

4 Express $\frac{4x^2 - 13x + 13}{(2x - 1)(x - 3)}$ in partial fractions.

[5]

[illegible]

4 Let $f(x) = \frac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions.

[3]

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(b) Hence find $\int_1^2 f(x) dx$, giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a and b are integers. [4]

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6 Let $f(x) = \frac{5a}{(2x-a)(3a-x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions.

[3]

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(b) Hence show that $\int_a^{2a} f(x) \, dx = \ln 6$.

[4]

[illegible]

9 Let $f(x) = \frac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

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(b) Hence show that $\int_0^1 f(x) \, dx = \frac{5}{2} - \ln 72$.

[5]

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8 Let $f(x) = \frac{3 - 3x^2}{(2x + 1)(x + 2)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

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- (b)** Hence find the exact value of $\int_0^4 f(x) \, dx$, giving your answer in the form $a + b \ln c$, where a , b and c are integers. [5]

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7 Let $f(x) = \frac{2}{(2x-1)(2x+1)}$.

(a) Express $f(x)$ in partial fractions.

[2]

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(b) Using your answer to part (a), show that

$$(f(x))^2 = \frac{1}{(2x-1)^2} - \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{(2x+1)^2}.$$

[2]

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(c) Hence show that $\int_1^2 (f(x))^2 dx = \frac{2}{5} + \frac{1}{2} \ln\left(\frac{5}{9}\right)$. [5]

[illegible]

11 Let $f(x) = \frac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$.

(a) Express $f(x)$ in partial fractions.

[5]

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(b) Hence show that $\int_0^2 f(x) \, dx = \ln 54 - \frac{1}{8}\pi$.

[5]

[illegible]

9 Let $f(x) = \frac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

(b) Hence find the exact value of $\int_0^2 f(x) \, dx$.

[6]

[illegible]

- [illegible]

[illegible]

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[5]

[illegible]

8 Let $f(x) = \frac{x^2 + 9x}{(3x - 1)(x^2 + 3)}$.

(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

(b) Hence find $\int_1^3 f(x) \, dx$, giving your answer in a simplified exact form.

[5]

[illegible]

[5]

[illegible]

[illegible]

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(b) Hence find the exact value of $\int_0^{\frac{1}{2}} \frac{8x^3 + 4x^2 + 2x + 7}{4x^2 + 1} dx$. [5]

[illegible]

10 Let $f(x) = \frac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

(a) Express $f(x)$ in partial fractions.

[5]

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11 Let $f(x) = \frac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}$.

(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

[illegible]

5 Find the exact value of $\int_0^6 \frac{x(x+1)}{x^2+4} dx$.

[6]

[illegible]

10 Let $f(x) = \frac{36a^2}{(2a+x)(2a-x)(5a-2x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions.

[5]

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- (b) Hence find the exact value of $\int_{-a}^a f(x) dx$, giving your answer in the form $p \ln q + r \ln s$ where p and r are integers and q and s are prime numbers. [5]

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