4 Express $\dfrac{4x^2 - 13x + 13}{(2x - 1)(x - 3)}$ in partial fractions. [5]

4 Let $\mathrm{f}(x) = \dfrac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [3]

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(b) Hence find $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $\ln\left(\dfrac{a}{b}\right)$, where a and b are integers. [4]

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6 Let $\mathrm{f}(x) = \dfrac{5a}{(2x-a)(3a-x)}$, where a is a positive constant.

(a) Express $\mathrm{f}(x)$ in partial fractions. [3]

(b) Hence show that $\int_a^{2a} \mathrm{f}(x)\,\mathrm{d}x = \ln 6$. [4]

9 Let $\mathrm{f}(x) = \dfrac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \frac{5}{2} - \ln 72$. [5]

8 Let $f(x) = \dfrac{3 - 3x^2}{(2x + 1)(x + 2)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $a + b\ln c$, where a, b and c are integers. [5]

7 Let $\mathrm{f}(x) = \dfrac{2}{(2x-1)(2x+1)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [2]

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(b) Using your answer to part **(a)**, show that

$$\big(\mathrm{f}(x)\big)^2 = \frac{1}{(2x-1)^2} - \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{(2x+1)^2}.$$ [2]

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(c) Hence show that $\int_1^2 \left(\mathrm{f}(x)\right)^2 \mathrm{d}x = \frac{2}{5} + \frac{1}{2}\ln\left(\frac{5}{9}\right)$. [5]

11 Let $\mathrm{f}(x) = \dfrac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x = \ln 54 - \frac{1}{8}\pi$. [5]

9 Let $f(x) = \dfrac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$. [6]

5 **(a)** Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

(b) Using your answer to part **(a)**, find the exact value of $\int_1^3 \frac{2x^3 - x^2 + 6x + 3}{x^2 + 3}\,\mathrm{d}x$. [5]

9 **(a)** Find the quotient and remainder when $x^4 + 16$ is divided by $x^2 + 4$. [3]

(b) Hence show that $\int_2^{2\sqrt{3}} \frac{x^4+16}{x^2+4}\,\mathrm{d}x = \frac{4}{3}(\pi+4)$. [5]

8 Let $\mathrm{f}(x) = \dfrac{x^2 + 9x}{(3x - 1)(x^2 + 3)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence find $\int_1^3 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in a simplified exact form. [5]

10 Let $f(x) = \dfrac{-7x^2 + 2x - 6}{(1+x)(4+x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$. Give your answer in the form $a\pi - \ln b$, where a and b are constants. [6]

8 **(a)** Find the quotient and remainder when $8x^3 + 4x^2 + 2x + 7$ is divided by $4x^2 + 1$. [3]

(b) Hence find the exact value of $\int_0^{\frac{1}{2}} \frac{8x^3 + 4x^2 + 2x + 7}{4x^2 + 1} \, \mathrm{d}x$. [5]

10 Let $\mathrm{f}(x) = \dfrac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Find the exact value of $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x$. Give your answer as a single logarithm. [5]

11 Let $\mathrm{f}(x) = \dfrac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Find the exact value of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$, simplifying your answer. [5]

5 Find the exact value of $\int_0^6 \frac{x(x+1)}{x^2+4}\,\mathrm{d}x$. [6]

10 Let $\mathrm{f}(x) = \dfrac{36a^2}{(2a+x)(2a-x)(5a-2x)}$, where a is a positive constant.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_{-a}^{a} \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $p\ln q + r\ln s$ where p and r are integers and q and s are prime numbers. [5]