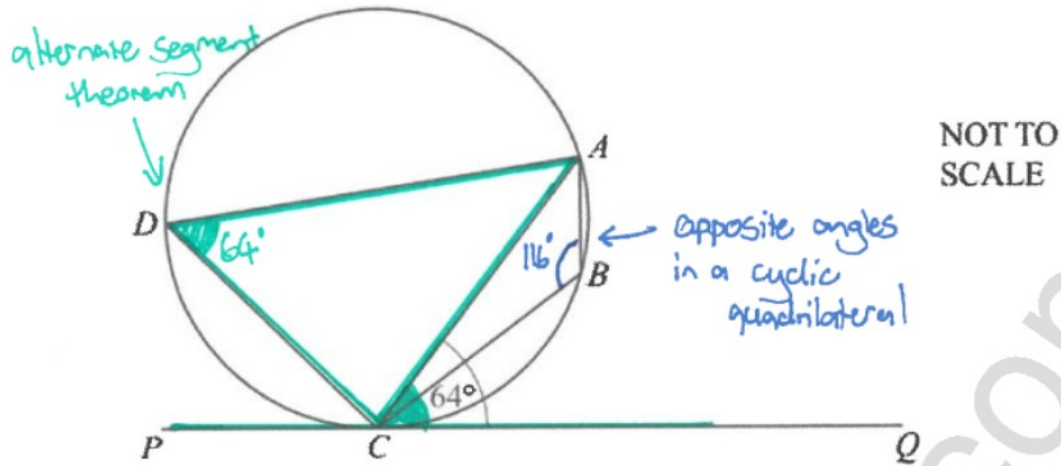
Work out the value of x .

$$x = 136 \div 2$$

$$= 68^\circ$$

(angle at the centre is twice
 the angle at the circumference.)

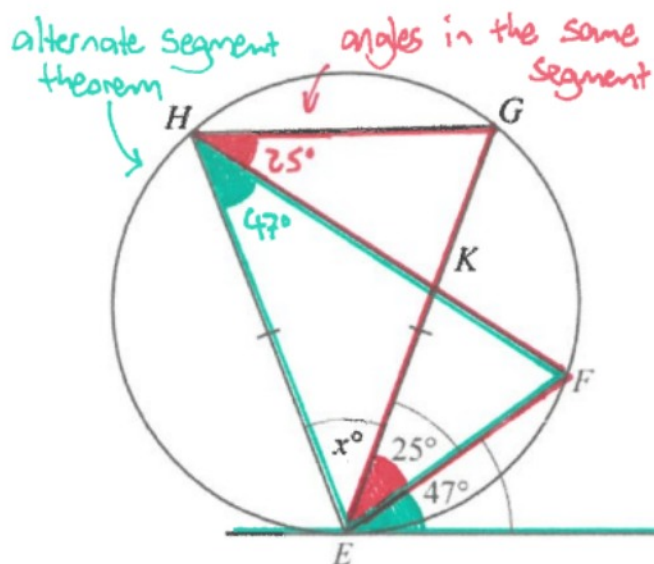
$$x = 68^\circ \quad [3]$$



A, B, C and D lie on the circle.
 PCQ is a tangent to the circle at C .
 Angle $ACQ = 64^\circ$.

Work out angle ABC , giving reasons for your answer.

Angle $ABC = 116^\circ$ because $ADC = 64^\circ$ from the alternate segment theorem. $ABC = 180 - 64 = 116$ because opposite angles in a cyclic quadrilateral sum to 180° . [3]



Points E, F, G and H lie on the circle and $EG = EH$.
 HF and EG intersect at K .
 ET is a tangent to the circle at E .
 Angle $FET = 47^\circ$ and angle $FEG = 25^\circ$.

Find the value of x .

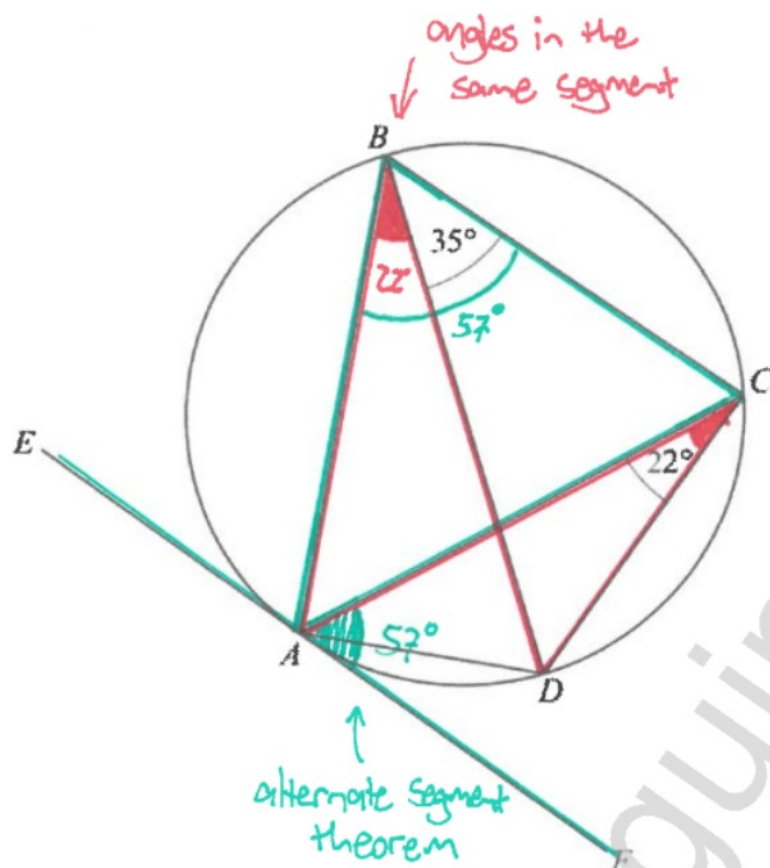
$$\begin{aligned}\hat{EHG} &= 47 + 25 \\ &= 72^\circ\end{aligned}$$

$$\hat{EHG} = 72^\circ \text{ (isosceles triangle)}$$

$$x = 180 - (72 + 72)$$

$$= \underline{36^\circ} \text{ (angles in a triangle)}$$

$$x = \underline{36^\circ} \quad [2]$$



A, B, C and D are points on the circle.
 EF is a tangent to the circle at A .
 Angle $DBC = 35^\circ$ and angle $ACD = 22^\circ$.

Find

(a) angle ABD

Angle $ABD = 22^\circ$ [1]

(b) angle ADC

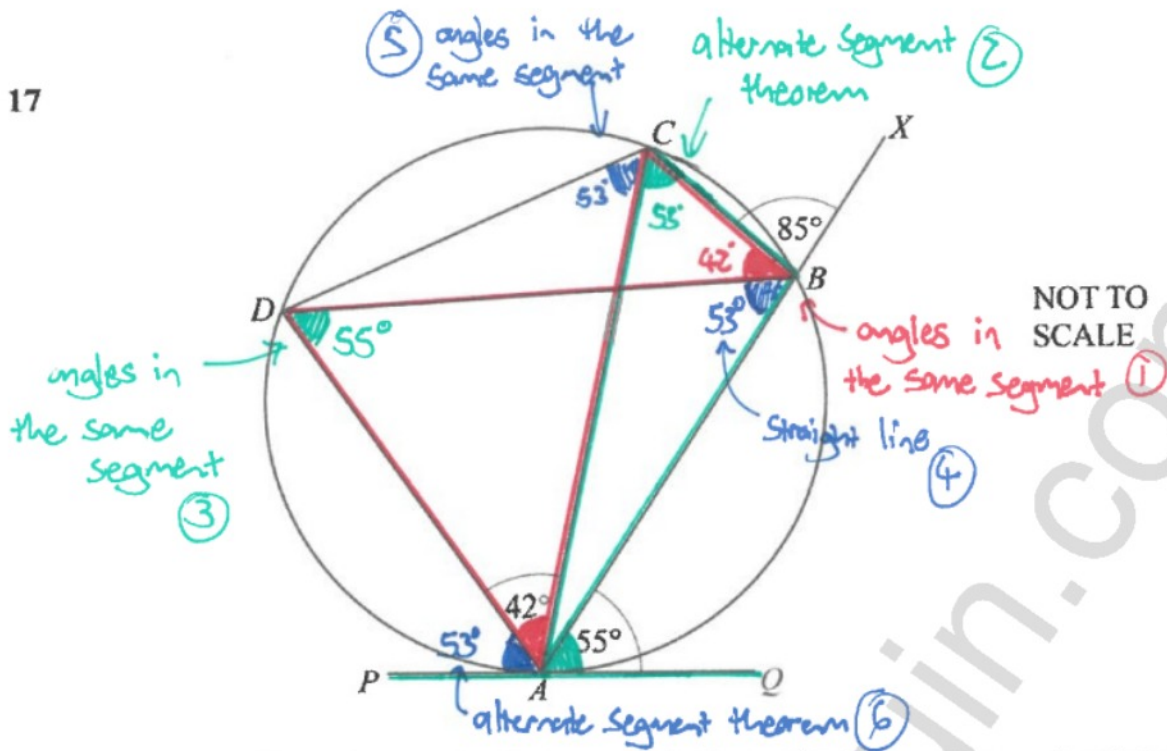
$$\begin{aligned} \angle ABC &= 22 + 35 \\ &= 57^\circ \end{aligned}$$

$$\begin{aligned} \angle ADC &= 180 - 57 \\ &= 123^\circ \end{aligned}$$

Angle $ADC = 123^\circ$ [1]

(c) angle CAF .

Angle $CAF = 57^\circ$ [1]



$ABCD$ is a cyclic quadrilateral, ABX is a straight line and PQ is a tangent to the circle at A .
Angle $CBX = 85^\circ$, angle $BAQ = 55^\circ$ and angle $CAD = 42^\circ$.

Find

(a) angle CBD

Angle $CBD = 42^\circ$ [1]

(b) angle ACB

Angle $ACB = 55^\circ$ [1]

(c) angle ADC

Angle $ADC = 55^\circ$ [1]

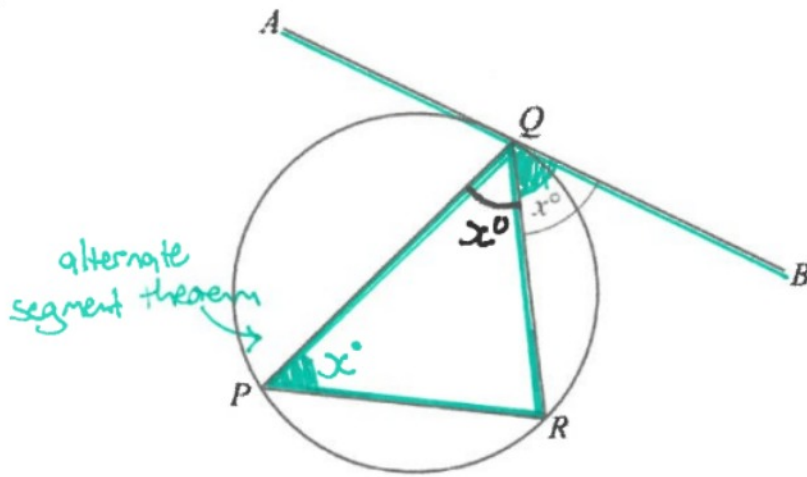
(d) angle BCD

$$53 + 55 = 108^\circ$$

Angle $BCD = 108^\circ$ [2]

(e) angle PAD

Angle $PAD = 53^\circ$ [1]



P , R and Q are points on the circle.
 AB is a tangent to the circle at Q .
 QR bisects angle PQB .
 Angle $BQR = x^\circ$ and $x < 60$.

Use this information to show that triangle PQR is an isosceles triangle.
 Give a geometrical reason for each step of your work.

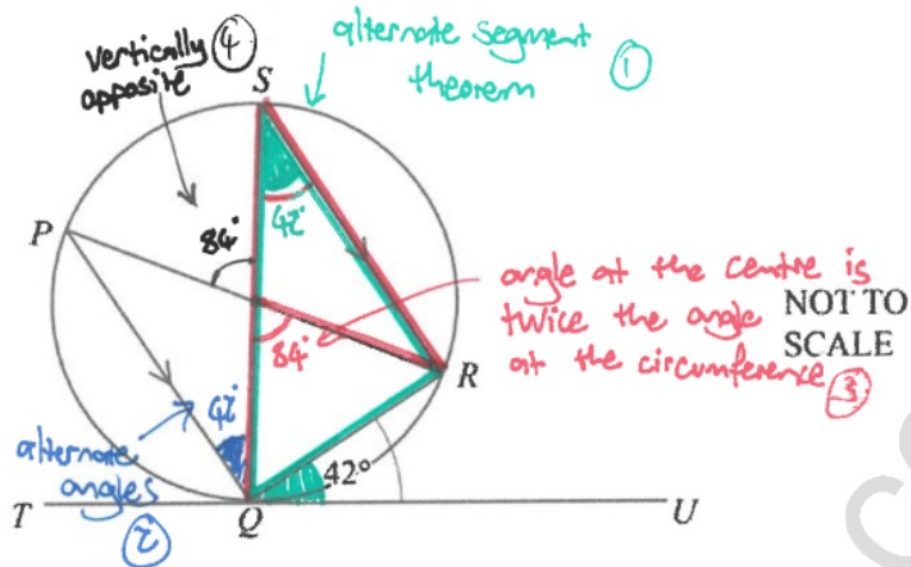
QR bisects PQB , so angle $PQR = x$.

$\angle RPQ = \angle BQR$ using the alternate segment theorem.

$\angle PQR = \angle RPQ = x$, so the triangle is isosceles.

[3]

(b)



P, Q, R and S are points on the circle and TQU is a tangent to the circle at Q .
 PR and SQ intersect at the centre of the circle, O , and PQ is parallel to SR .
Angle $RQU = 42^\circ$.

Calculate

(i) angle QSR

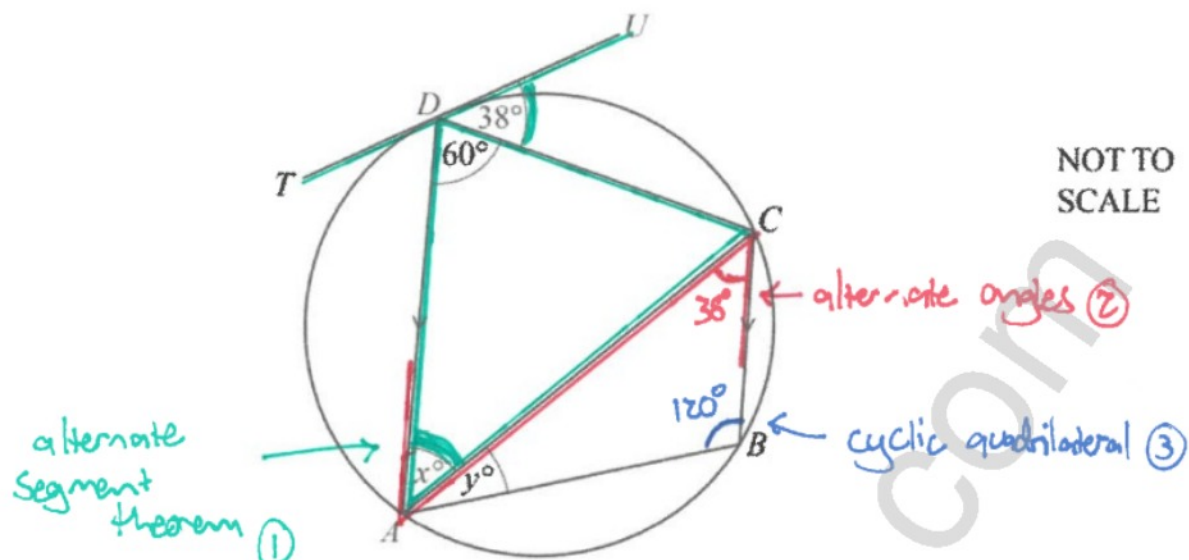
Angle $QSR = 42^\circ$ [1]

(ii) angle PQS

Angle $PQS = 42^\circ$ [1]

(iii) angle POS .

Angle $POS = 84^\circ$ [1]



A, B, C and D are points on a circle.
 TU is a tangent to the circle at D .
 DA is parallel to CB .

Find the value of x and the value of y .

$$x = 38^\circ \text{ (alternate segment theorem)}$$

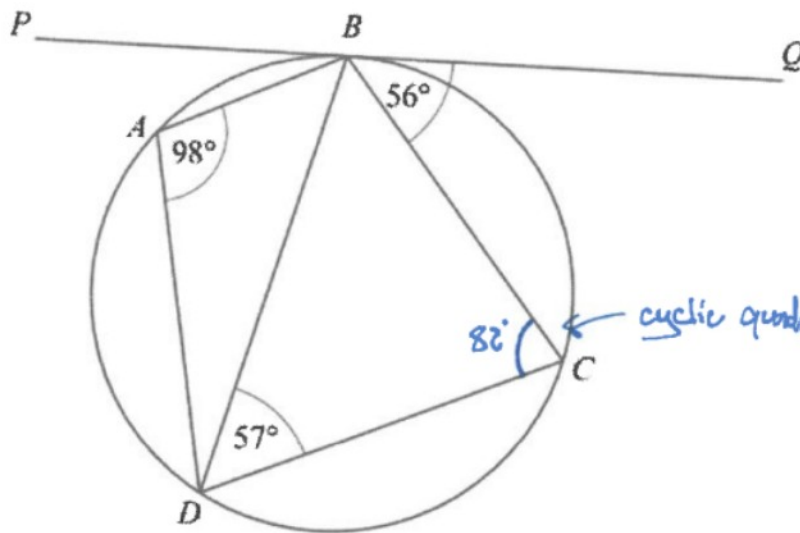
$$y = 180 - (120 + 38)$$

$$= 22^\circ$$

(angles in a triangle)

$$x = 38^\circ$$

$$y = 22^\circ \quad [3]$$



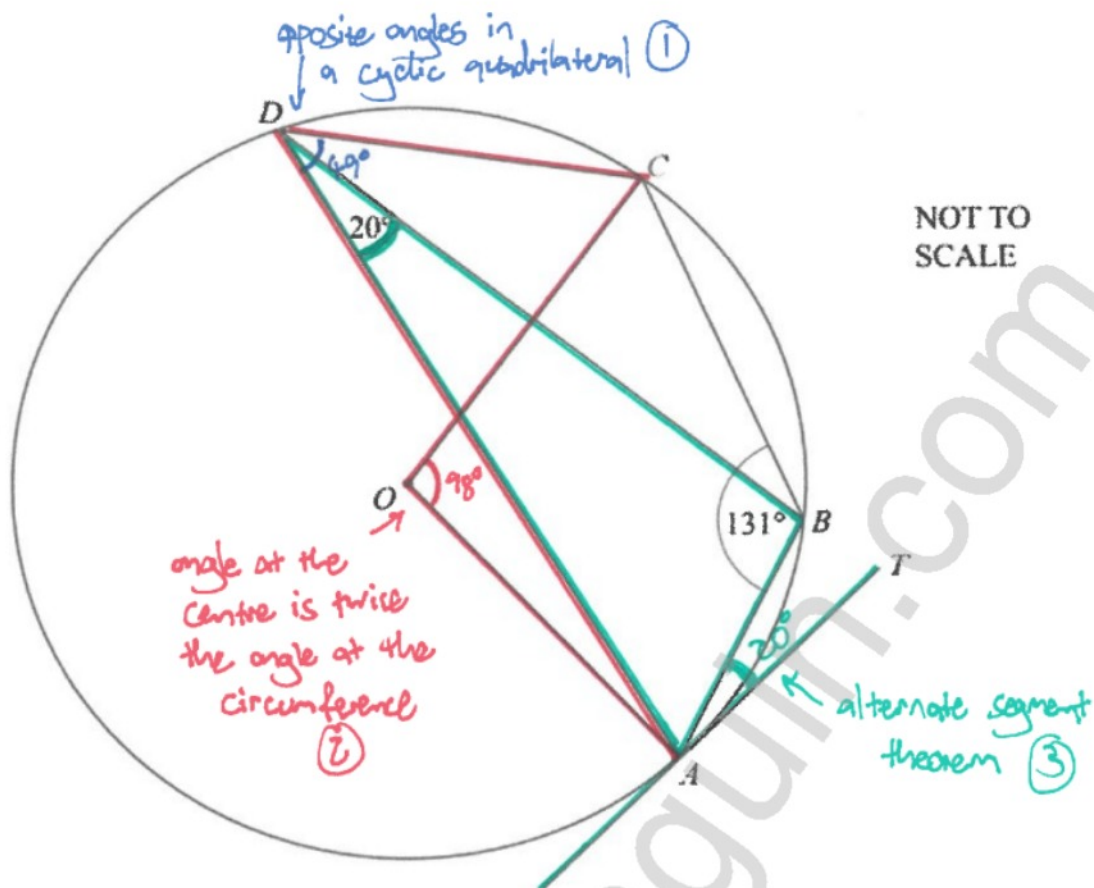
A, B, C and D are points on the circle.
 PBQ is a straight line.

- (a) Find angle DCB , giving a reason for your answer.

Angle $DCB = 82^\circ$ because opposite angles in a cyclic quadrilateral sum to 180° . [2]

- (b) Is PBQ a tangent to the circle?
 Give a reason for your answer.

No because if it was, angle BDC (57°) would be equal to angle QBC (56°) from the alternate segment theorem. [1]



A, B, C and D lie on the circle, centre O .
 TA is a tangent to the circle at A .
 Angle $ABC = 131^\circ$ and angle $ADB = 20^\circ$.

Find

- (a) angle ADC , $180 - 131 = 49^\circ$
 (cyclic quadrilateral)

Angle $ADC = 49^\circ$ [1]

- (b) angle AOC ,

Angle $AOC = 98^\circ$ [1]

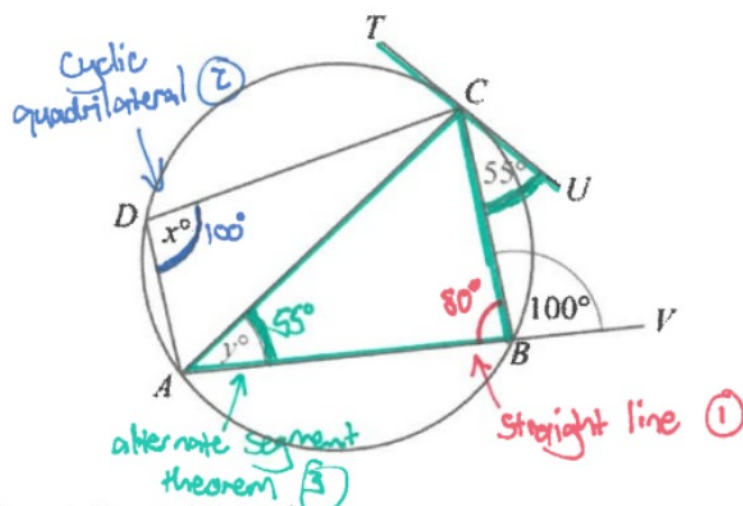
- (c) angle BAT ,

Angle $BAT = 20^\circ$ [1]

- (d) angle OAB , $90 - 20 = 70^\circ$

Angle $OAB = 70^\circ$ [1]

↑
 angle between tangent
 and radius = 90°



NOT TO
SCALE

$ABCD$ is a cyclic quadrilateral.

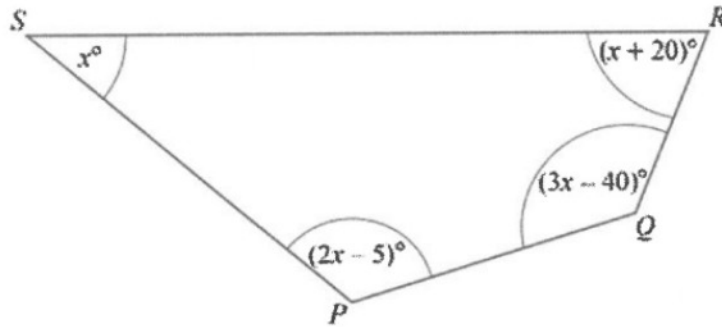
ABV is a straight line and TU is a tangent to the circle at C .

Find the value of x and the value of y .

$$x = 100^\circ$$

$$y = 55^\circ \quad [2]$$

(d)



NOT TO
SCALE

Show that $PQRS$ is a cyclic quadrilateral.

① Find x by using the fact that the interior angles sum to 360 :

$$x + x + 20 + 3x - 40 + 2x - 5 = 360$$

$$7x - 25 = 360$$

$$+ 25 \quad + 25$$

$$7x = 385$$

$$\div 7 \quad \div 7$$

$$\underline{x = 55}$$

② Show that opposite angles
sum to 180 :

$$x = 55^\circ$$

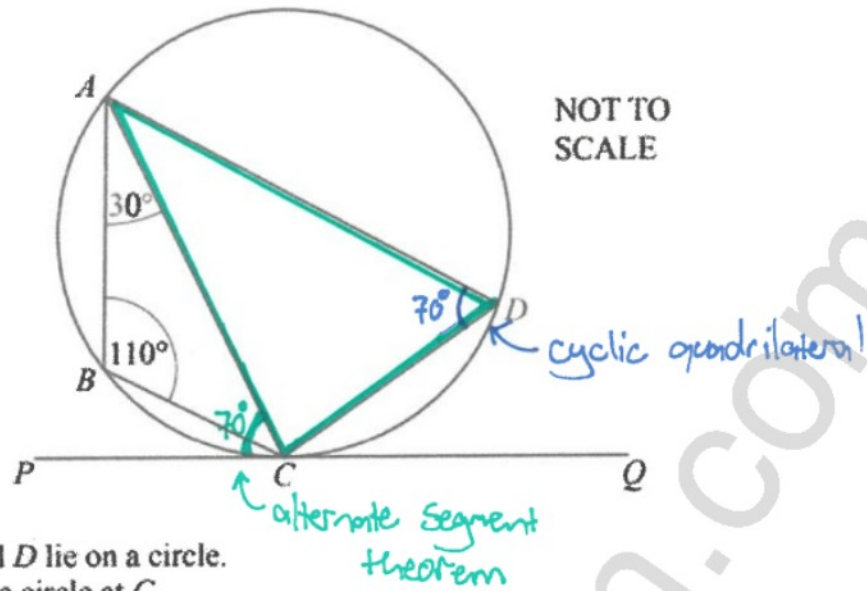
$$3x - 40 = 3(55) - 40 = 125$$

$$\left. \begin{array}{l} x = 55^\circ \\ 3x - 40 = 125 \end{array} \right\} 55 + 125 = 180$$

$$x + 20 = 55 + 20 = 75$$

$$2x - 5 = 2(55) - 5 = 105$$

$$\left. \begin{array}{l} x + 20 = 75 \\ 2x - 5 = 105 \end{array} \right\} 75 + 105 = 180$$



The points A, B, C and D lie on a circle.
 PCQ is a tangent to the circle at C.
 Angle $ABC = 110^\circ$ and angle $BAC = 30^\circ$.

Find

(a) angle ADC,

$$180 - 110 = 70^\circ$$

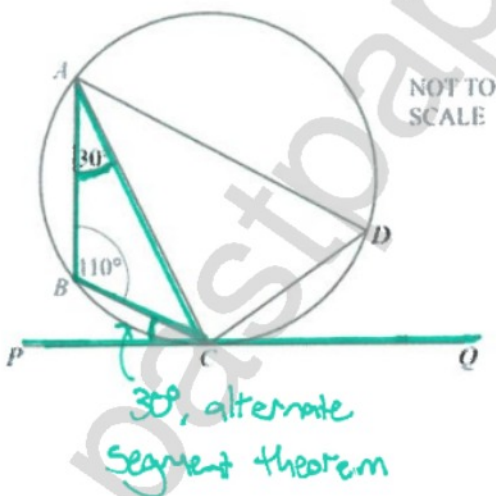
(cyclic quadrilateral)

Angle ADC = 70° [1]

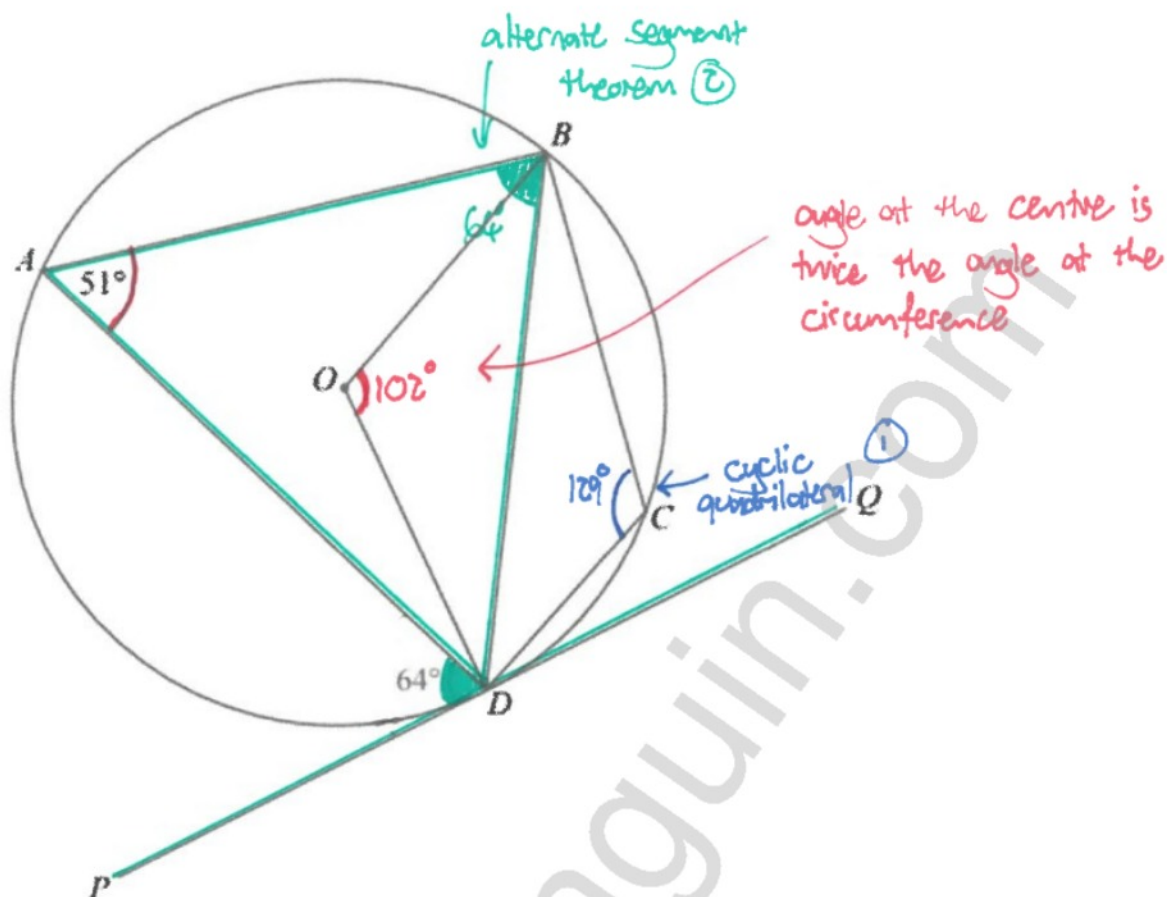
(b) angle ACP,

Angle ACP = 70° [1]

(c) angle PCB.



Angle PCB = 30° [1]



A, B, C and D are points on the circle centre O .
 PDQ is a tangent to the circle at D .
 Angle $BAD = 51^\circ$ and angle $PDA = 64^\circ$.

Find

(a) angle BCD

$$180 - 51 = 129^\circ$$

(cyclic quadrilateral)

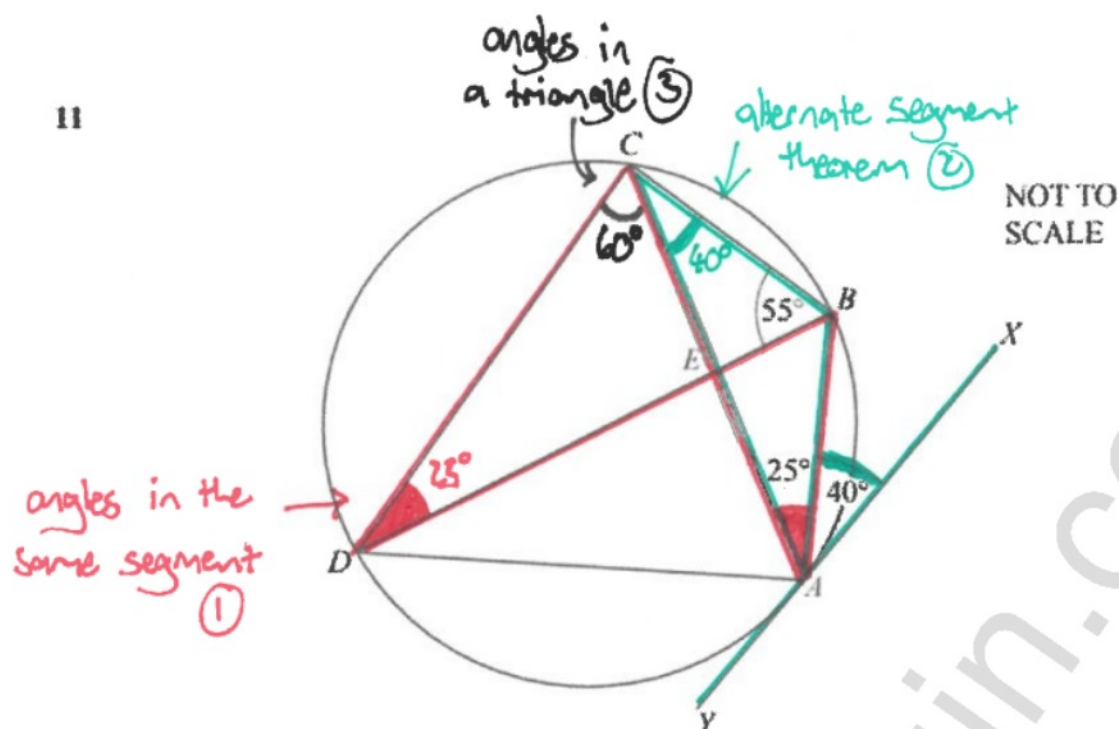
Angle $BCD = 129^\circ$ [1]

(b) angle ABD

Angle $ABD = 64^\circ$ [1]

(c) the obtuse angle BOD .

Angle $BOD = 102^\circ$ [1]



A, B, C and D are four points on a circle.
 AC and BD meet at E .
 XY is a tangent to the circle at A .

Find

(a) angle CDB ,

Angle $CDB = 25^\circ$ [1]

(b) angle ACB ,

Angle $ACB = 40^\circ$ [1]

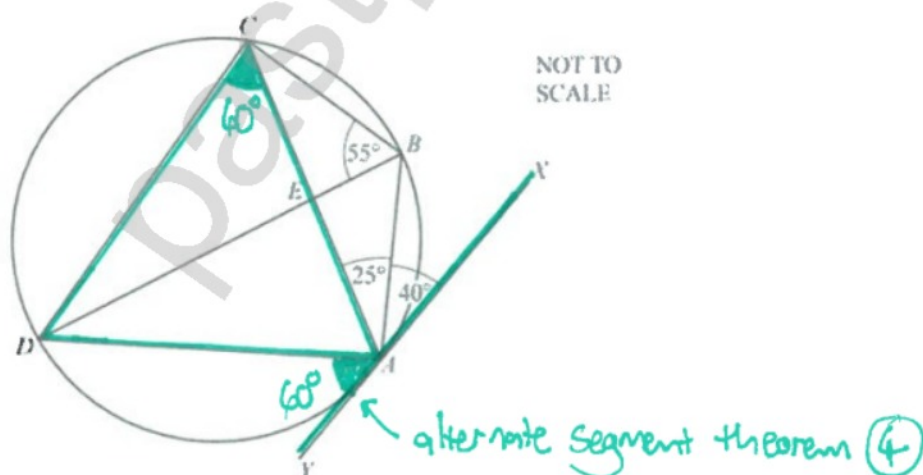
(c) angle DCE ,

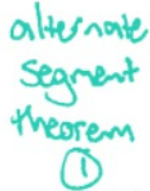
$$180 - (25 + 55 + 40) = 60^\circ$$

Angle $DCE = 60^\circ$ [1]

(d) angle YAD .

Angle $YAD = 60^\circ$ [1]





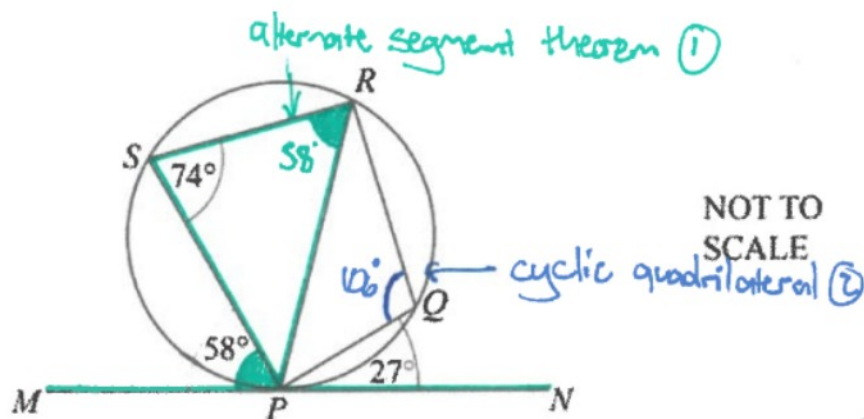
angle at the centre is
twice the angle at the
circumference (2)

Find the value of x .

↑ angles in a quadrilateral

33° [3]

(b)



P, Q, R and S lie on a circle.

MPN is a tangent to the circle at P .

Angle $MPS = 58^\circ$, angle $PSR = 74^\circ$ and angle $QPN = 27^\circ$.

(i) Find angle PRS .

Angle $PRS = 58^\circ$ [1]

(ii) Find angle PQR .

$$180 - 74 = 106^\circ$$

(cyclic quadrilateral)

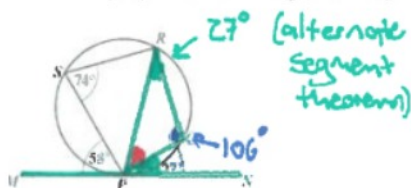
Angle $PQR = 106^\circ$ [1]

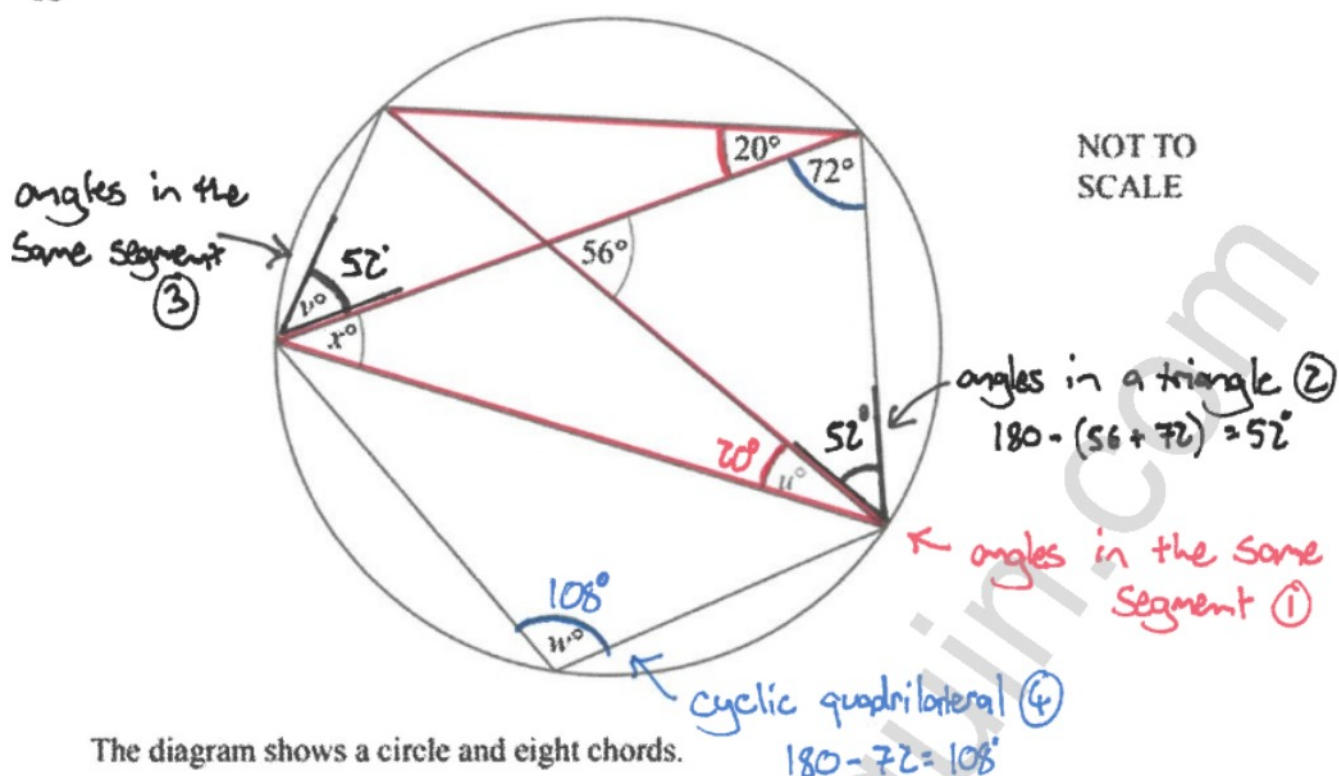
(iii) Find angle RPQ .

$$180 - (27 + 106) = 47^\circ$$

(angles in a triangle)

Angle $RPQ = 47^\circ$ [2]





The diagram shows a circle and eight chords.

Calculate the values of u , v , w and x .

$$x = 180 - (72 + 52 + 20)$$

$$= 36^\circ$$

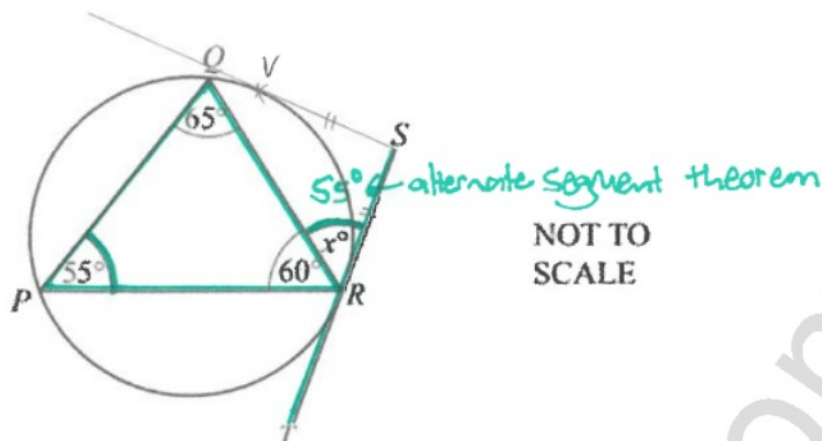
(angles in a triangle = 180°)

$$u = 20^\circ$$

$$v = 52^\circ$$

$$w = 108^\circ$$

$$x = 36^\circ \quad [4]$$



P , Q and R are points on a circle.
 ST is a tangent to the circle at R .

- (a) Write down the value of x .
 Give a geometrical reason for your answer.

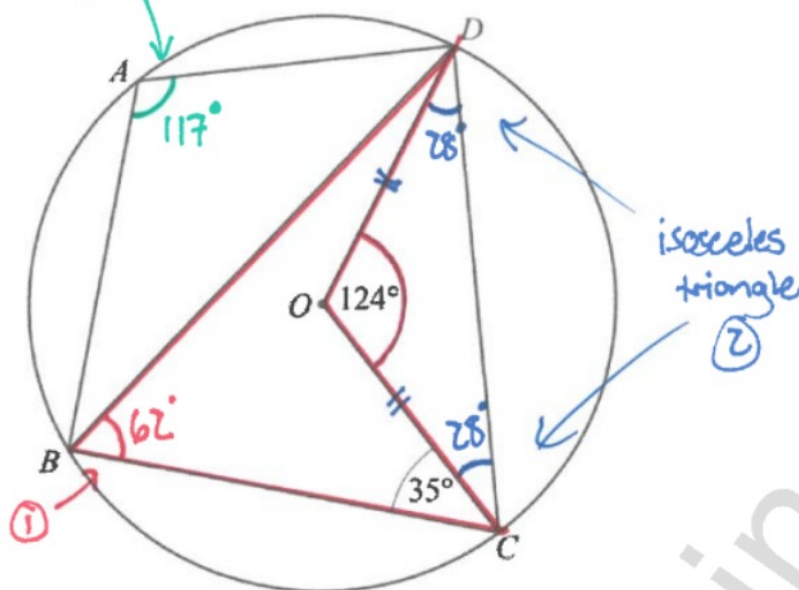
$x = 55^\circ$ because angles in the alternate segment are equal. [2]

- (b) Another tangent from the point S touches the circle at V .

Give a geometrical reason why triangle SVR is isosceles.

Tangents from an external point (S) are equal in length,
 so $VS = RS$, meaning SVR is isosceles. [1]

cyclic quadrilateral (3)



NOT TO
SCALE

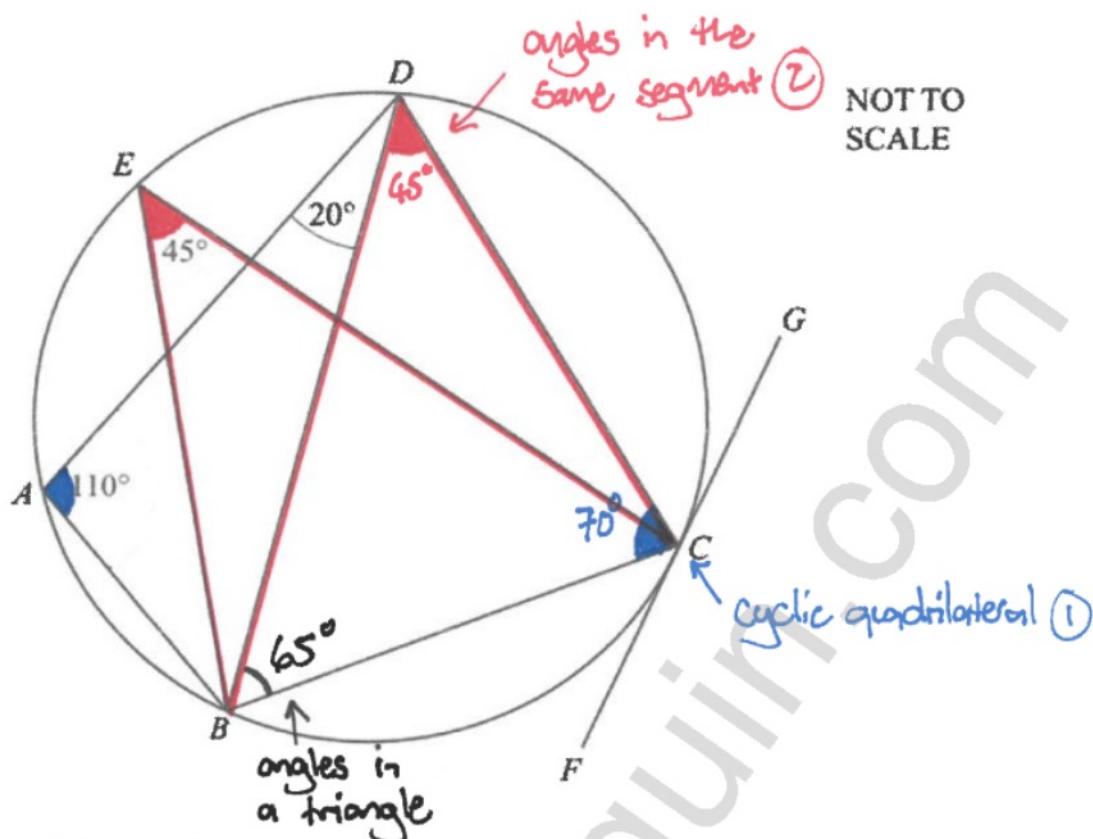
isocceles
triangle
②

- (i) Work out angle CBD .
Give a geometrical reason for your answer.

Angle $CBD = 62^\circ$ because the angle at the centre (124°) is twice the angle at the circumference [2]

- (ii) Work out angle BAD .
Give a geometrical reason for each step of your working.

Angle $BAD = 117^\circ$ because $\hat{OCD} = 28^\circ$ (base angles in an isosceles triangle are equal), $\hat{BCD} = 35 + 28 = 63^\circ$, $\hat{BAD} = 180 - 63 = 117^\circ$ (opposite angles in a cyclic quadrilateral sum to 180°).



A, B, C, D and E lie on a circle.

FG is a tangent to the circle at C .

Angle $BAD = 110^\circ$, angle $ADB = 20^\circ$ and angle $BEC = 45^\circ$.

(a) Find angle BCD .

Give a geometrical reason for your answer.

Angle $BCD = 70^\circ$ because opposite angles in a cyclic quadrilateral sum to 180° . [2]

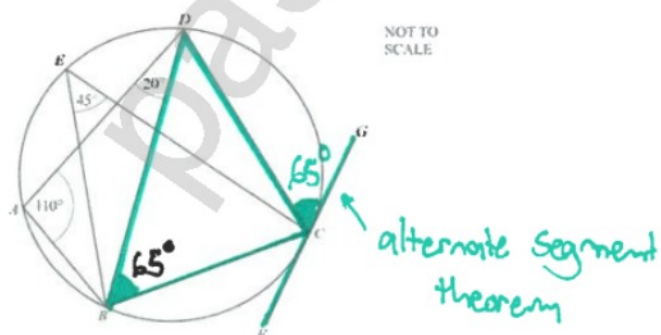
(b) (i) Find angle DBC .

$$180 - (45 + 70) = 65^\circ$$

(angles in a triangle)

Angle $DBC = 65^\circ$ [2]

(ii) Find angle DCG .



Angle $DCG =$ [1]