7 Let $\mathrm{f}(x) = \dfrac{\cos x}{1 + \sin x}$.

(b) Find $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \mathrm{f}(x)\,\mathrm{d}x$. Give your answer in a simplified exact form. [4]

4 **(a)** Prove that $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} \equiv \tan^2\theta$. [2]

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(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1-\cos 2\theta}{1+\cos 2\theta}\,\mathrm{d}\theta$. [4]

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6 **(a)** Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\tfrac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x. \quad [3]$$

(b) Hence show that $\int_0^{\frac{1}{4}\pi} \sin 3x \cos 2x \, \mathrm{d}x = \frac{1}{5}(3 - \sqrt{2})$. [3]

6 **(a)** Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$. [3]

(b) Hence show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\operatorname{cosec} 2\theta - \cot 2\theta)\, \mathrm{d}\theta = \frac{1}{2} \ln 2$. [4]

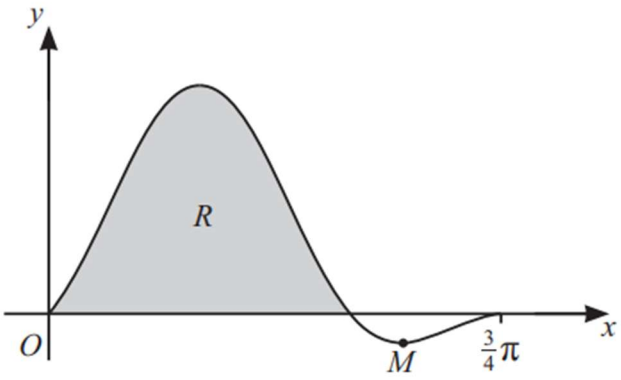
7 **(a)** Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$. [3]

(b) Hence find the exact value of $\int_{-\frac{1}{8}\pi}^{\frac{1}{8}\pi} \left(\cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta\right) \mathrm{d}\theta$. [6]

8 **(a)** Express $3\cos 2x - \sqrt{3}\sin 2x$ in the form $R\cos(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and α. [3]

(b) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{3}{\left(3\cos 2x - \sqrt{3}\sin 2x\right)^2}\,\mathrm{d}x$, simplifying your answer. [5]

6



The diagram shows the curve $y = \sin 2x(1 + \sin 2x)$, for $0 \leqslant x \leqslant \frac{3}{4}\pi$, and its minimum point M. The shaded region bounded by the curve that lies above the x-axis and the x-axis itself is denoted by R.

(b) Find the exact area of the region R. [4]