

- 7** Two lines have equations $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$.

(a) Show that the lines are skew.

[5]

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[illegible]

- (a) Find the position vector of D .

[3]

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- (b) The angle between BA and BC is θ .

Find the exact value of $\cos \theta$.

[3]

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- (c) Hence find the area of $ABCD$, giving your answer in the form $p\sqrt{q}$, where p and q are integers. [4]

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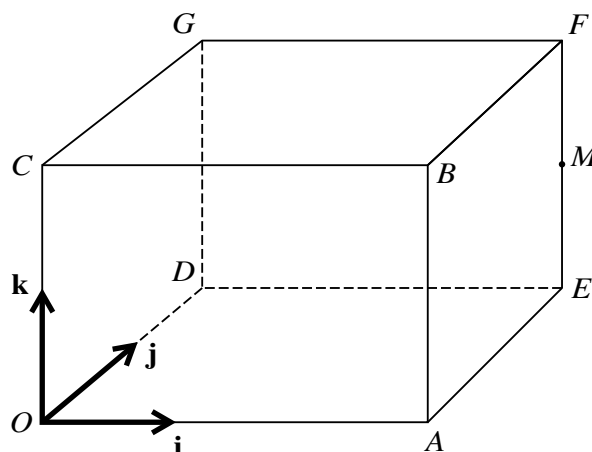
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In the diagram, $OABCDEFG$ is a cuboid in which $OA = 3$ units, $OC = 2$ units and $OD = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA , OD and OC respectively. M is the midpoint of EF .

- (a) Find the position vector of M . [1]

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The position vector of P is $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

- (b) Calculate angle PAM . [4]

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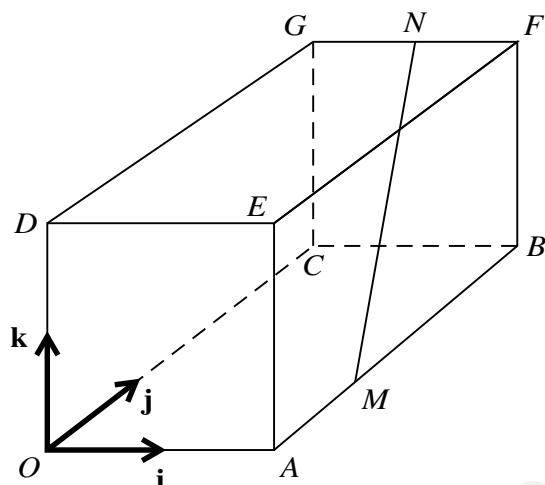
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$$l: \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad m: \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ c \end{pmatrix},$$

(a) Find the value of c .

[4]

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dotted lines spaced evenly down the page. A large, light gray watermark with the text "pastpaperpenquinn.com" is oriented diagonally from the bottom left towards the top right, spanning across the entire page. The background is white, and there are no other markings or text present.



In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 3$ units and $OD = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA , OC and OD respectively. The point M on AB is such that $MB = 2AM$. The midpoint of FG is N .

- (a) Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

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- (b) Find a vector equation for the line through M and N . [2]

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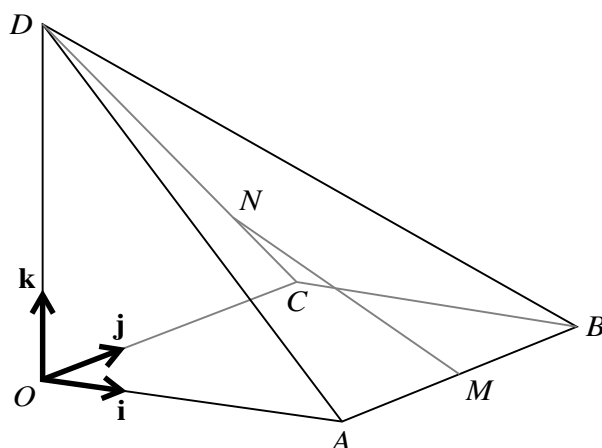
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The midpoint of AB is M and the point N on CD is such that $DN = 3NC$.

- (a) Find a vector equation for the line through M and N . [5]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. A faint, diagonal watermark reading "Pastpaperberk" is visible across the center of the page.

[4]

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- 9 With respect to the origin O , the position vectors of the points A , B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC , between B and C , and is such that $BN = 2NC$.

- (a) Find the position vectors of M and N . [3]

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- (b) Find a vector equation for the line through M and N . [2]

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[illegible]

- 9 With respect to the origin O , the vertices of a triangle ABC have position vectors

$$\vec{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \vec{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \vec{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

- (a) Using a scalar product, show that angle ABC is a right angle. [3]

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- (b) Show that triangle ABC is isosceles. [2]

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[illegible]

- 8 With respect to the origin O , the position vectors of the points A , B , C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

- (a) Show that $AB = 2CD$.

[3]

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- (b) Find the angle between the directions of $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

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[illegible]

- (a) Find a vector equation for the line through M and N . [5]

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The line through M and N intersects the line through O and B at the point P .

- (b) Find the position vector of P . [3]

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- (c) Calculate angle OPM , giving your answer in degrees. [3]

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$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$
respectively, where a and b are constants.

[4]

- (b) Given also that l and m are perpendicular, find the values of a and b . [4]

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- (c) When a and b have these values, find the position vector of the point of intersection of l and m . [2]

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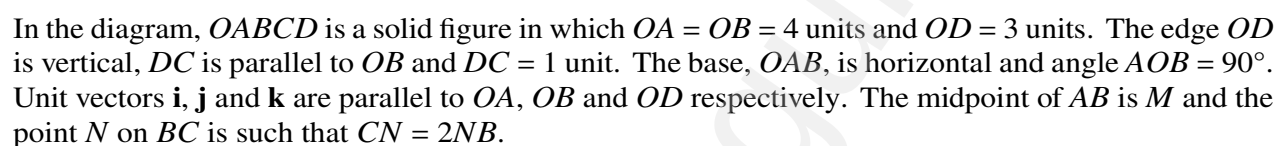
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- (b) Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

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- (c) Show that the length of the perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. [4]

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