4 A particle travels in a straight line. The velocity of the particle at time t s after leaving a point O is v m s^{-1}, where

$$v = kt^2 - 4t + 3.$$

The distance travelled by the particle in the first 2 s of its motion is 6 m. You may assume that $v > 0$ in the first 2 s of its motion.

(a) Find the value of k. [4]

(b) Find the value of the minimum velocity of the particle. You do **not** need to show that this velocity is a minimum. [3]

4 A cyclist starts from rest at a point A and travels along a straight road AB, coming to rest at B. The displacement of the cyclist from A at time t s after the start is s m, where

$$s = 0.004(75t^2 - t^3).$$

(a) Show that the distance AB is 250 m. [4]

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(b) Find the maximum velocity of the cyclist. [3]

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5 A particle moves in a straight line starting from a point O. The velocity $v\,\text{m}\,\text{s}^{-1}$ of the particle t s after leaving O is given by

$$v = t^3 - \tfrac{9}{2}t^2 + 1 \text{ for } 0 \leqslant t \leqslant 4.$$

You may assume that the velocity of the particle is positive for $t < \frac{1}{2}$, is zero at $t = \frac{1}{2}$ and is negative for $t > \frac{1}{2}$.

(a) Find the distance travelled between $t = 0$ and $t = \frac{1}{2}$. [4]

(b) Find the positive value of t at which the acceleration is zero. Hence find the total distance travelled between $t = 0$ and this instant. [4]

6 A particle moves in a straight line AB. The velocity v m s^{-1} of the particle t s after leaving A is given by $v = k(t^2 - 10t + 21)$, where k is a constant. The displacement of the particle from A, in the direction towards B, is 2.85 m when $t = 3$ and is 2.4 m when $t = 6$.

(a) Find the value of k. Hence find an expression, in terms of t, for the displacement of the particle from A. [7]

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(b) Find the displacement of the particle from A when its velocity is a minimum. [4]

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6 A particle travels in a straight line PQ. The velocity of the particle t s after leaving P is v m s^{-1}, where

$$v = 4.5 + 4t - 0.5t^2.$$

(a) Find the velocity of the particle at the instant when its acceleration is zero. [3]

The particle comes to instantaneous rest at Q.

(b) Find the distance PQ. [6]

5 A particle moving in a straight line starts from rest at a point A and comes instantaneously to rest at a point B. The acceleration of the particle at time t s after leaving A is a m s^{-2}, where

$$a = 6t^{\frac{1}{2}} - 2t.$$

(a) Find the value of t at point B. [3]

(b) Find the distance travelled from A to the point at which the acceleration of the particle is again zero. [5]

7 A particle P travels in a straight line, starting at rest from a point O. The acceleration of P at time t s after leaving O is denoted by a m s^{-2}, where

$$a = 0.3t^{\frac{1}{2}} \quad \text{for } 0 \leqslant t \leqslant 4,$$
$$a = -kt^{-\frac{3}{2}} \quad \text{for } 4 < t \leqslant T,$$

where k and T are constants.

(a) Find the velocity of P at $t = 4$. [2]

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(b) It is given that there is no change in the velocity of P at $t = 4$ and that the velocity of P at $t = 16$ is 0.3 m s^{-1}.

Show that $k = 2.6$ and find an expression, in terms of t, for the velocity of P for $4 \leqslant t \leqslant T$. [4]

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(c) Given that P comes to instantaneous rest at $t = T$, find the exact value of T. [2]

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(d) Find the total distance travelled between $t = 0$ and $t = T$. [4]

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3 A particle moves in a straight line starting from rest from a point O. The acceleration of the particle at time t s after leaving O is a m s^{-2}, where $a = 4t^{\frac{1}{2}}$.

(a) Find the speed of the particle when $t = 9$. [2]

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(b) Find the time after leaving O at which the speed (in metres per second) and the distance travelled (in metres) are numerically equal. [3]

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5 A particle P moves in a straight line. It starts at a point O on the line and at time t s after leaving O it has velocity v m s^{-1}, where $v = 4t^2 - 20t + 21$.

(a) Find the values of t for which P is at instantaneous rest. [2]

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(b) Find the initial acceleration of P. [2]

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(c) Find the minimum velocity of P. [2]

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(d) Find the distance travelled by P during the time when its velocity is negative. [4]

6 A particle starts from a point O and moves in a straight line. The velocity $v\,\text{m}\,\text{s}^{-1}$ of the particle at time $t\,\text{s}$ after leaving O is given by

$$v = k(3t^2 - 2t^3),$$

where k is a constant.

(a) Verify that the particle returns to O when $t = 2$. [4]

(b) It is given that the acceleration of the particle is $-13.5\,\text{m s}^{-2}$ for the positive value of t at which $v = 0$.

Find k and hence find the total distance travelled in the first two seconds of motion. [6]

6 A particle moves in a straight line. At time t s, the acceleration, $a\,\text{m s}^{-2}$, of the particle is given by $a = 36 - 6t$. The velocity of the particle is $27\,\text{m s}^{-1}$ when $t = 2$.

(a) Find the values of t when the particle is at instantaneous rest. [4]

(b) Find the total distance the particle travels during the first 12 seconds. [4]

6 A cyclist starts from rest at a fixed point O and moves in a straight line, before coming to rest k seconds later. The acceleration of the cyclist at time t s after leaving O is $a\,\text{m s}^{-2}$, where $a = 2t^{-\frac{1}{2}} - \frac{3}{5}t^{\frac{1}{2}}$ for $0 < t \leqslant k$.

(a) Find the value of k. [4]

(b) Find the maximum speed of the cyclist. [3]

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(c) Find an expression for the displacement from O in terms of t. Hence find the total distance travelled by the cyclist from the time at which she reaches her maximum speed until she comes to rest. [4]

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7 A particle moves in a straight line starting from a point O before coming to instantaneous rest at a point X. At time t s after leaving O, the velocity v m s^{-1} of the particle is given by

$$v = 7.2t^2 \qquad 0 \leqslant t \leqslant 2,$$
$$v = 30.6 - 0.9t \qquad 2 \leqslant t \leqslant 8,$$
$$v = \frac{1600}{t^2} + kt \qquad 8 \leqslant t,$$

where k is a constant. It is given that there is no instantaneous change in velocity at $t = 8$.

Find the distance OX. [9]

7 A particle moves in a straight line. The velocity $v\,\text{m}\,\text{s}^{-1}$ of the particle $t\,\text{s}$ after leaving a fixed point O is given by $v = k(20 + pt - 6t^2)$, where k and p are constants. The acceleration of the particle at $t = 1$ is $42\,\text{m}\,\text{s}^{-2}$, and the displacement of the particle from O at $t = 1$ is 93 m.

(a) Show that $k = 3$ and $p = 26$. [6]

(b) Find the distance moved by the particle between the time at which its acceleration is zero and the time at which its velocity is zero. [5]

7 A particle moves in a straight line through the point O. The displacement of the particle from O at time t s is s m, where

$$s = t^2 - 3t + 2 \qquad \text{for } 0 \leqslant t \leqslant 6,$$

$$s = \frac{24}{t} - \frac{t^2}{4} + 25 \quad \text{for } t \geqslant 6.$$

(a) Find the value of t when the particle is instantaneously at rest during the first 6 seconds of its motion. [2]

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At $t = 6$, the particle hits a barrier at a point P and rebounds.

(b) Find the velocity with which the particle arrives at P and also the velocity with which the particle leaves P. [3]

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(c) Find the total distance travelled by the particle in the first 10 seconds of its motion. [5]

6 A particle P moves in a straight line starting from a point O and comes to rest 14 s later. At time t s after leaving O, the velocity v m s^{-1} of P is given by

$$v = pt^2 - qt \qquad 0 \leqslant t \leqslant 6,$$
$$v = 63 - 4.5t \qquad 6 \leqslant t \leqslant 14,$$

where p and q are positive constants.

The acceleration of P is zero when $t = 2$.

(a) Given that there are no instantaneous changes in velocity, find p and q. [3]

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(b) Sketch the velocity-time graph. [3]

(c) Find the total distance travelled by P during the 14 s. [5]

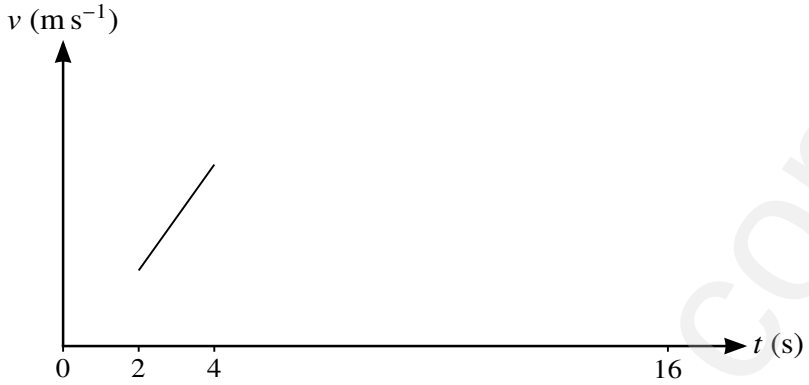
7 A particle P moving in a straight line starts from rest at a point O and comes to rest 16 s later. At time t s after leaving O, the acceleration a m s^{-2} of P is given by

$$\begin{aligned} a &= 6 + 4t \qquad 0 \leqslant t < 2, \\ a &= 14 \qquad\quad 2 \leqslant t < 4, \\ a &= 16 - 2t \qquad 4 \leqslant t \leqslant 16. \end{aligned}$$

There is no sudden change in velocity at any instant.

(a) Find the values of t when the velocity of P is 55 m s^{-1}. [5]

(b) Complete the sketch of the velocity-time diagram. [2]



(c) Find the distance travelled by P when it is decelerating. [3]