

- 3 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{2}x + \frac{72}{x^4}$. The curve passes through the point $P(2, 8)$.

(a) Find the equation of the normal to the curve at P . [2]

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(b) Find the equation of the curve. [4]

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- 11 The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a .

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(b) Determine the nature of this stationary point.

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(c) Find the equation of the curve.

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(d) Find the coordinates of any other stationary points on the curve.

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- (a)** Find the x -coordinates of the stationary points of the curve and determine their nature. [6]

(b) State the set of values for which the function f is increasing.

[1]

- 10** A curve has a stationary point at $(2, -10)$ and is such that $\frac{d^2y}{dx^2} = 6x$.

(a) Find $\frac{dy}{dx}$. [3]

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(b) Find the equation of the curve. [3]

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- (c) Find the coordinates of the other stationary point and determine its nature. [3]

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- (d) Find the equation of the tangent to the curve at the point where the curve crosses the y-axis. [2]

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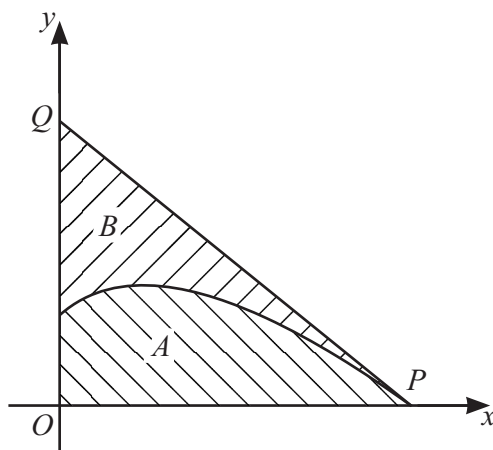
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The diagram shows the curve with equation

$$y = 4(3x + 4)^{\frac{1}{2}} - 2x - 6$$

for values of x such that $0 \leq x \leq 7$. The tangent to the curve at the point $P(7, 0)$ meets the y -axis at the point Q . Region A is bounded by the curve and the two axes. Region B is bounded by the curve, the line segment PQ and the y -axis.

- (a) Find the area of region A .

[4]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. A faint, diagonal watermark reading "Pastpaper" is visible across the center.



[5]

[illegible]

10 The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $(-1, \frac{9}{2})$.

(a) Determine the nature of the stationary point at $(-1, \frac{9}{2})$. [1]

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(b) Find the equation of the curve. [5]

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- (c) Show that the curve has no other stationary points.

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- (d) A point A is moving along the curve and the y -coordinate of A is increasing at a rate of 5 units per second.

Find the rate of increase of the x -coordinate of A at the point where $x = 1$.

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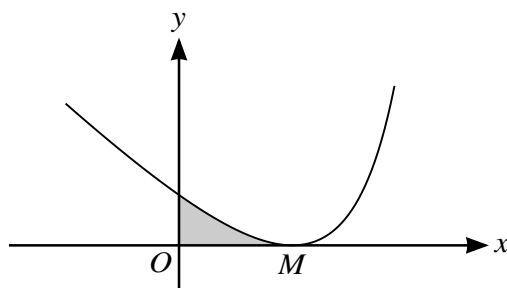
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(a) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y \, dx$. [6]

[illegible]

- (b) Find, by calculation, the x -coordinate of M . [2]

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- (c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

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- 9** A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \frac{4}{x^2}$.

- (a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$.

[4]

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(b) Find the coordinates of the stationary points on the curve.

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(c) Find $f''(x)$.

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(d) Hence, or otherwise, determine the nature of each of the stationary points.

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- 10** A curve has equation $y = f(x)$ and it is given that

$$f'(x) = \left(\frac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

- (a)** Find $f''(x)$ in terms of k and x , and hence find the set of possible values of k . [3]

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It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

- (b)** Find $f(x)$. [4]

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- (c) Find the coordinates of the other stationary point and determine its nature. [4]

- 10 A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(b) It is given instead that $\int_{\frac{1}{4k^2}}^{k^2} \left(\frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2} \right) dx = \frac{13}{12}$.

Find the value of k .

[5]



10 A function f with domain $x > 0$ is such that $f'(x) = 8(2x-3)^{\frac{1}{3}} - 10x^{\frac{2}{3}}$. It is given that the curve with equation $y = f(x)$ passes through the point $(1, 0)$.

(a) Find the equation of the normal to the curve at the point $(1, 0)$. [3]

[illegible]

(b) Find $f(x)$. [4]

[illegible]



(c) Determine, making your reasoning clear, whether f is an increasing function, a decreasing function or neither. [3]

This image shows a blank sheet of white paper designed for handwriting practice. It features approximately 20 horizontal dotted lines spaced evenly down the page. A large, light gray watermark is oriented diagonally from the bottom-left towards the top-right, containing the text "pastpaperpenquil.com". The watermark is semi-transparent, allowing the ruled lines to remain visible beneath it. There are no margins, text, or other markings on the page.

- (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of k . [2]

Handwriting practice lines (dotted lines on a solid background).

- Find the coordinates of the stationary point and determine its nature. [4]

This image shows a blank sheet of white paper designed for handwriting practice. It features multiple horizontal rows of small, evenly spaced dots. A prominent, light gray watermark with the text "pastpaperpen9" is oriented diagonally from the bottom-left towards the top-right, spanning most of the page. The background is plain white, and there are no other markings or text present.



- Find this value of k .

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