

- 5** The function f is defined by $f(x) = 2x^2 + 3$ for $x \geq 0$.

- (a) Find and simplify an expression for $\text{ff}(x)$. [2]

[illegible]

- (b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

[illegible]

5 The function f is defined for $x \in \mathbb{R}$ by

$$f : x \mapsto a - 2x,$$

where a is a constant.

(a) Express $ff(x)$ and $f^{-1}(x)$ in terms of a and x . [4]

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(b) Given that $ff(x) = f^{-1}(x)$, find x in terms of a . [2]

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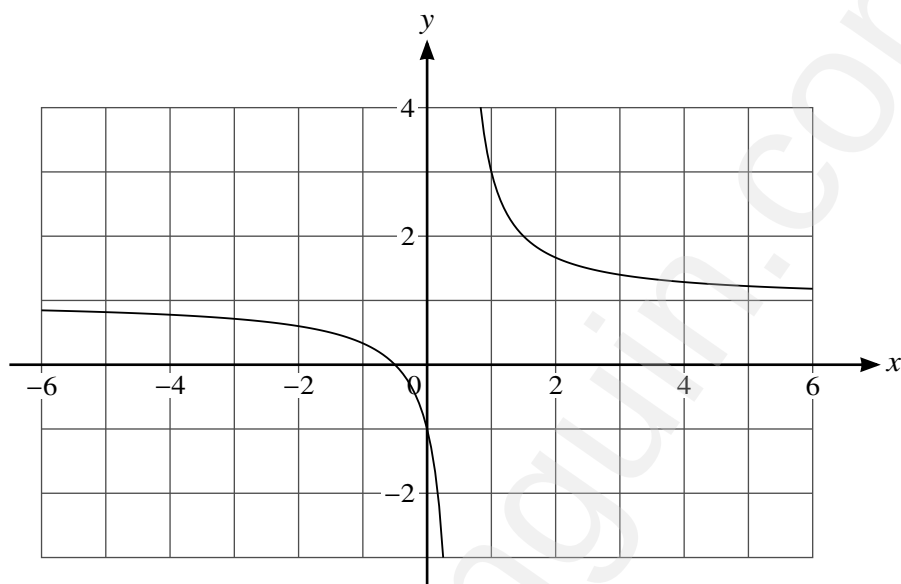
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10 Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \frac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1} .

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(b) Find an expression for $f^{-1}(x)$.

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(c) Find $gf^{-1}(3)$.

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(d) Explain why $g^{-1}(x)$ cannot be found.

[1]

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dotted lines spaced evenly down the page. A large, light gray watermark with the text "pastpaperpenguin.com" is oriented diagonally from the bottom left towards the top right, spanning across the entire page. The background is white, and there are no other markings or text present.

8 Functions f and g are defined by

$$f(x) = (x + a)^2 - a \text{ for } x \leq -a,$$

$$g(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $f^{-1}(x)$. [3]

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(b) (i) State the domain of the function f^{-1} . [1]

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(ii) State the range of the function f^{-1} . [1]

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[3]

[illegible]

2 The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$.

(a) Express $f(x)$ in the form $-2(x + a)^2 + b$, where a and b are integers. [2]

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(b) Find the range of f . [1]

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(c) Find an expression for $f^{-1}(x)$. [3]

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- 6 The function f is defined by $f(x) = \frac{2x}{3x-1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

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(b) Show that $\frac{2}{3} + \frac{2}{3(3x-1)}$ can be expressed as $\frac{2x}{3x-1}$. [2]

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(c) State the range of f . [1]

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(a) Solve the equation $fg(x) = 3$.

[4]

$$g : x \mapsto \frac{1}{2x+1} \text{ for } x < -\frac{1}{2}.$$

(a) Solve the equation $fg(x) = 3$.

[4]

(b) Find an expression for $(fg)^{-1}(x)$.

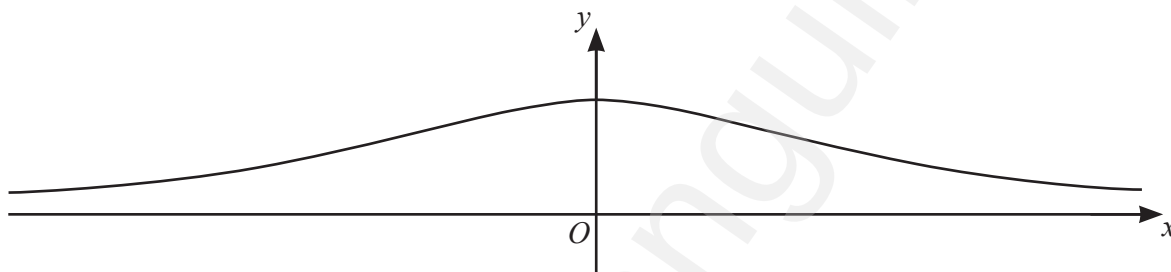
[illegible]

4 The function f is defined as follows:

$$f(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for $f^{-1}(x)$.

[1]



The diagram shows the graph of $y = g(x)$ where $g(x) = \frac{1}{x^2 + 2}$ for $x \in \mathbb{R}$.

(b) State the range of g and explain whether g^{-1} exists.

[2]

The function h is defined by $h(x) = \frac{1}{x^2 + 2}$ for $x \geq 0$.

- (c) Solve the equation $hf(x) = f\left(\frac{25}{16}\right)$. Give your answer in the form $a + b\sqrt{c}$, where a , b and c are integers. [4]

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dashed lines for writing. A large, light gray watermark with the text "pastpaperpenquin.com" is oriented diagonally across the page from the bottom left towards the top right. The background is white, and there are no other markings or text present.



5 The function f is defined by $f(x) = \frac{2x+1}{2x-1}$ for $x < \frac{1}{2}$.

(a) (i) State the value of $f(-1)$.

[1]

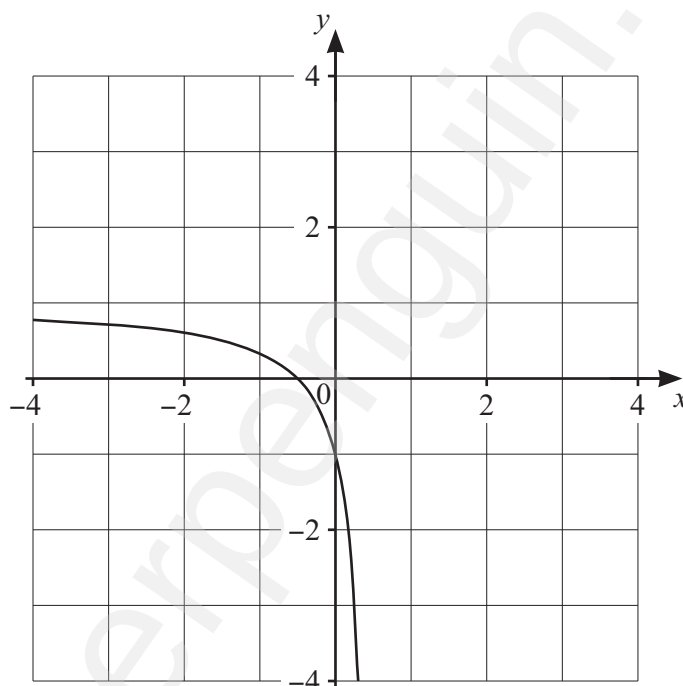
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(ii)



The diagram shows the graph of $y = f(x)$. Sketch the graph of $y = f^{-1}(x)$ on this diagram. Show any relevant mirror line. [2]

(iii) Find an expression for $f^{-1}(x)$ and state the domain of the function f^{-1} . [4]

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The function g is defined by $g(x) = 3x + 2$ for $x \in \mathbb{R}$.

- (b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$. [3]

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(a) Given that $f(7) = \frac{5}{7}$ and $gf(5) = 4$, find the values of a and b . [4]

[illegible]

[1]

Handwriting practice lines with a large watermark reading 'd.com' diagonally across the page.

[3]

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7 Functions f and g are defined as follows:

$$f : x \mapsto x^2 + 2x + 3 \text{ for } x \leq -1,$$

$$g : x \mapsto 2x + 1 \text{ for } x \geq -1.$$

- (a) Express $f(x)$ in the form $(x + a)^2 + b$ and state the range of f . [3]

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- (b) Find an expression for $f^{-1}(x)$. [2]

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- (c) Solve the equation $gf(x) = 13$. [3]

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- 7 The function f is defined by $f(x) = 2 - \frac{5}{x+2}$ for $x > -2$.

(a) State the range of f .

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

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The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax+b}{cx+d}$, where a , b , c and d are integers. [3]

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- 9 Functions f and g are defined as follows:

$$f(x) = (x - 2)^2 - 4 \text{ for } x \geq 2,$$

$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

- (a) State the range of f .

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- (b) Find $f^{-1}(x)$.

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- (c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$.

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[illegible]

- 9** The functions f and g are defined for all real values of x by

$$f(x) = (3x-2)^2 + k \quad \text{and} \quad g(x) = 5x-1,$$

where k is a constant.

- (a) Given that the range of the function gf is $gf(x) \geq 39$, find the value of k . [4]

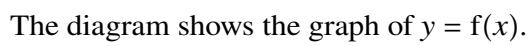
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- (b)** For this value of k , determine the range of the function f_g . [2]

Handwriting practice lines (dotted lines on a solid background).

- (c) The function h is defined for all real values of x and is such that $gh(x) = 35x + 19$.

Find an expression for $g^{-1}(x)$ and hence, or otherwise, find an expression for $h(x)$. [3]



- It is now given that $f(x) = -\frac{x}{\sqrt{4-x^2}}$ where $-2 < x < 2$.

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- This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. A faint, diagonal watermark reading "Pastpaperbe" is visible across the center of the page.

The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

- (c) State the maximum possible value of a for which fg can be formed. [1]

- (d) Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]