2 A particle P moves in a straight line. At time t s after leaving a point O on the line, P has velocity $v\,\text{m s}^{-1}$, where $v = 44t - 6t^2 - 36$.

(a) Find the set of values of t for which the acceleration of the particle is positive. [2]

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(b) Find the two values of t at which P returns to O. [3]

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3 A particle moves in a straight line starting from rest. The displacement s m of the particle from a fixed point O on the line at time t s is given by

$$s = t^{\frac{5}{2}} - \frac{15}{4}t^{\frac{3}{2}} + 6.$$

Find the value of s when the particle is again at rest. [4]

6 A particle moves in a straight line. It starts from rest from a fixed point O on the line. Its velocity at time t s after leaving O is v m s^{-1}, where $v = t^2 - 8t^{\frac{3}{2}} + 10t$.

(a) Find the displacement of the particle from O when $t = 1$. [4]

(b) Show that the minimum velocity of the particle is $-125\,\text{m}\,\text{s}^{-1}$. [7]

6 A particle, P, travels in a straight line, starting from a point O with velocity $6\,\text{m}\,\text{s}^{-1}$. The acceleration of P at time t s after leaving O is $a\,\text{m}\,\text{s}^{-2}$, where

$$a = -1.5t^{\frac{1}{2}} \qquad \text{for } 0 \leqslant t \leqslant 1,$$

$$a = 1.5t^{\frac{1}{2}} - 3t^{-\frac{1}{2}} \qquad \text{for } t > 1.$$

(a) Find the velocity of P at $t = 1$. [3]

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(b) Given that there is no change in the velocity of P when $t = 1$, find an expression for the velocity of P for $t > 1$. [3]

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(c) Given that the velocity of P is positive for $t \leqslant 4$, find the total distance travelled between $t = 0$ and $t = 4$. [4]

6 A particle P starts at rest and moves in a straight line from a point O. At time t s after leaving O, the velocity of P, $v\,\text{m s}^{-1}$, is given by $v = bt + ct^{\frac{3}{2}}$, where b and c are constants. P has velocity $8\,\text{m s}^{-1}$ when $t = 4$ and has velocity $13.5\,\text{m s}^{-1}$ when $t = 9$.

(a) Show that $b = 3$ and $c = -0.5$. [1]

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(b) Find the acceleration of P when $t = 1$. [2]

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(c) Find the positive value of t when P is at instantaneous rest and find the distance of P from O at this instant. [5]

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(d) Find the speed of P at the instant it returns to O. [3]

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4 A particle P moves in a straight line. It starts from rest at a point O on the line and at time t s after leaving O it has acceleration a m s^{-2}, where $a = 6t - 18$.

Find the distance P moves before it comes to instantaneous rest. [6]

6 A particle moves in a straight line. It starts from rest, at time $t = 0$, and accelerates at $0.6t\,\text{m s}^{-2}$ for 4 s, reaching a speed of $V\,\text{m s}^{-1}$. The particle then travels at $V\,\text{m s}^{-1}$ for 11 s, and finally slows down, with constant deceleration, stopping after a further 5 s.

(a) Show that $V = 4.8$. [1]

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(b) Sketch a velocity-time graph for the motion. [3]

(c) Find an expression, in terms of t, for the velocity of the particle for $15 \leqslant t \leqslant 20$. [2]

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(d) Find the total distance travelled by the particle. [4]

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5 A particle P moves in a straight line, starting from rest at a point O on the line. At time t s after leaving O the acceleration of P is $k(16 - t^2)$ m s^{-2}, where k is a positive constant, and the displacement from O is s m. The velocity of P is 8 m s^{-1} when $t = 4$.

(a) Show that $s = \frac{1}{64}t^2(96 - t^2)$. [5]

(b) Find the speed of P at the instant that it returns to O. [3]

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(c) Find the maximum displacement of the particle from O. [3]

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7 A particle P moves in a straight line, starting from a point O with velocity $1.72\,\text{m s}^{-1}$. The acceleration $a\,\text{m s}^{-2}$ of the particle, t s after leaving O, is given by $a = 0.1t^{\frac{3}{2}}$.

(a) Find the value of t when the velocity of P is $3\,\text{m s}^{-1}$. [4]

(b) Find the displacement of P from O when $t = 2$, giving your answer correct to 2 decimal places. [3]

8 A particle P moves in a straight line, passing through a point O with velocity $4.2\,\text{m}\,\text{s}^{-1}$. At time t s after P passes O, the acceleration, $a\,\text{m}\,\text{s}^{-2}$, of P is given by $a = 0.6t - 2.7$.

Find the distance P travels between the times at which it is at instantaneous rest. [7]

5 A particle starts from rest from a point O and moves in a straight line. The acceleration of the particle at time t s after leaving O is $a\,\text{m s}^{-2}$, where $a = kt^{\frac{1}{2}}$ for $0 \leqslant t \leqslant 9$ and where k is a constant. The velocity of the particle at $t = 9$ is $1.8\,\text{m s}^{-1}$.

(a) Show that $k = 0.1$. [3]

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For $t > 9$, the velocity $v\,\text{m s}^{-1}$ of the particle is given by $v = 0.2(t - 9)^2 + 1.8$.

(b) Show that the distance travelled in the first 9 seconds is one tenth of the distance travelled between $t = 9$ and $t = 18$. [4]

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(c) Find the greatest acceleration of the particle during the first 10 seconds of its motion. [3]

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6 A particle moves in a straight line, starting from a point O. The velocity of the particle at time t s after leaving O is $v\,\text{m s}^{-1}$. It is given that $v = kt^{\frac{1}{2}} - 2t - 8$, where k is a positive constant. The maximum velocity of the particle is $4.5\,\text{m s}^{-1}$.

(a) Show that $k = 10$. [5]

(b) (i) Verify that $v = 0$ when $t = 1$ and $t = 16$. [1]

(ii) Find the distance travelled by the particle in the first 16 s. [5]

6 A particle moves in a straight line and passes through the point A at time $t = 0$. The velocity of the particle at time t s after leaving A is v m s^{-1}, where

$$v = 2t^2 - 5t + 3.$$

(a) Find the times at which the particle is instantaneously at rest. Hence or otherwise find the minimum velocity of the particle. [4]

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(b) Sketch the velocity-time graph for the first 3 seconds of motion. [3]

(c) Find the distance travelled between the two times when the particle is instantaneously at rest. [3]

5 Particles X and Y move in a straight line through points A and B. Particle X starts from rest at A and moves towards B. At the same instant, Y starts from rest at B.

At time t seconds after the particles start moving

- the acceleration of X in the direction AB is given by $(12t + 12)\,\text{m s}^{-2}$,
- the acceleration of Y in the direction AB is given by $(24t - 8)\,\text{m s}^{-2}$.

(a) It is given that the velocities of X and Y are equal when they collide.

Calculate the distance AB. [6]

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(b) It is given instead that $AB = 36$ m.

Verify that X and Y collide after 3 s. [2]

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5 A particle P moves on the x-axis from the origin O with an initial velocity of $-20\,\text{m s}^{-1}$. The acceleration $a\,\text{m s}^{-2}$ at time t s after leaving O is given by $a = 12 - 2t$.

(a) Sketch a velocity-time graph for $0 \leqslant t \leqslant 12$, indicating the times when P is at rest. [5]

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(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 12$. [5]

7 A particle X travels in a straight line. The velocity of X at time t s after leaving a fixed point O is denoted by v m s^{-1}, where

$$v = -0.1t^3 + 1.8t^2 - 6t + 5.6.$$

The acceleration of X is zero at $t = p$ and $t = q$, where $p < q$.

(a) Find the value of p and the value of q. [4]

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It is given that the velocity of X is zero at $t = 14$.

(b) Find the velocities of X at $t = p$ and at $t = q$, and hence sketch the velocity-time graph for the motion of X for $0 \leqslant t \leqslant 15$. [3]

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(c) Find the total distance travelled by X between $t = 0$ and $t = 15$. [5]

7 A particle P moves in a straight line through a point O. The velocity $v\,\text{m s}^{-1}$ of P, at time $t\,\text{s}$ after passing O, is given by

$$v = \frac{9}{4} + \frac{b}{(t+1)^2} - ct^2,$$

where b and c are positive constants. At $t = 5$, the velocity of P is zero and its acceleration is $-\frac{13}{12}\,\text{m s}^{-2}$.

(a) Show that $b = 9$ and find the value of c. [5]

(b) Given that the velocity of P is zero only at $t = 5$, find the distance travelled in the first 10 seconds of motion. [5]

7 A particle P moves in a straight line. The velocity $v\,\text{m s}^{-1}$ at time t seconds is given by

$$v = 0.5t \qquad \text{for} \quad 0 \leqslant t \leqslant 10,$$
$$v = 0.25t^2 - 8t + 60 \qquad \text{for} \quad 10 \leqslant t \leqslant 20.$$

(a) Show that there is an instantaneous change in the acceleration of the particle at $t = 10$. [3]

(b) Find the total distance covered by P in the interval $0 \leqslant t \leqslant 20$. [6]

6 A particle P moves in a straight line. The velocity v m s^{-1} at time t s is given by

$$\begin{aligned} v &= 2t + 1 \qquad \text{for } 0 \leqslant t \leqslant 5, \\ v &= 36 - t^2 \qquad \text{for } 5 \leqslant t \leqslant 7, \\ v &= 2t - 27 \qquad \text{for } 7 \leqslant t \leqslant 13.5. \end{aligned}$$

(a) Sketch the velocity-time graph for $0 \leqslant t \leqslant 13.5$. [3]

(b) Find the acceleration at the instant when $t = 6$. [2]

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(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 13.5$. [5]